A Note on Incentive Functions in Government Procurement Contracts

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April 2, 2013
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Abstract

Government agencies use variants of the first-price sealed-bid auction mechanism to realize low cost, high quality, and on-time delivery when procuring goods and services. In a commonly used approach, agencies announce an incentive function that determines the amount they will pay based on the realized value of an attribute of the work performed, but bids do not explicitly include this attribute. The value of this attribute to the agency may be different in different situations. For example, in the procurement of transportation construction services, state transportation agencies make ex-post payments related to fuel or steel price escalation that have no value. They also make payments related to pavement density that have value because density is correlated with longer pavement life. This paper focuses on linear incentives and identifies situations in which an agency would benefit from incentives that favor high-cost high-quality contractors versus incentives that favor low-cost low-quality contractors. Although in this paper the procurement of transportation construction services is used as a motivating example, our results are applicable to a whole host of fixed price contracts in which extra payments are based either on changes in the price of a key input or the realized performance of the winning bidder on an attribute of value to the government.

Keywords: Incentive Functions, Cost and Quality Tradeoffs, Procurement, First-Price Sealed Bid Auctions, Transportation Construction Management

1 Introduction

In the 2010 fiscal year, the US Federal Government contracted approximately $539 billion for procurement of goods and services of which $293 billion were awarded via the full and open competition. Furthermore, more than half of all such contracts by value (approximately $167 billion) were awarded on a fixed price basis (www.usaspending.gov). The most common mechanism used for achieving full and open competition is the first-price sealed-bid mechanism (Federal Acquisition Regulation 2011). If within this mechanism contracts are awarded on fixed-price basis, then qualified bidders submit sealed bids that are opened on the day of contract letting and the contract is awarded to the lowest bidder. However, total payments are not equal to the contracted amount in many fixed price contracts issued by the government. In particular, two types of adjustments are common – economic price adjustments and incentives.

We are interested in this paper in instances where economic price adjustments are proposed by the government. In such cases, bids are evaluated on the basis of quoted prices without the allowable adjustments being added (Federal Acquisition Regulation 2011, Section 14.408-4). We are also interested in fixed-price contracts with incentives in which incentives are based on either product or delivery performance (Federal Acquisition Regulation 2011, Sections 16.402-2, 16.402-3). Similar to the case of economic price adjustments, bids are evaluated based on quoted prices without adding performance incentives in such instances as well. Among US Federal Government’s
fixed price contracts in 2010, 2.7% (in dollar value) involved some form of incentives and 9.6% (in dollar value) had economic price adjustments (www.usaspending.gov). Although the percent amounts appear small, the total dollar value of such contracts were, respectively, 4.5 and 16 billion dollars. That is, such contracts are economically significant.

A specific example that motivates the models presented in this paper concerns the use of fixed price contracts in the procurement of transportation-related construction services by state transportation agencies, which we refer to as agencies. Highway and street construction is an important economic activity. According to the US Census Bureau (2012), the December 2011 seasonally adjusted annual rate of state and local construction put in place for highway and street construction was approximately $83.43 billion. Agencies propose fuel and steel price escalation clauses as well as quality incentives with a linear payment structure. For example, the Minnesota Department of Transportation (Mn/DOT) covers a percent of the increase in fuel cost\(^1\) based on actual consumption, and offers up to 4\% higher payment per ton for higher density pavement\(^2\). Fuel price related adjustments are based on changes in the value of a particular fuel price index and the calculated amount of fuel used by the contractor during project execution. Pavement density related incentives are similarly calculated at the completion of each day’s work by taking core samples and testing them in materials’ labs.

In this paper, we refer to all payments in excess of the contracted fixed price as incentives and to the attribute that triggers extra payments as quality. Viewed from this perspective, economic price adjustments are triggered by a non-value attribute, whereas payments based on product quality or delivery performance are triggered by a value-adding attribute. We focus on agencies’ choice of incentive functions when incentive payments are not included in bid selection and bidders have private information about their cost (denoted by \(c\)) and quality (denoted by \(q\)). Parameters \(c\) and \(q\) may be arbitrarily correlated. The agency chooses an incentive function \(I(q)\) such that a winner’s profit equals \(b + I(q) - c\), if it wins the contract upon bidding \(b\) and \((c, q)\) are the realized values of its cost and quality. The agency’s cost, given winner’s parameters \((b, q)\), is \(b + I(q) - \lambda q\), where \(\lambda\) is the value that the agency derives from quality \(q\). Note that the agency does not observe \(c\), setting \(\lambda = 0\) obtains the economic price adjustment contract, and \(\lambda > 0\) obtains contracts with performance incentives. Cases in which \(\lambda\) could be negative are not considered in this paper as they are mathematically analogous to cases with \(\lambda > 0\).

The most commonly used incentive functions are linear functions (McAfee and McMillan 1987a), i.e. \(I(q) = r \cdot q\), where \(r\) is the incentive rate. The widespread use of linear incentives gives rise to a number of research questions that are addressed in this paper. Is the linear incentive function an optimal incentive function? If an agency were to use \(I(q) = r \cdot q\), what would be the optimal incentive rate \(r^*\)? What is the magnitude of additional cost of using a linear function, as opposed to an optimal function? Which factors affect the cost difference between the best linear function and an optimal function? If contractors could control quality via costly effort, how would that

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\(^1\)see, e.g. Minnesota Department of Transportation (2009) for the terms of Mn/DOT’s fuel escalation clause.

\(^2\)see, e.g. Table 2360.6-B4, Minnesota Department of Transportation (2005, p. 309), for Mn/DOT’s payment schedule for maximum pavement density.
affect optimal linear incentive rate relative to the situation in which quality cannot be controlled? These questions have not been considered in settings where incentive payments are not included in bid selection. Note that our models are not specific to a particular procurement environment, although they are motivated by the transportation construction environment.

What is novel in this paper is that it allows investigation of the effect of incentive functions in a commonly-encountered setting in which there are two attributes that matter to the agency – cost and quality. For example, if cost and quality are positively correlated and the agency does not offer any incentive for quality, then the expected quality of the winner will be less than the average quality among bidders. By offering some incentive, the agency improves expected quality of the winner. Although it pays for quality, its overall cost net of incentives is still lower. The agency cost under optimal linear incentives increases in the correlation between cost and quality. At the same time, the winner’s expected profit decreases in the correlation between cost and quality because the agency intensifies competition among contractors via incentives by making higher quality (and higher cost) contractors more competitive. Our model allows agencies to identify situations in which they should select the high cost - high quality contractor, or choose the low cost - low quality contractor by adjusting how much they pay for quality.

We present a summary of the broader literature and the relationship of this paper to that literature in Section 2. In what follows, we highlight the contribution of this paper relative to the most closely related work. The use of incentives in procurement auctions is equivalent to royalty payments in the sale by the government of oil and mineral rights to government-owned land (McAfee and McMillan 1987a). These payments are based on the value of oil extracted, denoted by some variable $\tilde{v}$, which also determines the winner’s valuation $v$. If the distribution of $\tilde{v}$ is exogenous (i.e. independent of the royalty rate set by the seller), then McAfee and McMillan (1987b) show that the seller’s expected revenue is an increasing function of the royalty rate. That is, in the limit, royalties should be close to 100%. McAfee and McMillan (1987a) further argue that this does not happen in practice because of moral hazard, i.e. because the winner can change the amount of oil extracted by choosing its effort. Unit extraction costs increase as more oil is extracted, which when coupled with high royalty payments cause the winners to extract only the cheaply-accessible oil.

In papers dealing with royalty payments, each bidder is characterized by a single parameter – its valuation (which is analogous to bidder’s cost in our model). In contrast, we present a more general framework in which contractors’ cost and quality are separate parameters. The agency in our setting does not observe the true cost of any bidder. The value of quality ($\lambda$) could be either zero or a positive number. We do not a priori assume a linear incentive function and obtain an upper bound on the incremental cost from using a linear incentive function. We fully characterize the agency’s optimal incentive function when cost and quality are either independent or perfectly positively correlated. For arbitrarily correlated cost and quality, we perform numerical experiments to illustrate how incentive rate varies with the degree of correlation between cost and quality.

