Contractors’ and Agency Decisions and Policy Implications in A+B Bidding

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Abstract

The focus of this paper is on the A+B transportation procurement mechanism that uses the proposed cost (A component) and the proposed time (B component) to score contractors’ bids. This mechanism has been adopted by several state transportation agencies, and empirical studies have shown that it shortens project duration. We use normative decision models to study the effect of certain discretionary parameters set by state transportation agencies on contractors’ equilibrium bidding strategies, winner selection, and actual completion times. These models are then used to inform policy questions. We model the bidding environment in detail including multidimensional bids, contractors’ uncertainty about completion times, and reputation cost. The latter refers to a private penalty that accrues to tardy contractors from increased cost of posting bonds and reduced prospects of winning future projects. Our model features explain why line-item bids may not reflect contractors’ true costs of associated activities and why winners frequently finish earlier than bid. Our analysis shows that in order to maximize social welfare, agencies should equate all time-based parameters and not cap incentives. This is a departure from current practice, where incentives are weaker than penalties and incentives are often capped.

Keywords: Auctions, Time-based incentives, A+B bidding, Procurement policy

1 Introduction

Transportation-related construction is an 84-billion dollar a year industry (US Census Bureau 2011). State transportation agencies, hereafter referred to as agencies, are required by state legislation to exclusively use the first-price sealed-bid mechanism and to award construction contracts to the lowest bidder (see, e.g. Minnesota Department of Transportation 2011). A+B bidding is one of several project-letting mechanisms used by state transportation agencies to provide incentives for faster project completion (see Innovative Contracting in Minnesota 2000–2005 for examples). To date A+B contracts have been quite successful: contractors have offered short completion times and frequently completed projects on time or earlier than bid (Herbsman 1995).

The A+B mechanism is a first-price sealed bid scoring auction that selects the winner as follows: each contractor’s multidimensional sealed bid consists of unit prices for line items included in the request for bids (RFB) and a time bid. When bids are opened, unit prices are multiplied by engineers’ estimated quantities to obtain line-item bids, whose sum is referred to as the A component. The B component is the product of the daily road user cost (RUC) and the time bid. The bidder with the lowest total score (sum of A and B components) is declared the winner. Agencies announce the value of daily RUC and estimated quantities when the RFB is posted. RUC is based on estimated cost of longer commute times and economic loss resulting from road closures. It may not equal the above-mentioned estimate, which we call the true RUC.
The payment to the winner has two parts as well. The first part is the sum of the product of actual quantities consumed in project execution and winner’s unit bids. Note that actual quantities may be different from estimated quantities that are used to select the winner. The second part is either an early completion incentive or a lateness penalty, which is calculated relative to the winner’s time bid. For this purpose, agencies announce daily incentive and disincentive rates when issuing a RFB. They also announce minimum and maximum time bids permitted and the maximum number of incentive days. The latter caps the winner’s incentive from early completion.

Based on interactions with agency engineers and contractors who are experienced in preparing bids for A+B projects (see Section 3 for details), we present a normative model of the A+B bidding environment that explains some commonly observed results — that line-item bids do not reflect contractors’ true costs of associated activities and that time bids are shorter than engineers’ estimates, but bidders complete on time or earlier than bid. We also demonstrate that the common practices of setting daily incentive rate lower than the disincentive rate or capping incentives lowers social welfare\(^1\) and leads to the possibility that bids are not sorted correctly. There are three features of our model that set it apart from previous attempts at modeling time incentives in construction projects. These features are (1) multi-attribute private information and multidimensional bids, (2) contractors’ uncertainty about the effectiveness of their effort to expedite completion, and (3) reputation costs. We explain each feature and its significance in the sequel.

Contractors’ private information consists of actual material quantities, cost and effectiveness of expediting effort, and reputation costs (see Section 3). Differences in quantities estimated by engineers and contractors affect contractors’ bidding strategy. By modeling multi-attribute private values, we are able to characterize each contractor’s individually optimal multidimensional bid as a function of its private signal and target A+B score. We show that by and large, if the contractor’s estimate is smaller (respectively, larger) than the engineers’ estimate for a line item, then it should offer a low (high) price for that item. However, which line-item bids a contractor should inflate or deflate also depends on time incentives and the target A+B score (see Theorem 1).

Before project award, winners are required to post bonds equal to the bid prices to assure completion. Nearly all contractors use approved third-party sureties to post bonds. Sureties use contractors’ performance history including all instances of having paid delay costs (also called liquidated damages (LD)) to agencies to assess their credit worthiness and set bond rates\(^2\). Lower credit worthiness lowers the bonding offered by the surety, and also increases the bonding cost. Contractors estimate that frequent project delays could increase their bonding costs by between 0.75% and 1.5% of their entire project portfolio. In addition, all past instances of LD need to be reported when bidding on Design-Build type projects. The prospects of bidding on Design-Build projects are different for different contractors. For these reasons, we model the reputation cost as heterogeneous, private fixed cost, which may be substantial for some contractors.

\(^1\)The social-welfare-maximizing effort is the amount of effort that a single entity consisting of the commuting public and the winning bidder exerts. We provide a formal definition of welfare-maximizing effort in Section 7.

\(^2\)Although the cost of posting performance bonds varies based on the contractor’s performance history, it is usually less than 5% of the bid price – see, e.g. http://www.bryantsuretybonds.com/Surety_Bond_Information/Surety_Bond_Cost.html.
The presence of time incentives causes contractors to choose competitive time bids. Without such incentives, contractors would have no reason to choose time bids shorter than engineers' estimates. At the time of bidding, they face significant uncertainty about the effectiveness of their efforts to expedite completion. This stems in part from the fact that some portions of the work are performed by subcontractors and in part from the fact that extra crews and equipment may not be available when needed. The prospect of incurring a reputation cost lengthens time bids. Actual expediting effort is chosen during project execution after uncertainty about cost and effectiveness of effort is resolved. Longer initial bids cause contractors to frequently finish earlier than bid, as observed in data. In absence of the three features we consider, a simpler normative model would show that either contractors should bid the minimum allowed time bid or be indifferent to their choice of time bid, and finish late. By modeling multi-dimensional bids and reputation cost, we also show that a contractor's profit function is not concave in its time bid, which is assumed in all previous papers dealing with time incentives (see discussion following Equation (9)).

Agencies have multiple objectives when conducting an auction. While overall cost reduction is the traditional objective, encouraging contractors to exert greater effort on expediting completion and adherence to the cost and time bids are also important for budgetary reasons, public confidence, and scheduling of complementary projects. Agencies’ choice of auction parameters presents a trade off among these objectives. If the daily incentive and disincentive rates are set equal to the true RUC and incentive payments are not capped, then the effort exerted by the winning contractor maximizes social welfare. However, whether the contractor completes as bid depends on its private reputation and expediting costs. Consequently, agencies find it difficult to forecast actual completion times, complicating scheduling of complementary projects and budgeting of project engineers’ time for project supervision. In contrast, if the incentive rate is smaller than the disincentive rate, incentives are capped, the disincentive rate is equal to the RUC and the RUC selected by the agency is smaller than the true RUC, then contractors exert less than social-welfare maximizing effort, although they are more likely to complete as bid.

We obtained data from the Minnesota Department of Transportation (MnDOT) for all 38 projects that were let using the A+B mechanism during April 2000 and August 2008 (see data summary in the Appendix). Winning bids were on average 20% shorter than engineers’ estimates, and completion times were 5% shorter than bids. Engineers’ estimates typically equalled maximum allowed time bid. RFBs included widely different choices of incentive and disincentive rates, and maximum number of incentive days, confirming the lack of clear guidance to project engineers on how to set these parameters. In particular, engineers frequently set incentive rates smaller than disincentive rates, or capped incentive days.