McAfee and McMillan (1987b)’s model is analogous to a case in which the agency’s extra payments are based on observed cost. The payments serve to intensify competition among contractors
by reducing their base-cost differences. In terms of findings, similar to McAfee and McMillan (1987a), we also find that an optimal linear incentive rate is generally less than $\lambda$. However, this is not because of moral hazard. The optimal incentive rate is smaller than $\lambda$ because cost and quality are typically not perfectly positively correlated. In fact, the optimal incentive rate approaches $\lambda$ as cost and quality become more strongly correlated. Note that in the limit, one would have a single parameter and our model will converge to the special case discussed in McAfee and McMillan (1987a). Finally, we show that when the contractor can affect quality through effort, the optimal linear incentive rate and realized quality are higher.

The remainder of this paper is organized as follows. We present a summary of relevant literature in Section 2, notation and a formal model in Section 3, and analytical results in Sections 4 and 5. Section 6 contains numerical examples that help bring out additional insights. Finally, concluding remarks are presented in Section 7. All proofs are presented in an online Appendix.

2 Literature Review

Literature related to incentives in procurement auctions falls into two groups – economics and operations management. Surveys on procurement auctions, bids, and incentives are available in the economics literature in Engelbrecht-Wiggans et al. (1983), Krishna (2002), McAfee and McMillan (1987a), and Laffont and Tirole (1993). We focus in this paper on the sealed-bid first-price auction mechanism because that is the mechanism of choice for letting transportation construction projects and in government procurement contracts more generally.

Unlike our setting where contractor selection is based entirely on bid price, agencies may include quality as a criterion in contractor selection. This can be accomplished via either a scoring rule, or a minimum quality threshold, or both. In the former approach, contractors’ bids include cost and quality parameters. A scoring function is used to determine the overall best bid and the quality level included in the winner’s bid is binding on the winner. If realized quality falls short of the amount bid, then the contractor may be asked to either perform additional work to improve quality to the contracted level, or pay penalties. In some cases, agencies may also pay incentives for exceeding the contracted level of quality.

Naegelen (2002) observes that if quality is exogenous and common knowledge, a discriminatory scoring rule, where the rule depends on the quality, is optimal. Che (1993) analyzes auctions with multi-attribute scoring rules and finds that if the quality bid is binding on the winning contractor, the agency’s optimal scoring rule will score quality less than the agency’s marginal utility. Several recent papers (see, e.g. Gupta et al. 2013, Lewis and Bajari 2011) apply similar approach to time incentives offered by agencies. A key difference in our paper is that quality is not a component of bids. Furthermore, in Che (1993) and Lewis and Bajari (2011), the bidder’s private information is scalar – it consists of the bidder’s cost of quality. This approach does not allow investigation of the effect of the dependence between cost and quality, which is the main focus of our paper. One of our results, however, is similar to that obtained in Che (1993). In particular, we too find that the linear incentive rate, if cost and quality are either independent or linearly dependent, should not
exceed the agency’s benefit from each unit of extra quality.

This paper is related to works in which bids consist of unit prices and final payments are based on realized quantities; see, for example, Ewerhart and Fieseler (2003), and Maskin (1992). The first two papers show that these adjustments induce strategic bidders to place unbalanced bids. Maskin shows that multi-attribute signals result in allocations in which the low-cost bidder is not necessarily awarded the contract. Ewerhart and Fieseler (2003) argue that unit-price auctions can reduce buyer’s cost, since they subsidize high-cost contractors, requiring more aggressive bids by low-cost providers. The similarity between these works and our paper lies in the fact that payments depend on some realized attribute of the project. We too find that such payments can reduce buyer’s costs. However, in our setting, bids are not scored on the basis of the attribute of interest.

Among papers that focus on quality thresholds, Estache and Iimi (2009) examines the effect of a threshold quality requirement on the number of bidders and the subsequent level of competition in electricity infrastructure projects. It finds that if the quality threshold is flexible, agencies may loosen the quality requirement to increase competition. Asker and Cantillon (2008) shows that from the agency perspective, a price-only auction with a quality threshold is strictly dominated by an auction with a scoring rule.

Papers in the Operations Management literature generally assume direct negotiations between the supplier and the buyer and focus on either verifying and enforcing quality (see, e.g. Baiman et al. 2000, Balachandran and Radhakrishnan 2005, and Chao et al. 2009) or division of quality improvement benefits and costs through contract design (see, e.g. Reyniers and Tapiero 1995 and Lim 2001). Kostamis et al. (2009) model a situation in which the buyer incurs supplier-specific costs of doing business. The buyer knows these costs for each potential supplier, but not the suppliers’ production costs. Each supplier knows its own production cost and cost adjustment but not those of other suppliers. Suppliers incorporate their beliefs about other bidders’ costs into their equilibrium bids. The authors propose two mechanisms: first-price sealed-bid auction and descending open-bid auction. The similarity with our setting is that contractors in our model also do not know incentives that competitors might receive and must incorporate such adjustments into their bids. The difference is that the agency does not know supplier-specific incentives and awards contracts on the basis of bid prices rather than total payments net of any quality benefits.

In other related work, Chen et al. (2008) consider the use of audits to mitigate the impact of information asymmetry between the buyer and the sellers. They show that the buyer can optimize its profit and coordinate the channel by using a combination of auctions, audits, and profit sharing. The similarity with our setting is that in our model, the buyer is able to observe one attribute of the project (such as fuel consumption or pavement density), which affects its total cost. However, the problem settings are otherwise quite different.

Chen et al. (2010) analyze a variation of scoring auctions in which private information is two dimensional – cost and probability of success. The bids include a proposed non-completion penalty, which the buyer uses to score bids. The probability of success, which is non-contractible, affects
buyer’s total cost (just as quality does in our paper). The authors study the impact of different scoring rules on equilibrium bids, efficiency and social welfare. The differences between this approach and our setting are that (1) agencies do not utilize a scoring rule to select the winner, and (2) quality is a continuous attribute rather than a binary outcome with either success or failure in a project. As such, buyer’s cost varies continuously with the realization of quality.

Papers in the construction management literature are largely empirical. They make recommendations about the choice of contract mechanism based on experience. For example, Ellis et al. (2007) studies the use of alternative contracting techniques in Florida. It discusses results of previous projects and the arguments used to decide which contract mechanism should be used for which projects. Anderson and Damnjanovic (2008) surveys agency engineers to determine which contract mechanisms have the greatest potential to accelerate project completion. Fick et al. (2010) focuses on techniques for implementation and evaluation of incentives/disincentives provisions.

In summary, the novelty in this paper lies in modeling a commonly occurring situation that has not received attention in the literature. We provide both analytical and numerical analyses of the bidder’s and the agency’s problems.

3 Notation & Formulation

We consider the following sequence of events. The agency’s request for bids includes details of the work that must be completed as well as any incentive structure that will be used. Contractors estimate their costs and incentives and submit sealed bids consisting of offered prices. The agency awards the contract to the lowest bidder. The contractors and the agency are risk neutral.

Upon performing the work, the winner receives its bid amount and an incentive based on the realized value of the relevant attribute, which we call quality. Contractors’ estimates of their costs and quality are private. We use $C_b$ to denote cost, $Q$ to denote quality, and $\lambda$ to denote agency’s value from quality. We assume that $(C_b, Q)$ are revealed to each contractor just before bidding in the form of a signal, which is referred to as the contractor’s type. Essentially, type is a label that we attach to each realization of the two-dimensional vector of private values. A type is denoted by $t$ and the type space is denoted by $\mathcal{T}$. We emphasize that $\mathcal{T}$ may consist of a continuum of types. Our analysis does not assume a discrete number of types. A complete set of notation used to develop the models presented in this paper can be found in Table 1.

The agency’s goal is to select a real-valued bounded incentive function $I(q)$ that minimizes its expected cost, net of any quality benefits. The class of all such functions is denoted by $\mathcal{I}$. Note that the agency does not observe contractor type, which complicates the problem of choosing an appropriate incentive function. Because linear functions are often used in practice, and because they lead to tractable models, we focus in particular on linear incentive functions of the form $I(q) = r \cdot q$, where $r$ can be interpreted as the incentive rate. The agency’s problem in that case is to find the optimal incentive rate $r^*$. The key assumptions in this paper are as follows.

**Assumption 1.** Contractors’ types are private information. Before submitting a bid, each con-
tractor observes its own type and has knowledge of the type space \( \mathcal{T} \) from which other contractors draw their types. However, it does not observe the types of other contractors. The agency cannot observe any contractor’s type.

**Assumption 2.** Upon learning its type, a contractor knows its cost and quality with certainty.

**Assumption 3.** Given a particular request for bids and project specifications, contractor types are independent. This also means that each contractor’s cost and incentive attribute are independent of all other contractors’ costs and incentive attributes.