The remainder of this paper is organized as follows. In Section 2, we explain this paper’s contribution relative to the literature. This is followed by institutional background and model formulation in Sections 3 and 4, respectively. We analyze a contractor’s choice of bid parameters in two parts. By selecting the winner according to the total score but making payments that depend on individual bid parameters, the A+B mechanism naturally permits this two-part approach. In
the first part, presented in Section 5, we assume that the contractor has selected its A+B score and observed a signal containing its multi-attribute private information. In Theorem 1, we obtain the optimal multi-dimensional bid from the contractor’s perspective by solving a stochastic nonlinear program. Proposition 1 obtains the contractor’s contingent actual completion time as a function of bid parameters and residual uncertainty at the time of bidding. In the second part, presented in Section 6, we demonstrate how to obtain equilibrium A+B scores, after transforming the multi-attribute private signal into a scalar and adapting the classical auction approach from Milgrom and Weber (1982). This transformation is made possible by contractors’ choice of optimal bid parameters. The applicability of Milgrom and Weber (1982) and later works of Esö and White (2004) and Lu and Perrigne (2008) is justified under the assumption that mobilization costs are sufficiently large to assure that the contractors’ conditional expected profits are linearly increasing in their A+B scores. We discuss policy implications in Section 7 and conclude the paper in Section 8.

2 Literature Review

There are three bodies of literature related to A+B bidding. These are construction management and practitioner literature, economics literature, and Operations Research (OR) literature. Significant amount of practitioner literature can be found on the web in the reports published by the Federal Highway Administration (FHWA) and the American Association of State Highway and Transportation Officials (AASHTO) (fhwa.dot.gov and transportation1.org/aashtonew/default.aspxaashto.gov). Reports posted on these web pages describe various experimental contracting schemes and the experiences of early adopters.

Among construction management articles, Gransberg and Riemer (2009) study the effect of inflated engineers’ quantity estimates, due to the need for contingency budgets\(^3\), on unbalanced unit bids. Unbalancing occurs when a contractor bids an amount that is significantly different from its true cost and markup. The authors argue that quantity inflation leads to higher bids. In contrast, it can be inferred from our model that payments resulting from obvious inaccuracies in quantity estimates would be factored into the bids with no effect on project costs.

Anderson and Damnjanovic (2008) report a survey of state transportation departments’ (DOTs’) employees to evaluate processes adopted by DOTs to select from a set of contracting methods and the effectiveness of different approaches to accelerate project completion. El-Rayes (2001) develops a model for optimum resource utilization in the A+B environment. Herbsman (1995) presents empirical evidence of early completion from the use of the A+B mechanism. In the vast majority of the projects studied there, the completion time was shorter than the time bid. The articles mentioned above ignore strategic bidding, and do not directly address the problems of choosing bid prices and agency parameters.

There is significant economics literature on procurement auctions. Excellent surveys on auc-

\(^3\) The contingency budget is used to account for differences between actual quantities used and engineers’ estimates.
tions, bids, and the use of incentives in procurement can be found in Engelbrecht-Wiggans et al. (1983), McAfee and McMillan (1987), Laffont and Tirole (1993), and Krishna (2002). Economics literature also contains papers that are focused specifically on construction procurement auctions (e.g., Hong and Shum 2002, Krasnokutskaya 2011). Therefore, we present a brief survey of these works and focus primarily on papers that analyze the A+B mechanism.

Among relevant papers from economics literature, Ewerhart and Fieseler (2003) model unit-price auctions, Maskin (1992) consider multi-attribute private information, and Asker and Cantillon (2008) and Che (1993) study multi-dimensional auctions with scoring rules. Our model has all of these features. In addition, we consider the fact that bidders’ effort costs and actual completion time remain uncertain at the time of bidding. The vast majority of previous papers in the auction literature assume that for each bidder, all private information is known with certainty before it submits its bids. Esö and White (2004) and Lu and Perrigne (2008) extend auction models discussed above by explicitly incorporating ex-post uncertainty in deriving bidding strategies. This is also an important aspect of construction auctions, and we build on this research in our model.