**Assumption 4.** Bidders are symmetric, i.e. each draws its type from the same probability distribution \( P(t), t \in \mathcal{T} \). The distribution \( P(t) \) is common knowledge among bidders and the agency.

**Assumption 5.** Contractors do not collude.

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**Table 1: Notation**

<table>
<thead>
<tr>
<th>Basic parameters</th>
<th>( n )</th>
<th>Number of contractors; ( i = 1, \ldots, n ) denotes ( i )-th contractor</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathcal{T} )</td>
<td>( \mathcal{T} )</td>
<td>Type space from which a contractor type is drawn, ( t ) denotes an arbitrary type</td>
</tr>
<tr>
<td>( T )</td>
<td>( T )</td>
<td>Random type of an arbitrary contractor</td>
</tr>
<tr>
<td>( P(t) )</td>
<td>( P(t) )</td>
<td>Probability distribution of contractor types, ( t \in \mathcal{T} )</td>
</tr>
<tr>
<td>( C_b(t) )</td>
<td>( C_b(t) )</td>
<td>Random base cost of an arbitrary contractor, ( C_b(t) \in [c_{lb}, c_{ub}] )</td>
</tr>
<tr>
<td>( Q(t) )</td>
<td>( Q(t) )</td>
<td>Random quality of an arbitrary contractor, ( Q(t) \in [q_{lb}, q_{ub}] )</td>
</tr>
<tr>
<td>( I )</td>
<td>( I )</td>
<td>The set of incentive functions</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>( \lambda )</td>
<td>The agency’s unit value from quality</td>
</tr>
</tbody>
</table>

| Type-\( t \) contractor’s parameters | \( c_b(t) \) | Realized base cost |
|---------------------------------------|\( c_b(t) \)|Realized base cost|
|                                       | \( c_m(t) \) | Realized modified cost, \( c_m(t) = c_b(t) - I(q(t)), \forall t \) |
|                                       | \( q(t) \) | Realized quality |

| Agency’s decisions                   | \( I(q) \) | Selected incentive function, \( I \in I \) |
|--------------------------------------|\( r \)|Linear incentive rate when \( I(q) = r \cdot q \) |

| Calculated parameters               | \( T^{(i)}_b \) | The type of the \( i \)-th lowest base cost contractor |
|-------------------------------------|\( T^{(i)}_m \)|The type of the \( i \)-th lowest modified cost contractor|
|                                     | \( T^{(i)} \) | The type of the \( i \)-th lowest net cost contractor, where net cost = \( c_b(t) - \lambda q(t) \) for each \( t \) |
|                                     | \( C_b^{(i)} \) | \( C_b(T^{(i)}_b) \) = The \( i \)-th order statistic of \( n \) base costs |
|                                     | \( C_m^{(i)} \) | \( C_m(T^{(i)}_m) \) = The \( i \)-th order statistic of \( n \) modified costs |
|                                     | \( Q^{(i)} \) | \( Q(T^{(i)}_m) \) = The quality of the \( i \)-th lowest modified cost contractor |
|                                     | \( \pi_C \) | The expected contract cost to the agency |
|                                     | \( \pi_N \) | The expected incentive cost to the agency |
|                                     | \( \pi_Q \) | The expected quality benefit to the agency |
|                                     | \( \pi(\cdot) \) | The expected net project cost as a function of chosen incentive function |

These assumptions can be justified as follows. Contractors do not reveal more information than they are obligated to include in their bids. Therefore, it is reasonable to assume that their cost and
incentive attribute remain private information. The agency observes the winning’s contractor’s quality only upon completion of the project. Contractors’ cost and quality remain unknown to the agency at the time when bids are evaluated. Assumption 2 is made primarily for expositional clarity. Under certain conditions, which are discussed in Section 7, it is possible to model some types of uncertainty about cost and quality at the time of bidding without affecting the qualitative results of this paper. Assumption 3 of independent costs and quality across contractors follows from the fact that any correlation in project costs and quality of contractors is largely due to the fact that they are considering the same project. This is already accounted for in the definition of \( T \). Assumption 4 is made to keep notation simple and the model tractable. Assumption 5 draws upon recent findings in the literature, which show that there is insufficient evidence of collusion and bid rigging in transportation construction industry (Bajari and Ye 2003). It is also consistent with the vast majority of auction literature that ignores collusion.

3.1 The Contractor’s Problem

In this section, we consider a particular contractor’s problem, which is referred to as the contractor. The contractor observes its type \( t \) and calculates its modified cost as:

\[
c_m(t) = c_b(t) - I(q) \tag{1}
\]

Each contractors’ problem is to determine its bidding function \( \beta(t) \), which returns the equilibrium amount that a contractor ought to bid if its type were \( t \). This can be accomplished with the help of standard arguments from auction theory. We present an intuitive explanation below.

If a type-\( t \) contractor bids \( \beta(t) \) and wins, its profit would be \((\beta(t) - c_m(t))\), and it would win with probability \( P(T_m^{(1)} = t) \). Clearly, the contractor’s expected profit increases in \( \beta(t) \) so long as the probability that it is the lowest modified cost contractor remains unchanged. This suggests that \( \beta(t) \) can be set as high as the conditional expected value of \( C_m^{(2)} \) given that type \( t \) is the type of the lowest modified cost contractor. That is,

\[
\beta(t) = E[C_m^{(2)} | T_m^{(1)} = t]. \tag{2}
\]

A formal proof that the above bidding strategy is optimal can be found in several publications, including for example, Krishna (2002).

3.2 The Agency’s Problem

The agency’s problem is to minimize overall cost, which consists of contract cost plus the cost of incentives minus the benefit from quality. Using (2) and Assumptions 1-5, it is straightforward to calculate project cost as follows.

\[
\pi_C = E[\beta(T_m^{(1)})] = E[C_m^{(2)}] \\
= E[c_b(T_m^{(2)})] - E[I(q(T_m^{(2)}))]. \tag{3}
\]
The agency’s incentive cost $\pi_N$ is equal to the expected value of the incentive paid to the winning contractor. That is,

$$\pi_N = E[I(q(T_m^{(1)}))] .$$  \hfill (4)

The third component of agency’s cost is the expected benefit from the winner’s quality. That equals

$$\pi_Q = E[Q^{(1)}] = \lambda E[q(T_m^{(1)})] .$$  \hfill (5)

Upon combining the above terms, the agency’s objective function is as shown below.

$$\pi(I) = E[c_b(T_m^{(2)})] - E[I(q(T_m^{(2)})]) + E[I(q(T_m^{(1)}))] - \lambda E[Q^{(1)}]$$  \hfill (6)

The agency’s problem is to choose an incentive function $I(\cdot) \in I$ that minimizes $\pi(I)$. It should pay incentives only when $\pi(I)$ is smaller than $\pi(0)$, where the latter denotes the agency’s expected cost when no incentive is offered. Clearly,

$$\pi(0) = E[c_b(T_b^{(2)})] - \lambda E[q(T_b^{(1)})] .$$  \hfill (7)

If cost and quality are independent, then with zero incentives the expected quality of the winner is equal to the average quality among all bidders, i.e. $E[q(T_b^{(1)})] = E(Q)$. However, if cost and quality are positively correlated, then $E[q(T_b^{(1)})] \leq E(Q)$.

### 4 Optimal Incentive Functions

Given contractor types $\{T_1, \cdots, T_n\}$ in a particular auction, the agency’s lowest expected cost is $E[\min_T (c_b(T) - \lambda q(T))]$ because for each bid, the agency will pay the winner at least that contractor’s base cost and the agency’s benefit from quality is $\lambda q(T)$. In this section, we use $T^{(i)}$ to denote the $i$-th order statistic of $(c_b(t) - \lambda q(t))$ among $n$ bidders.

Suppose the agency were able to observe contractor type upon completion of a project. Then, it could choose a type-specific incentive function of the form $I(t) = r(t)q(t)$, where $r(t)$ could be interpreted as the incentive rate for a type-$t$ contractor. It is straightforward to show that with the ability to observe contractor types, an agency can realize the minimum project cost $E[\min_T (c_b(T) - \lambda q(T))]$ by choosing type-specific incentive rates $r(\cdot)$. We omit a formal proof of this argument because it is intuitive. In practice, a type-specific incentive rate is not implementable for two reasons: (1) agencies only observe realized quality $q$ and not the contractor type$^3$, and (2) specifying a different rate for each contractor type would be cumbersome. Therefore, it is important to consider other structures. A particularly attractive choice is $I(q) = r \cdot q$, i.e. the case when incentive rate is independent of type and therefore realized quality. Such functions are used widely.

Next, we prove two results. First, if $I^*(q)$ denotes an optimal incentive function in class $I$,

$^3$Quality provides partial information about type because type includes cost as well, and cost and quality may not be perfectly positively correlated.
then we provide bounds on the agency’s expected cost if it were to use $I^*$. These bounds also extend to the special case when $I(q) = r \cdot q$ and the agency picks an optimal $r^*$. We use $\pi(I^*)$ and $\pi(r^*)$ to denote agency’s expected cost under the two types of incentive functions. Second, if cost and quality are not perfectly correlated, we show that no incentive function can achieve minimum project cost.

**Lemma 1.** The expected cost of an agency upon choosing an optimal incentive function is bounded as follows:

$$E[c_b(T^{(1)})] - \lambda E[q(T^{(1)})] \leq \pi(I^*) \leq \pi(r^*) \leq E[c_b(T^{(2)})] - \lambda E[q(T^{(2)})].$$

Furthermore, the first inequality above is strict when cost and quality are not perfectly correlated.

Lemma 1 states that unless cost and quality are strongly correlated, the agency cannot achieve minimum cost. This occurs because observed quality does not provide complete information about contractor’s cost. It also implies that among all functions in class $\mathcal{I}$, the agency’s expected cost lies between the lowest and the second lowest expected cost. Hence, the agency’s excess cost (relative to the minimum) is smaller when no contractor has significant total cost advantage. In the remainder of this section, we consider two special cases: linear dependence, where quality is a linear function of cost, and independence, where cost and quality are independent of each other. Whereas linearly dependent and independent cost and quality represent the two bookend scenarios, cost and quality may be arbitrarily correlated in reality. We analyze such cases via numerical examples.

### 4.1 Linearly Dependent Cost and Quality

A linear relationship between cost and quality can be expressed as $q(t) = a + b c_b(t)$ for each fixed $t$, where $a$ and $b$ are constants. Lemma 2 in this section establishes that it would serve the agency well to restrict attention to linear incentive functions when cost and quality are linearly dependent. To the extent that cost and quality are typically assumed to be strongly positively correlated (in fact, higher cost is often a signal for higher quality), the analysis of linearly dependent cost and quality serves as an important benchmark. We consider only those cases in which $b \geq 0$ because otherwise the optimal incentive rate would be negative. Negative quality incentives are not practical and therefore in such cases no incentives would be offered. On an intuitive level, the reason why a linear function suffices is that upon observing quality, the agency also observes cost. Therefore, it is able to choose the desired combination of cost and quality via a single parameter $r$.

**Lemma 2.** Suppose $q(t) = a + b c_b(t)$, $b \geq 0$, and $g(q)$ is an arbitrary non-decreasing incentive function. Then, $\pi(g) \geq \pi(r^*)$.

Note that $t$ can be inferred from $q$ in this case and therefore if $g^*$ is an optimal type-specific incentive function mentioned at the beginning of this section, then $\pi(g^*) = \pi(r^*)$. Our main result of this section obtains a closed-form expression for the optimal incentive rate $r^*$, which we present in Theorem 1.
Theorem 1. If \( q(t) = a + b c_b(t) \) for all types \( t \in T \), then \( r^* \) is obtained as follows:

- If \( b = 0 \), then the choice of \( r^* \) is arbitrary because incentive does not affect project cost.
- If \( 0 < b < \frac{1}{\lambda} \), then \( r^* \to \frac{1}{b} \) \(^- \), where superscript "-'" indicates approach from the left.
- If \( b = \frac{1}{\lambda} \), then the agency should offer an incentive of \( \frac{1}{b} \).
- If \( b > \frac{1}{\lambda} \), then \( r^* \to \frac{1}{b} \) \(^+ \), where superscript "+'" indicates approach from the right.

Theorem 1 can be explained on an intuitive level as follows. When \( b = 0 \), all contractors have the same quality, therefore quality-based incentives are irrelevant. In all other cases, the agency generates maximum competition among contractors by setting \( r^* = \frac{1}{b} \). This comes from the fact that when \( q(t) = a + b c_b(t) \), the modified cost \( c_m(t) = (1 - rb)c_b(t) - ra \). When \( b \) is between 0 and \( \frac{1}{\lambda} \), the contractor with the lowest base cost also has the lowest quality, but the difference in cost between contractors is higher than the difference in their quality (i.e. the low-cost contractor’s cost is sufficiently low to make up for the low quality), so the low-cost, low-quality contractor is selected. Conversely, when \( b > \frac{1}{\lambda} \), the difference between qualities is larger and the high-cost, high-quality contractor has large enough additional quality to make the higher cost worth it to the agency, and the optimal incentive therefore reverses bid ordering.

Next, we discuss a particular example to illustrate the application of Theorem 1. Suppose each contractor’s base cost is \( c_b(t) = a + b' q(t) \), where \( q(t) \) is the amount of fuel consumed and \( b' \) is the expected price of fuel including the anticipated increase. Then, we can reorganize the terms above to obtain \( a = -a'/b' \) and \( b = 1/b' \) to obtain the relationship between \( q(t) \) and \( c_b(t) \) discussed in Theorem 1. Agencies typically pay a fraction of \( (b' - b_0) \) where \( b_0 \) is the prevailing unit price at the time of contract letting. Theorem 1 implies that an optimal linear incentive rate would be \( \frac{1}{b} = b' \) and \( r^* \) would approach \( \frac{1}{b} \) from below (because \( \lambda = 0 \)).

In the example presented above, the optimal linear incentive rate is 100% of the unit fuel price. This does not happen in practice. We offer two reasons to explain the fact that \( r^* \) is typically less than the amount by which unit fuel or steel price increases. First, the agency does not know the relationship between the base cost and increase in cost due to fuel or steel consumed. In fact, the component denoted by \( a \) is typically different for different contractors. Second, if contractors knew that they would receive incentives that equal 100% of their cost of fuel or steel consumed, then that would lead to moral hazard by removing contractors’ incentive to use efficient production methods. The moral hazard issue that occurs in this instance is similar to the case discussed in McAfee and McMillan (1987a) and McAfee and McMillan (1987b) when royalty payments are charged on the basis of oil or minerals extracted from government land.

### 4.2 Independent Cost and Quality

In contrast to Theorem 1, which shows that the optimal linear incentive rate is either equal to or approaches \( \lambda \) with linearly dependent cost and quality, we find that if base cost and quality are
independent, the optimal incentive should not be as large as the value of quality to the agency, i.e. $r^* \leq \lambda$. This result, formalized in Theorem 2, is consistent with the result proved in Che (1993) for procurement auctions with a scoring rule.

**Theorem 2.** If the distributions of $c_b(T)$ and $q(T)$ are independent, continuous, and bounded, then $r^* \in [0, \lambda]$.

Furthermore, if for sufficiently small $\delta > 0$,

$$\delta [E[q(T_m^{(1)} (r + \delta))] - E[q(T_m^{(2)} (r))]] + (r - \lambda)E[q(T_m^{(1)} (r + \delta))] - q(T_m^{(1)} (r))]$$ (8)

is increasing in $r$, then $\pi(r)$ is convex in $r$ and there exists a unique $r^*$ in $[0, \lambda]$.

Theorem 2 can be explained as follows. Quality matters to the agency. Contractors pay attention to quality when choosing a bidding strategy only because of incentives. Agencies need to avoid overcompensating for quality. When cost and quality are independent, the winner’s quality is higher than the average quality among all contractors. If the agency were to place too much emphasis on quality, then that would lead to less aggressive bidding by the contractor that happens to have high quality, increasing the agency’s cost more than the quality benefit. That is, the agency needs to find a balance between emphasizing competition and emphasizing quality. This balance is achieved by choosing an incentive rate in $[0, \lambda]$.

Because paying for fuel or steel price increases does not add value to agency, Theorem 2 also shows that payments based on such factors do not lower agency’s expected cost if they are independent of contractors’ base costs. In reality, there is some correlation, though not perfect, between a contractor’s base cost and the amount of fuel or steel it uses in its production methods. We investigate partial correlation via numerical examples because the analysis of such cases is mathematically difficult. Before doing so, we discuss a different case in which contractors can affect quality by exerting costly effort.