Papers in the auction literature typically assume independent private values (IPV) for reasons of tractability. We assume that contractors’ signals are affiliated private values (APV). The APV framework applies when contractors estimate their costs based on their signals, which are accurate reflections of true costs, but the signals themselves may be affiliated across contractors. For example, if there is ample supply of construction materials and labor for a project, that reduces cost for all contractors. IPV are a special case of APV in which the affiliation across contractors’ signals are negligibly small. Evidence of IPV in pavement rehabilitation projects that are typically let using the A+B mechanism can be found in Hong and Shum (2002) and Krasnokutskaya (2011).

Among papers focusing on time incentives in construction contracts, Lewis and Bajari (2011a) estimate the benefits to commuters from expediting, and compare these to the marginal cost curve they derive from data on A+B contracts in California. Based on their empirical analysis they recommend expanding the use of A+B to all projects, but scoring bids using a RUC much lower than commuters’ true cost. As a basis for estimating contractors’ cost, the authors also propose a relatively simple theoretical model for A+B bidding. Lewis and Bajari (2011a) assume scalar private information that reveals to each contractor the relationship between its chosen actual completion time and project cost. They do not model unit bids, contractor effort, reputation cost, and possible discrepancy between time bids and actual completion times. These parameters are important because they affect contractors’ decisions.

Lewis and Bajari (2011b) expands on the framework of Lewis and Bajari (2011a) by incorporating uncertainty that is unresolved at the time of bidding. However, this paper is confined to standard (A-only) contracts. Through an empirical analysis of standard projects, the authors model the impact of uncertainty on work rate to estimate contractors’ responses to uncertainty. They assume a quadratic cost-of-expediting function, and that capital and equipment are fixed for the duration of the project but labor investments may vary. In contrast, we allow for a general expediting cost function, potentially with capital as well as labor components, and evaluate the
impact of incentives in A+B bidding environments.

Recently, several OR papers have investigated alternate mechanism design for multi-attribute procurement auctions to achieve specific objectives – see, e.g., Beil and Wein (2003), Parkes and Kalagnanam (2005), and Kostamin et al. (2009). However, the objectives and mechanisms described in these papers (e.g., multi-round auctions) are either not relevant for transportation construction auctions or are not permitted under FHWA and state regulations, which require the use of single round first-price sealed bid auctions for the award of such contracts.

3 Institutional Background

This section is based on repeated interactions with MnDOT engineers and interviews of three contractors who have tendered multiple bids on A+B projects. Some of the institutional background is included in the Introduction section where we describe winner selection and payment mechanisms. In this section, we do not repeat information that was presented earlier.

The agency’s request for bids (RFB) includes a detailed product design and a line-item list of all pieces of work along with engineers’ estimates of materials required to complete each line item. There are two types of line items – those that are bid on a lump sum basis (i.e., quantity equals 1) and those that are bid on a per-unit basis. Examples of lump-sum items are mobilization costs, contractors’ costs of setting up field offices, traffic control systems, and construction area signs. In this paper, we put all lump-sum items into a single category and call them mobilization costs. Mobilization costs are a large fraction of the $A$ component. For items that are bid on a per-unit basis, agency engineers may be uncertain about the amount of materials that will be needed to complete each task at the time of issuing a request for bids. However, agencies announce firm estimates because they must; recall that quantity estimates are needed to evaluate bids. In fact, agencies do not expect the project to consume precisely the estimated amount of materials for each line item, and routinely set aside contingency funds to account for discrepancies that arise.

Agency engineers also estimate lump-sum and per-unit costs for all line items. These estimates are used to determine which bids are unbalanced. If a contractor bids very high unit prices on some items and very low unit prices on others, then the bid is declared unbalanced. Such bids are disqualified. Increasingly, bids are submitted via the Internet, which allows contractors convenient access to the range of prices bid by contractors on similar items in earlier projects. Therefore unbalanced bids are rare – there were no unbalanced bids in our data sample of 38 projects, which contained a total of 188 bids. We use these facts to justify our assumption in Section 4 that contractors know the ranges of values of unit bids that would be considered acceptable.