## 5 Contractor Effort

Often contractors have the ability to affect quality by following best practices in production methods, instituting strict quality assurance guidelines, training workers in the correct production procedures and using high quality materials. In fact, one of the primary goals of quality incentives is to induce contractors to produce high quality work. A type-$t$ contractor’s quality has two components – an exogenous component $q(t)$ and a component $h_t(x)$ that it can affect via effort costing $x$. Each contractor faces a cost-benefit tradeoff. Producing higher quality work costs more, but it also gives rise to greater incentives at the end of the project.

In the contract setting we model, contractors neither reveal their target quality levels nor commit to particular levels. Therefore, contractors would choose a quality level that maximizes the difference between the quality incentive received and the cost to achieve that quality. Each contractor determines the additional payout it will receive if the contract is won, and uses that to
determine its bid. It is generally accepted in the construction management literature that initial gains in quality can be achieved by using cheaper approaches, but successive quality improvements become increasingly more expensive. Evidence of this phenomenon from construction management literature can be found in Shr and Chen (2004), Pyeon and Park (2010) and Sillars and Riedl (2007). Consistent with these observations, we assume that \( h_t \) is nondecreasing, concave and bounded. For tractability, we also assume that \( h_t \) is continuous and differentiable, and that \( h_t(0) = 0 \). The latter assumptions are made to simplify analysis. Each contractor decides \( x \) upon observing its type \( t \).

It is straightforward to see that for a given incentive rate \( r \), a type-\( t \) contractor will choose \( x \) to maximize \( rh_t(x_t(r)) - x \). This function is concave in \( x \). Therefore, an optimal \( x_t(r) \) is such that \( h_t'(x_t(r)) = 1/r \). Mathematically, this means that the distributions of cost and quality are now functions of \( r \) and \( t \), whereas up until now cost and quality depended only on \( t \). Specifically, a type-\( t \) contractor’s modified cost with a linear reward for quality becomes

\[
c_m(t, r) = c_b(t) - r(q(t) + h_t(x_t(r))) + x_t(r).
\]  

(9)

This development makes mathematical analysis more difficult. To keep the analysis tractable, we further assume that functions that relate effort cost and additional quality are independent of \( t \). This means that for a fixed \( r \), each contractor will choose to increase quality by the same amount \( x(r) \) because this decision is now independent of \( t \). The modified cost is then

\[
c_m(t, r) = c_b(t) - r(q(t) + h(x(r))) + x(r).
\]  

(10)

From (10), we note that the ordering of \( c_m(t, r) \) for each sequence \((t_1, \cdots, t_n)\) of realized contractor types is identical to the ordering of modified costs in situations in which contractors cannot choose effort (see Equation 1). To stress this point, we hereafter use \( T_{m,0}^{(i)} \) to denote the \( i \)th lowest modified cost contractor when the modified cost is as shown in (1). The agency’s expected cost with contractor effort, denoted \( \pi_e \), is

\[
\pi_e(r) = \mathbb{E}[C_b(T_{m,0}^{(2)})] - r\mathbb{E}[q(T_{m,0}^{(2)})] + (r - \lambda)\mathbb{E}[q(T_{m,0}^{(1)})] - \lambda h(x(r)) + x(r)
\]  

(11)

\[
\pi_e(r) = \pi(r) + x(r) - \lambda h(x(r))
\]  

(12)

In the above expression, \( q(\cdot) \) continues to mean the native quality of a contractor that depends only on its type. In particular, \( q(\cdot) \) does not include changes to quality brought about by contractor effort. Recall that \( \pi(r) \) is the agency’s expected profit when contractors cannot choose effort (see Equation 6). We now present the key result of this section.

**Theorem 3.** If all contractors have the same functional relationship between effort cost and quality, then the optimal incentive rate with contractor effort, \( r_e^* \), lies in \([0, \lambda] \). Furthermore, if the agency’s expected cost without contractor effort is convex in \( r \), then \( r_e^* \geq r^* \).

The practical implication of this theorem is that if contractors can affect quality via effort, agencies should offer a larger incentive than would be appropriate without contractor control on
quality. However, the incentive rate should still be less than the value of quality to the agency.

6 Examples

It is difficult to obtain analytical results when cost and quality are arbitrarily correlated. Therefore, we carried out numerical experiments and determined optimal incentive rate, contractor expected profit and agency's expected cost under different values of the correlation between cost and quality. We set $\lambda = 1$ and calculated the agency's overall minimum cost, $E[\min_T(c_b(T) - q(T))]$, as a benchmark. In this section, we describe the results of this study and its implications.

In our experiments, there were 5 bidders. For each contractor, we generated correlated values of bidders' cost and quality as follows. We began by sampling pairs $(x_1, x_2)$ from a bivariate normal distribution with correlation $\rho$. Each $X_i$ was normally distributed with parameters $(\mu_i, \sigma_i)$. Next, we calculated $\Phi(x_i)$, the marginal cumulative distribution function. Clearly, $\Phi(x_1)$ and $\Phi(x_2)$ were also correlated. Then, we used inverse transform to generate random samples $\hat{c}_b$ and $q$, where $\hat{c}_b$ and $q$ were uniform $[0, 1]$. It is possible to use any marginal distribution for this purpose. We used uniform distribution for convenience. Finally, we set $c_b = 10 + (1.5)(\hat{c}_b)$ to create different scaling of cost and quality parameters. Note that when $\hat{c}_b$ and $q$ are perfectly positively correlated, this model reduces to linearly dependent case with $a = -10/1.5$ and $b = 1/1.5$.

Our results are reported in the four panels of Figure 1. In each panel, one parameter is varied while keeping all other parameters fixed, and an outcome of interest is calculated by averaging the results over 5000 auctions. In Figure 1(a), we varied the correlation coefficient and calculated agency's expected cost as a function of different values of incentive rate. The agency's expected cost corresponding to the optimal incentive rate is shown in Figure 1(d). This figure also shows the overall minimum cost to the agency if it were to use a type-specific incentive rate.

Figures 1(a) and 1(d) show that as correlation becomes stronger up to a point, the agency's cost increases. This pattern also holds when the agency uses an optimal incentive rate. The overall minimum cost, which may be achieved by using a type-specific incentive rate also increases. However, for very high correlation, the cost associated with optimal single incentive rate starts to decline and eventually converges to the overall minimum cost. The latter occurs because when correlation is perfect, this example reduces to the linear dependent case with $b = 1/1.5$ and in that case linear incentive function is optimal. Because $b < 1$, it follows from Theorem 1 that the optimal incentive rate is $\frac{1}{b}^\gamma$. Figure 1(a) shows the discontinuity at $\frac{1}{b}$ as predicted by Theorem 1, and confirms that optimal value of the incentive rate approaches $\frac{1}{b}$ from below.

Figure 1(b) shows that up to a point, the optimal incentive rate increases with stronger correlation between cost and quality. This is because high quality contractors have higher cost and the agency reduces its total cost by offering to pay more for quality. In the limit as $\rho \to 1$, cost and quality become linearly dependent. In this case, the agency knows precisely how much to pay for quality to minimize its cost as shown in Theorem 1. Because quality incentive can be determined more accurately, the incentive rate declines.

Figures 1(c) and 1(d) show how winner's profits and agency's minimum cost depend on the
correlation between cost and quality. The agency’s minimum cost increases in $\rho$ up to a point because the agency pays more for higher quality. However, when cost and quality are perfectly correlated, quality is a deterministic function of cost and the agency can target incentives more accurately. Therefore, agency’s cost declines. In contrast, the winner’s expected profit is decreasing in $\rho$ over its entire range.

**Figure 1:** Effect of Correlation Between $c_b$ and $q$.

### 7 Concluding Remarks

In this paper, we consider an agency’s problem of choosing incentive functions to realize low cost and high quality. This problem is complicated by the fact that contractors bid strategically and some incentive structures may increase agency’s cost without increasing quality. We provide a rigorous mathematical framework to support the following specific conclusions. We show that agencies will be able to achieve the minimum project cost only when cost and quality are strongly correlated. In particular, we show that agencies cannot achieve minimum cost if cost and quality are not perfectly correlated irrespective of the selected incentive function. We also obtain bounds on the minimum achievable cost under general and linear incentive structures. If the attribute that triggers incentive payments has no value to the agency but the attribute and the base cost are positively correlated, then the benefit of offering incentives comes from that fact that incentives intensify competition.
If a value-adding attribute triggers incentive payments, then a linear incentive smaller than benefit per unit of quality lowers agency’s net cost.