In a typical A+B project, the specified RUC is less than or equal to the true RUC to ensure that no legal challenges arise as a result of choosing unsupported values of daily RUC\(^4\). The incentive rate

\(^4\)In Milton Construction Company v. Alabama DOT, US 11th Circuit Court of Appeals 1990, the Alabama DOT set a disincentive rate higher than the RUC. The contractor, who was late, sued and won, setting a precedent that disincentive rates exceeding the daily RUC cannot be enforced in a court of law. Setting the RUC higher than the true road closure cost has similar implications, and is avoided.
is typically smaller than the disincentive rate and total incentive is capped at a maximum number of days. Agency engineers use standard project management software (e.g., Sure Track, Microsoft Project) to estimate the average project completion time before bids are invited. Software tools are also used for managing projects during execution.

From the time an RFB is issued, contractors typically have a couple of weeks to respond with their bids. Because the response time is short, contractors do not have sufficient time to independently estimate material quantities for all items. Therefore, they first identify from experience a few line items for which they suspect engineers’ estimates might be wrong. Our interviewees suggested that their estimates may differ from engineers’ estimates for up to 20% of line items and these differences could be significant for up to two percent of line items. There are a variety of reasons why estimates may differ. For example, a project may include two line items, one requiring excavation and another requiring fill. In such cases, the contractor may use material excavated from one spot as fill in another spot, effectively reducing the material required for fill. Agency engineers as well as some contractors may fail to adjust for this fact and estimate fill independently of other line items in the RFB. Later, as a consequence of material-use audit, the contractor is paid only for extra fill material required, causing engineers’ initial estimates to be too high. In this paper, each contractor’s estimates of quantities needed, its unit costs, and functions that relate its expediting effort to cost and actual completion time are private information.

Contractors use private but firm estimates of material quantities to place their bids. This is consistent with the independent private values assumption in this paper (Hong and Shum 2002, and Krasnokutskaya 2011). Also, A+B bidding is typically used for projects with no known utility work and low probability of requiring change orders. Discrepancy in quantities specified by agency engineers and contractor estimates arises because of oversight and errors. Therefore, change orders are not factored into contractors’ bidding strategy.

Contractors can affect project completion date by engaging in activities such as working longer hours or double shifts, working through weekends and holidays, and using extra crews, prefabricated components, and equipment capable of producing higher daily output. Like agencies, they also use standard project management software to develop an initial estimate of completion times. However, their time bids take into account competition and both positive and negative consequences of bidding an amount different from their initial estimate. Shorter time bids make proposals more competitive, but upon finishing later than bid contractors pay a penalty to the agency, and also experience a private tardiness penalty, which we refer to as the reputation cost. Therefore, they also need to factor cost of expediting, number of concurrent projects, availability of working capital, and uncertainty about subcontractors’ completion times in their decisions.

The contractors we interviewed told us that they do not commit to a particular level of effort at the time of bidding. Instead, they choose their effort after the resolution of some uncertainties, e.g., availability of extra crews and subcontractors’ ability to shorten completion of their tasks, which are not resolved at the time of submitting a bid. Nevertheless, such uncertainties and

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5The origins of the reputation cost are explained in detail in the Introduction section.
anticipated effort are factored into contractors’ bids. That is, contractors act rationally by choosing bid parameters that take into account their ex ante expected cost of effort. The effort cost is not observed by the agency.

4 Notation and Formulation

The notation used to describe a model of A+B bidding is divided into four time points, which correspond to four distinct events; namely, the posting of request for bids, the submission of sealed bids, the selection of the winner, and the realization of projects costs for the agency and either profit or loss for the contractor who won the contract (see Table 1). Within each time point, we present two types of notation representing ownership by either the agency or the contractors. Common-knowledge parameters are denoted by an asterisk (*). Contractors are assumed to be risk neutral. Throughout this section and in Sections 5 and 6, we refer to a tagged (arbitrary) contractor as the contractor and do not use contractor index.

In Table 1, all parameters are real valued (denoted by \( \mathbb{R} \)) unless stated otherwise. The agency’s decision variables are \( c_U \), \( c_I \), \( c_D \), \( t_I \), \( t \), and \( f \). We assume that \( c_I \leq c_D \leq c_U \leq c_T \), which is a common practice, for two reasons – (i) it lowers agency’s project costs and (ii) it prevents court challenges of the type discussed in Footnote 4. At the time of bidding, each contractor knows the number of bidders, the scope of work, task descriptions and estimated quantities \( q_e \), permissible ranges of unit bids \([c_i, c_i]\), and time related parameters \([t, t]\), \( c_U \), \( c_D \), \( c_I \), and \( t_I \). The contractor’s decision consists of bid parameters: i.e. time \( t_B \) and unit prices \( b = (b_0, \cdots, b_n) \). Bids are disqualified if the following constraints are not met: \( t_B \in [t, t] \) and \( b_i \in [c_i, c_i] \) for each \( i = 1, \cdots, n \). Note that item index 0 represents mobilization and \( b_0 \in [0, \infty) \). Each bid is evaluated on the basis of its score \( s(b, t_B) = c_U t_B + b \cdot q_e \). Sealed bids are opened on a pre-announced date and the bidder with the lowest score wins the contract.

Each bidder has private information about the quantity it would need for each line item \( (q) \), unit line-item costs \( (c) \), reputation cost \( (c_R) \), and the relationships between its effort and cost \( (h(\gamma, \xi)) \), and between its effort and completion time \( (t(\gamma, \xi)) \), where \( \gamma \) denotes contractor effort and the parameter \( \xi \in \Xi \) represents the contractor’s uncertainty about the effectiveness of its effort to reduce completion time. Consistent with practitioner literature, we assume that \( t(\gamma, \xi) \) is decreasing convex in \( \gamma \) (i.e. \( t'(\gamma, \xi) < 0, t''(\gamma, \xi) \geq 0 \)) and \( h(\gamma, \xi) \) is increasing convex (i.e. \( h'(\gamma, \xi) > 0, h''(\gamma, \xi) \geq 0 \)), for each fixed \( \xi \) (see, e.g. Shr and Chen 2004, Pyeon and Park 2010 and Sillars and Riedl 2007). For notational convenience, we denote the contractor’s private information by a multi-attribute signal \( \sigma \in \Sigma \), which it learns prior to placing its bids.

Each contractor submits its multi-dimensional bid consisting of \( (b, t_B) \) after observing its private signal \( \sigma \). If the contractor is awarded the contract, it then chooses individually optimal effort upon observing \( \xi \). Although uncertainty about effort cost and how effort affects completion time may unfold continuously over the course of the project, for the purpose of modeling we represent it as a
### Time Point 1: Request for Bids Posted