One of the key assumptions underlying our model is that contractors can accurately estimate their cost and incentive attribute before bidding. Here, we discuss which types of uncertainty can be handled within our model without significant change. Suppose that each contractor’s private information consists of its expected cost and attribute value, but that the realized values of these parameters remain uncertain at the time of bidding. Furthermore, the uncertainty is contractor specific. This means that the realizations of cost and incentive attribute value of one contractor are not affected by the realized values of these parameters for other contractors. In such cases, risk-neutral contractors’ equilibrium bidding strategies will remain unchanged, except that bids will be based on expected values. This generalization is afforded by the key assumption of independent private values (see, e.g. Krishna 2002 for details). The vast majority of rehabilitation projects in transportation construction have been found to satisfy the independent private values assumption—see e.g. Hong and Shum (2002) and Krasnokutskaya (2011).

In a different setting, all contractors may use information available in the public domain and the same forecast mechanism to estimate uncertain components of cost, e.g. future fuel prices, weather conditions, and acts of war. In this case, there may be common uncertainty but the independent private values structure is still applicable. Such instances can be handled with the help of our model as well. However, instances in which contractor’s believe their estimates to be inaccurate and correlated with other contractors’ estimates are significantly more challenging to model. Such models are the focus of ongoing and future research by the authors.

Acknowledgement: This material is based upon work supported in part by the National Science Foundation under Grant No. CMMI-0653451.

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Online Appendix

A Proof of Lemma 1

Because the function \( I(q) = r \cdot q \) is a particular instance of incentive functions in \( \mathcal{I} \), it follow immediately that \( \pi(I^*) \leq \pi(r^*) \). The fact that \( E[c_b(T^{(1)})] - \lambda E[q(T^{(1)})] \leq \pi(I^*) \) can be shown as follows. Using the fact that \( c_m(T^{(1)}_m) \leq c_m(T^{(2)}_m) \), we obtain

\[
c_b(T^{(1)}_m) - I(q(T^{(1)}_m)) \leq c_b(T^{(2)}_m) - I(q(T^{(2)}_m))
\]

\[
\Rightarrow c_b(T^{(1)}_m) \leq c_b(T^{(2)}_m) - I(q(T^{(2)}_m)) + I(q(T^{(1)}_m)).
\] (13)

But

\[
\pi(I^*) = E[c_b(T^{(2)}_m)] - E[I(q(T^{(2)}_m))] + E[I(q(T^{(1)}_m))] - \lambda E[q(T^{(1)}_m)]
\] (14)

Combining the two relationships above, we have

\[
\pi(I^*) \geq E[c_b(T^{(1)}_m) - \lambda q(T^{(1)}_m)]
\]

\[
\geq E[\min_T (c_b(T) - \lambda q(T))].
\] (15)

This completes the proof of the first three inequalities in the statement of the Lemma.

Next, we focus on the fourth inequality. Note that \( \pi(r^*) \leq \pi(r) \) for every \( r \geq 0 \) and that in particular, \( \pi(r^*) \leq \pi(\lambda) \). Furthermore, from Equation (6), we have \( \pi(\lambda) = E[c_m(T^{(2)}_m)] + \lambda E[q(T^{(1)}_m)] - \lambda E[q(T^{(1)}_m)] = E[c_b(T^{(2)}_m)] - \lambda E[q(T^{(2)}_m)] \). That is,

\[
\pi(r^*) \leq E[c_b(T^{(2)}_m)] - \lambda E[q(T^{(2)}_m)].
\]

From the definition of order statistics of modified costs, we have that \( c_m(T^{(i)}_m) \leq c_m(T^{(j)}_m) \), for every \( i < j \). In particular, when \( r = \lambda \), \( c_m(T^{(i)}_m) = c_b(T^{(i)}_m) - \lambda q(T^{(i)}_m) \leq c_m(T^{(j)}_m) = c_b(T^{(j)}_m) - \lambda q(T^{(j)}_m) \), for every \( i < j \), which means that \( c_b(T^{(i)}_m) - \lambda q(T^{(i)}_m) = c_b(T^{(j)}_m) - \lambda q(T^{(j)}_m) \) for each \( i \). Therefore, \( \pi(r^*) \leq E[c_b(T^{(2)}_m)] - \lambda E[q(T^{(2)}_m)] \).

Finally, consider the statement that \( E[c_b(T^{(1)}_m)] - \lambda E[q(T^{(1)}_m)] \leq \pi(I^*) \) unless cost and quality are perfectly correlated. Consider a particular auction with incentive function \( I^* \). There are two possibilities: either event \( E = \{q(T^{(1)}_m) = q(T^{(2)}_m), c_b(T^{(1)}_m) \neq c_b(T^{(2)}_m)\} \) or its complement, denoted by \( E^c \). When \( E^c \) occurs, base costs may or may not be equal, and the argument in expressions (13) – (15) remain intact. That is, \( E[\{c_b(T^{(2)}_m)\}] - E[I(q(T^{(2)}_m))] + E[I(q(T^{(1)}_m))] - \lambda E[q(T^{(1)}_m)] \mid E^c \geq E[\min_T (c_b(T) - \lambda q(T))] \mid E^c \).

Let \( P(E) \) denote the probability that event \( E \) occurs. If cost and quality are not perfectly
correlated, it should be clear that \( P(E) > 0 \). Then,

\[
\pi(I^*) = E[c_b(T_m^{(2)})] - E[I(q(T_m^{(2)}))] + E[I(q(T_m^{(1)}))] - \lambda E[q(T_m^{(1)})]
\]

\[
= E[c_b(T_m^{(2)})] - I(q(T_m^{(2)})) + I(q(T_m^{(1)})) - \lambda q(T_m^{(1)})[E^c](1 - P(E))
\]

\[
+ E[c_b(T_m^{(2)})] - \lambda q(T_m^{(1)})|E| \cdot P(E),
\]

Conditioning on event \( E \), we have

\[
E[c_b(T_m^{(2)})] - \lambda q(T_m^{(1)})|E| \geq E[c_b(T_m^{(1)})] - \lambda q(T_m^{(1)})|E| \geq E[\min(T_b(T) - \lambda q(T))|E].
\]

Combining the above arguments, if \( P(E) > 0 \), then we must have

\[
\pi(I^*) > E[\min(T_b(T) - \lambda q(T))].
\]

This completes the proof.

\section*{B Proof of Lemma 2}

From Equation (6), we obtain

\[
\pi(g) = E[c_m(T_m^{(2)})] + E[g(q(T_m^{(2)}))] - \lambda E[q(T_m^{(1)})]
\]

\[
= E[c_b(T_m^{(2)})] - E[g(q(T_m^{(2)}))] + E[g(q(T_m^{(1)}))] - \lambda E[q(T_m^{(1)})]
\]

(19)

Because \( c_m(T_m^{(1)}) \leq c_m(T_m^{(2)}) \) by definition, we obtain

\[
c_m(T_m^{(1)}) = c_b(T_m^{(1)}) - g(q(T_m^{(1)})) \leq c_m(T_m^{(2)}) = c_b(T_m^{(2)}) - g(q(T_m^{(2)}))
\]

\[
\Leftrightarrow c_b(T_m^{(1)}) - c_b(T_m^{(2)}) \leq g(q(T_m^{(1)})) - g(q(T_m^{(2)}))
\]

\[
\Leftrightarrow \frac{1}{b}[g(q(T_m^{(1)})) - g(q(T_m^{(2)}))] \leq g(q(T_m^{(1)})) - g(q(T_m^{(2)})).
\]

(20)

(21)

Substituting for \( g(q(T_m^{(1)})) - g(q(T_m^{(2)})) \) from (20) in (19), we obtain

\[
\pi(g) \geq E[c_b(T_m^{(2)})] + E[c_b(T_m^{(1)})] - E[c_b(T_m^{(2)})] - \lambda E[q(T_m^{(1)})]
\]

\[
= E[c_b(T_m^{(1)})] - \lambda E[q(T_m^{(1)})]
\]

\[
= (1 - \lambda b)E[c_b(T_m^{(1)})] - \lambda a
\]

(22)

If \( 0 < b < \frac{1}{\lambda} \), then a lower bound on \( \pi(g) \) is obtained when \( E[c_b(T_m^{(1)})] = E[c_b(T_b^{(1)})] \). Similarly, when \( b > \frac{1}{\lambda} \), a lower bound on \( \pi(g) \) is obtained when \( E[c_b(T_m^{(1)})] = E[c_b(T_b^{(n)})] \). This implies

\[
\pi(g) \geq \begin{cases} 
(1 - \lambda b)E[c_b(T_b^{(1)})] - \lambda a & \text{if } 0 < b < \frac{1}{\lambda}, \text{ and} \\
(1 - \lambda b)E[c_b(T_b^{(n)})] - \lambda a & \text{if } b > \frac{1}{\lambda}.
\end{cases}
\]

(23)
Upon substituting \( q(t) = a + b c_b(t) \) into Equation (6) and simplifying the resulting expression, we obtain

\[
\pi(r) = (1 - rb) \left( E[C_b(T_m^{(2)})] - E[C_b(T_m^{(1)})] \right) + (1 - \lambda b) E[C_b(T_m^{(1)})] - \lambda a,
\]

which is minimized by setting \( r = \frac{1}{b} \). However, the minimum value depends on whether \( b > \frac{1}{\lambda} \) or not. The minimum cost is as shown below (see Proof of Theorem 1 for complete details).

\[
\pi(r^*) = \begin{cases} 
(1 - \lambda b) E[c_b(T_b^{(1)})] - \lambda a & \text{if } 0 < b < \frac{1}{\lambda}, \text{ and} \\
(1 - \lambda b) E[c_b(T_b^{(n)})] - \lambda a & \text{if } b > \frac{1}{\lambda}.
\end{cases} 
\]

Intuitively, this comes from the fact that \( \pi(r) \) is minimized by setting \( E[C_b(T_m^{(1)})] = E[C_b(T_b^{(1)})] \) when \( 0 < b < \frac{1}{\lambda} \) and by \( E[C_b(T_m^{(1)})] = E[C_b(T_b^{(n)})] \) when \( b > \frac{1}{\lambda} \).

Therefore, \( \pi(g) \geq \pi(r^*) \) for all \( b > 0 \). When \( b = 0 \), all contractors have the same quality and incentive does not affect agency’s cost, which means that \( \pi(g) = \pi(r^*) \) in that case. Combining the observations above, we have proved that \( \pi(g) \geq \pi(r^*) \) for every value of \( b \geq 0 \).

C Proof of Theorem 1

Two cases arise — one in which bid order is preserved and the other in which it is reversed. We consider each case separately.

Case 1: Bid Order is Preserved

When bid order is preserved, i.e. \( (1 - rb) > 0 \), the agency’s cost function from (6) reduces to:

\[
\pi(r) = E[c_b^{(2)}] - \lambda (a + b E[c_b^{(1)}]) - rb(E[C_b^{(2)}] - E[C_b^{(1)}])
\]

The first two terms are independent of \( r \), and \( E[c_b^{(2)}] - E[c_b^{(1)}] \) is positive. Now the following additional cases arise:

1.(a) If \( b = 0 \), the last term equals 0, and the project cost is independent of \( r \). In this case, the choice of \( r^* \) is irrelevant. This explains the second bullet in the statement of the theorem.

1.(b) If \( b > 0 \), the last term is decreasing in \( r \). Therefore, \( r^* = \sup_{r \geq 0} (1 - rb) \), subject to \( r < 1/b \). Clearly, in this case \( r^* = \frac{1}{b} \).

Case 2: Order is Reversed

If \( (1 - rb) < 0 \), bid ordering is reversed. That is, the contractor with the highest base cost wins the contract because quality incentives are high enough to make its modified cost the lowest among all bidders. This can only happen if \( b > 0 \). Now, the project cost is

\[
\pi(r) = E[(1 - rb)C_b^{(n-1)} - ra] + (r - \lambda) E[(a + bC_b^{(n)})] \\
= E[C_b^{(n-1)}] - \lambda (a + b E[C_b^{(n)}]) - rb(E[C_b^{(n-1)}] - E[C_b^{(n)})]
\]

(27)
The first two terms are again independent of \( r \), but now \( (E[C_b^{(n-1)}] - E[C_b^{(n)}]) \) is negative, so the last term is increasing in \( r \). That is, the agency would want to choose the smallest possible value of \( r \), which implies \( r^* = \inf_{r \geq 0} (1 - rb) \), subject to \( r > 1/b \). Clearly, in this case \( r^* = \frac{1}{b} \).

In cases 1.(b) and 2, the optimal value of \( r \) approaches \( \frac{1}{b} \) and the project cost to the agency approaches

\[
\lim_{r \to \frac{1}{b}^-} \pi(r) = -\lambda a - (\lambda b - 1)E[C_b^{(1)}] \quad \text{if } (1 - rb) > 0, \quad (28)
\]

\[
\lim_{r \to \frac{1}{b}^+} \pi(r) = -\lambda a - (\lambda b - 1)E[C_b^{(n)}] \quad \text{if } (1 - rb) < 0. \quad (29)
\]

The limiting values of agency cost are equal when \( b = \frac{1}{\lambda} \), the limit in (28) is smaller if \( b < \frac{1}{\lambda} \), and the limit in (29) is smaller if \( b > \frac{1}{\lambda} \). The arguments above explain the last three bullets in the proof of the theorem.

D Proof of Theorem 2

The ensuing proof requires two intermediate results. First, we argue that \( E[q(T^{(1)}_m)] \) is nondecreasing in \( r \). Second, we prove that \( E[C_m^{(2)}(r + \delta)] - E[C_m^{(2)}(r)] = -\delta E[q(T^{(2)}_m(r))] + o(\delta) \), as \( \delta \downarrow 0 \). Then we use these two facts to show that for every \( r > \lambda \) and small \( \delta > 0 \), \( \pi(r + \delta) - \pi(r) \geq 0 \), and that \( \pi(\delta) - \pi(0) \leq 0 \).

For the first required result, fix \( r \) and for a set of sampled contractor types, determine which contractor would win the contract. As \( r \) increases, the winner will remain unchanged at first until a different contractor with higher cost and higher quality will become cheaper in modified cost due to quality incentives. Note that with linear incentives, neither a contractor with higher cost but lower quality, nor a contractor with lower cost but higher quality could beat the initial winner. Put differently, the quality of the new winner must be at least as high or higher for each fixed set of contractor types. This means that the expected value of quality of the winner also must be non-decreasing in \( r \).

For the second result, we need to define some more notation. Let

\[
s = \{(c_b(t_1), q(t_1)), \ldots, (c_b(t_n), q(t_n))\}
\]

denote a sample path of cost and quality pairs that emerge in a random draw of contractor types. We focus attention on a case in which incentive rate lies in the range \([r, r + \delta]\), where \( \delta > 0 \) is a real number. Let \( S \) denote the set of all possible sample paths \( s \). We divide the set \( S \) into two non-overlapping subsets. The subset \( S_1(r, \delta) \) contains all those sample paths in which the index of the second lowest cost contractor does not change for any incentive rate in \([r, r + \delta]\), whereas \( S_2(r, \delta) \) contains those sample paths in which the index of the second lowest cost contractor is different. Let \( c_m^{(2)}(s, r) \) and \( q^{(2)}(s, r) \) denote, respectively, the second lowest modified cost and quality of the contractor that has the second lowest modified cost in sample \( s \) if incentive rate \( r \) is offered. From the definition of set \( S_1 \) and \( S_2 \), we have \( q^{(2)}(s, r) = q^{(2)}(s, r + \delta) \) and \( c_m^{(2)}(s, r + \delta) - c_m^{(2)}(s, r) = -\delta q^{(2)}(s, r) \).
if \( s \in S_1(r, \delta) \). Clearly, \( E[c^{(2)}_m(S, r)] = E[C^{(2)}_m(r)] \) and \( E[q^{(2)}(S, r)] = E[q(T^{(2)}_m(r))] \) because these are two different ways of specifying the same quantity. Next,

\[
E[c^{(2)}_m(S, r + \delta) - c^{(2)}_m(S, r)] = -\delta E[q^{(2)}(s, r) | s \in S_1(r, \delta)] \cdot P(s \in S_1(r, \delta))
\]

\[+ \quad P(s \in S_2(r, \delta)) \cdot E[c^{(2)}_m(s, r + \delta) - c^{(2)}_m(s, r) | s \in S_2(r, \delta)]
\]

\[= -\delta E[q^{(2)}(S, r)] + P(s \in S_2(r, \delta)) \cdot [E[c^{(2)}_m(s, r + \delta) - c^{(2)}_m(s, r) | s \in S_2(r, \delta)]
\]

\[+ \quad \delta E[q^{(2)}(s, r) | s \in S_2(r, \delta)]].
\]