<table>
<thead>
<tr>
<th>Agency</th>
<th>Contractor (index suppressed)</th>
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</thead>
<tbody>
<tr>
<td>(n) = Number of line items that are bid on a per-unit basis, (n \geq 1) &amp; integer (⋆)</td>
<td>(\nu) = number of bidders, (\nu &gt; 1) &amp; integer (⋆)</td>
</tr>
<tr>
<td>(q^e = (q_0^e, \cdots, q_n^e)) = Engineers’ quantity estimates, (q_0^e \equiv 1) and (q_i^e &gt; 0, q_i^e \in \mathbb{R}, 1 \leq i \leq n)</td>
<td>(q = (q_0, \cdots, q_n)) = Actual quantities used, (q_0 \equiv 1) and (q_i &gt; 0, q_i \in \mathbb{R}, 1 \leq i \leq n)</td>
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<tr>
<td>(c^e = (c_0^e, \cdots, c_n^e)) = Agency’s estimates of line-item costs, (c_i^e &gt; 0, c_i^e \in \mathbb{R}, 0 \leq i \leq n)</td>
<td>(c = (c_0, \cdots, c_n)) = Contractor’s estimates of line-item costs, (c_i &gt; 0, c_i \in \mathbb{R}, 0 \leq i \leq n)</td>
</tr>
<tr>
<td>([\underline{q}, \overline{q}]) = Acceptable ranges of unit prices, (0 &lt; \underline{q} \leq \overline{q}, (\underline{q}, \overline{q}) \in \mathbb{R}^2, 1 \leq i \leq n)</td>
<td>(c_R) = Contractor’s private reputation cost from late completion, (c_R \geq 0, c_R \in \mathbb{R})</td>
</tr>
<tr>
<td>(c_T) = True daily RUC, (c_T \geq 0)</td>
<td>(\xi) = Uncertain component of completion time and expediting cost, (\xi \in \Xi)</td>
</tr>
<tr>
<td>(t^e) = Engineers’ estimate of project completion time, (t^e &gt; 0, t^e \in \mathbb{R})</td>
<td>(\gamma(\xi)) = Contractor’s expediting effort, (\gamma \geq 0, \gamma(\xi) \in \mathbb{R}) for each (\xi)</td>
</tr>
<tr>
<td>(c_D) = Announced daily lateness penalty, (0 &lt; c_D \leq c_U, c_D \in \mathbb{R}) (⋆)</td>
<td>(t(\gamma, \xi)) = Completion time as a function of effort (\gamma) and uncertainty (\xi), (t(\gamma, \xi) \geq 0, t(\gamma, \xi) \in \mathbb{R})</td>
</tr>
<tr>
<td>(c_I) = Announced daily early-completion reward, (c_I \geq 0, c_I \in \mathbb{R}) (⋆)</td>
<td>(h(\gamma, \xi)) = Expediting cost as a function of effort (\gamma) and uncertainty (\xi), (h(\gamma, \xi) \geq 0, h(\gamma, \xi) \in \mathbb{R})</td>
</tr>
<tr>
<td>(c_U) = Announced daily RUC, (0 &lt; c_U \leq c_T, c_U \in \mathbb{R}) (⋆)</td>
<td>(\sigma = (q, c, c_R, t(\gamma, \xi), h(\gamma, \xi))) = Contractor’s private information, (\sigma \in \Sigma)</td>
</tr>
<tr>
<td>(t_1) = Maximum number of incentive days, (t_1 \geq 0, t_1 \in \mathbb{R}) (⋆)</td>
<td>(x(\sigma) = ) Zero-profit bid score, (x &gt; 0, x \in \mathbb{R})</td>
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<tr>
<td>((\underline{t}, \overline{t})) = Announced minimum and maximum bid days, (0 \leq \underline{t} &lt; \overline{t}, (\underline{t}, \overline{t}) \in \mathbb{R}^2) (⋆)</td>
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### Time Point 2: Sealed Bids Submitted

<table>
<thead>
<tr>
<th>Agency</th>
<th>Contractor (index suppressed)</th>
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<tbody>
<tr>
<td>(t^B = ) Contractor’s bid time</td>
<td>(b = (b_0, \cdots, b_n) = ) Contractor’s unit bids</td>
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### Time Point 3: Winner Selected

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<tr>
<th>Agency</th>
<th>Contractor (index suppressed)</th>
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<tbody>
<tr>
<td>Calculates contractor scores (⋆)</td>
<td>(s(b, t^B) = c_U t^B + b \cdot q^e = ) Contractor’s score (⋆)</td>
</tr>
<tr>
<td>Announces winner = low bidder (⋆)</td>
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### Time Point 4: Project Executed