For each sample path, the absolute difference \( |c^{(2)}_m(s, r + \delta) - c^{(2)}_m(s, r)| \) is bounded by the increase in incentive rate times the maximum value of quality among all contractors in that sample. That is \( |c^{(2)}_m(s, r + \delta) - c^{(2)}_m(s, r)| \leq \delta \bar{q}(s) \), where \( \bar{q}(s) \) is the highest quality of any contractor in sample path \( s \). So the second term of the last equation above will be bounded by:

\[
P(s \in S_2(r, \delta)) \cdot [E[c^{(2)}_m(s, r + \delta) - c^{(2)}_m(s, r) | s \in S_2(r, \delta)] + \delta E[q^{(2)}(s, r) | s \in S_2(r, \delta)]]
\]

\[\leq P(s \in S_2(r, \delta)) \cdot 2\delta E[q(S)]
\]

Next, observe that the \( P(s \in S_2(r, \delta)) \) approaches zero as \( \delta \downarrow 0 \). This happens because as \( \delta \downarrow 0 \), the relative number of sample paths in which the index of the second lowest modified cost contractor changes goes to zero. Therefore, the term (31) divided by \( \delta \) goes to zero as \( \delta \downarrow 0 \), i.e. it is \( o(\delta) \). In other words,

\[
E[C^{(2)}_m(T^{(2)}_m + \delta)] - E[C^{(2)}_m(T^{(2)}_m)] = -\delta E[q(T^{(2)}_m(T^{(2)}_m))] + o(\delta), \quad \text{as} \quad \delta \downarrow 0.
\]

We are now ready to prove the statement of Theorem 2. We have that

\[
\pi(r + \delta) - \pi(r)
\]

\[= E[C^{(2)}_m(T^{(2)}_m + \delta)] - E[C^{(2)}_m(T^{(2)}_m)] + (r + \delta - \lambda)E[q(T^{(1)}_m(T^{(2)}_m))] - (r - \lambda)E[q(T^{(1)}_m(T^{(2)}_m))] \]

\[= E[C^{(2)}_m(T^{(2)}_m + \delta)] - \delta E[q(T^{(1)}_m(T^{(2)}_m))] + (r + \delta - \lambda)E[q(T^{(1)}_m(T^{(2)}_m)) - q(T^{(1)}_m(T^{(2)}_m))] \]

\[= \delta [E[q(T^{(1)}_m(T^{(2)}_m))] - E[q(T^{(2)}_m(T^{(2)}_m)))] + (r - \lambda)E[q(T^{(1)}_m(T^{(2)}_m)) - q(T^{(1)}_m(T^{(2)}_m))] + o(\delta)
\]

The last equation above utilizes the relationship established in (32). Because \( C_b \) and \( Q \) are independent, \( E[q(T^{(1)}_m(T^{(2)}_m))] \geq E[q(T^{(2)}_m(T^{(2)}_m))] \). That is, the expected quality of the lowest modified cost bidder is at least as high as the expected quality of the second lowest modified cost bidder. In addition, as proved earlier, \( E[q(T^{(1)}_m(T^{(2)}_m))] \) is nondecreasing in \( r \). This means that the first term \( E[q(T^{(1)}_m(T^{(2)}_m))] - E[q(T^{(2)}_m(T^{(2)}_m))] \geq E[q(T^{(1)}_m(T^{(2)}_m))] - E[q(T^{(1)}_m(T^{(2)}_m))] \geq 0 \). The second term in (33) is also non-negative when \( r \geq \lambda \), which establishes that the agency’s expected cost is non-decreasing in \( r \) when \( r \geq \lambda \). This proves that an optimal value of \( r \) lies in \([0, \lambda] \).
E  Proof of Theorem 3

When proving Theorem 2, we worked with differences rather than derivatives because \( E[q(T_m^{(1)}(r))] \) may not be differentiable in \( r \). We take a different approach here, which results in a shorter proof. We show first that \( E[q(T_m^{(1)}(r))] \) can be approximated as closely as desired by a polynomial function. Thereafter, we take the derivative of this function with respect to \( r \) to prove the main result of Theorem 3.

Let \( S_1^{(1)}(r, \delta) \) denote all those sample paths in which the index of the lowest cost contractor does not change for any incentive rate in \([r, r+\delta]\) whereas \( S_2^{(1)}(r, \delta) \) denotes all those sample paths in which the index of the lowest cost contractor is different. Then,

\[
E[q(T_m^{(1)}(r + \delta))] - E[q(T_m^{(1)}(r))] = E[q(T_m^{(1)}(r + \delta)) - q(T_m^{(1)})|s \in S_1^{(1)}(r, \delta)] \cdot P(s \in S_2^{(1)}(r, \delta))
\]

Since \( P(s \in S_2^{(1)}(r, \delta)) \to 0 \) as \( \delta \to 0 \), \( E[q(T_m^{(1)}(r))] \) is continuous in \( r \). Moreover, we proved in Theorem 2 that \( E[q(T_m^{(1)}(r))] \) is increasing in \( r \). It is well known that an increasing and continuous function is differentiable almost everywhere, so \( E[q(T_m^{(1)}(r))] \) may be non-differentiable only on a countable set of points. Moreover, by Stone-Weierstrass Theorem, every continuous function defined on an interval \([a, b]\) can be uniformly approximated as closely as desired by a polynomial function. Because \( r \) is non-negative and finite, this means that \( E[q(T_m^{(1)}(r))] \) can be approximated by a polynomial function as closely as desired. Also this approximation will not change the monotonicity property of \( E[q(T_m^{(1)}(r))] \), e.g., upon using Bernstein polynomial functions. For these reasons, we assume hereafter that \( E[q(T_m^{(1)}(r))] \) and therefore \( \pi(r) \) is differentiable in \( r \).

For any \( y \), let \( h'(y) \) denote the derivative of \( h(x) \) at \( x = y \) and consider the case when \( h'(0) < \frac{1}{\lambda} \). Because \( h \) is concave, if \( h'(0) < \frac{1}{\lambda} \), then this means that \( h'(x) < \frac{1}{\lambda} \) for every \( x \geq 0 \). That is, for each incremental unit of effort cost, the contractor realizes less than a unit of quality, irrespective of the level of effort. This means that if \( 0 \leq r \leq \lambda \), then the contractor would choose not to improve quality. If \( r \) were sufficiently greater than \( \lambda \), then the contractor might choose a non-zero quality improvement effort. But that would not benefit the agency because the increase in benefit from quality will be less than the incentive paid to achieve that increase. This implies that \( r^*_e \in [0, \lambda] \).

If we restrict attention to the case when \( r^*_e \) is selected from \([0, \lambda]\), the option to increase quality via effort is not exercised by the contractors and the situation is identical to that in Theorem 2. This means \( r^*_e = r^* \) and confirms that \( r^*_e \geq r^* \) holds in this case.

Next consider the case when \( h'(0) \geq \frac{1}{\lambda} \). Now, there would exist some value of \( r \leq \lambda \) such that \( x(r) \geq 0 \) and \( \frac{d}{dr} h(x) = \frac{1}{r} \). Taking the derivative with respect to \( r \) in Equation (12) and using the fact that \( \frac{d}{dx} h(x) = \frac{1}{r} \), we obtain

\[
\frac{d}{dr}(\pi_e(r)) = \frac{d}{dr}(\pi(r)) + (1 - \frac{\lambda}{r}) \frac{d}{dr}(x(r))
\]

(35)

From Theorem 2, we know that \( \frac{d}{dr}(\pi(r)) \geq 0 \) for \( r \geq \lambda \). In the above expression, \((1 - \frac{\lambda}{r}) \) is non-
negative when \( r \geq \lambda \) and \( \frac{d}{d r}(x(r)) = (-1/r^2)(1/h''(x(r))) \geq 0 \) because \( h \) is concave. Therefore, agency’s expected cost increases in \( r \) for all values of \( r \geq \lambda \) and the agency would not choose that option. Recall from Theorem 2 that \( r^* \) is a solution to the equation \( \frac{d}{d r}(\pi(r)) = 0 \), which yields

\[
\frac{d}{d r}(\pi_e(r^*)) = (1 - \frac{\lambda}{r^*}) \frac{d}{d r}(x(r))|_{r=r^*} < 0. \tag{36}
\]

Finally, because \( \pi(r) \) is convex in \( r \) by assumption in Theorem 3, \( \pi_e \) is also convex and \( r_e^* \geq r^* \) must hold. The agency can afford to offer a higher incentive because incentives induce contractors to increase quality to a level more than what would occur when contractors cannot affect quality. Hence proved.