<table>
<thead>
<tr>
<th>Agency</th>
<th>Contractor (index suppressed)</th>
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</thead>
<tbody>
<tr>
<td>Contractor learns (\xi) and chooses (t^*(\xi) = ) optimal completion time</td>
<td></td>
</tr>
<tr>
<td>Agency realizes total project cost (⋆)</td>
<td>Contractor realizes profit or loss</td>
</tr>
</tbody>
</table>
realization of $\xi$ at a single time point\(^6\). That is, the pair $(\gamma, \xi)$ uniquely determines the completion time $t(\gamma, \xi)$. Note that for each realization of $\xi$, the choice of effort $\gamma$ is equivalent to the contractor choosing a completion time $t$. In a slight abuse of notation, we now perform a variable transform such that the argument of $h(\gamma, \xi)$ is changed to $h(t, \xi)$. Because the original function $h$ is increasing convex in $\gamma$ and $t(\gamma, \xi)$ is decreasing convex in $\gamma$, the transformed function $h(t, \xi)$ is decreasing convex in $t$ for each fixed $\xi$. We denote the winner’s individually optimal completion time by $t^*(\xi)$.

The distribution of contractors’ signals is common knowledge, but realized values are private. After the winner completes the work, it receives a payment of $b \cdot q$ for labor and materials and time based incentive or disincentive of $c_I \min\{t_1, (t^B - t)^+\} - c_D ((t - t^B)^+)$, where $t$ is the actual completion time. The contractor’s conditional profit function, $\Pi(b, t^B, t \mid s, \sigma, \xi)$, which is the amount that the contractor with signal $\sigma$ will earn if it wins the project upon bidding $s$ and $\xi$ realizes, can be written as follows.

$$\Pi(b, t^B, t \mid s, \sigma, \xi) = b \cdot q - c \cdot q - h(t, \xi) - c_D (t - t^B) \mathbb{1}_{\{t > t^B\}} - c_R \mathbb{1}_{\{t > t^B\}} + c_I t \mathbb{1}_{\{t < t^B - t_i\}} + c_I (t^B - t) \mathbb{1}_{\{t^B - t_i \leq t \leq t^B\}}$$

The term $[-c_R \mathbb{1}_{\{t > t^B\}}]$ is the contractor’s private reputation cost if it completes later than $t^B$. The term $[c_I t \mathbb{1}_{\{t < t^B - t_i\}} + c_I (t^B - t) \mathbb{1}_{\{t^B - t_i \leq t \leq t^B\}}]$ is the incentive payment, which is capped at $c_I t_1$, whereas $[-c_D (t - t^B) \mathbb{1}_{\{t > t^B\}}]$ is the disincentive payment. Both sets of terms cannot be simultaneously nonzero. We refer to these terms as time-related windfall. Similarly, $b \cdot q - b \cdot q^e$, the difference between the actual payment for material and labor and the A-component is the windfall due to unit bids, which may be positive or negative. The contractor chooses its multidimensional bid to maximize $E[\Pi(b, t^B \mid \sigma)]$ anticipating that it would choose an optimal completion time corresponding to each realization of $\xi$. We present this analysis next.

## 5 Optimal Bid Parameters

For ease of exposition, it helps to think of the contractor’s problem as consisting of two steps. In Step 1, the contractor determines optimal bid parameters $(b^*, t^{B*})$ as functions of signal $\sigma$ and score $s$. The bid parameters are chosen to maximize the contractor’s profit upon assuming that $\sigma$ and $s$ are fixed. In Step 2, the contractor determines its equilibrium bid score $s$ as a function of signal $\sigma$, assuming that it will choose contingent optimal bid parameters. Together these two parts characterize the contractor’s bidding strategy. In what follows, we obtain optimal bid parameters for a fixed $s$. Equilibrium values of $s$ are obtained in Section 6. We define additional notation (see Table 2) and make some assumptions. Note that the minimum time bid equals zero when the agency does not specify this parameter.

**Assumption 1.** $b_0 \geq 0; b_i \in [\underline{a}_i, \overline{a}_i]$ for $1 \leq i \leq n; t^B \in [\underline{t}, \overline{t}];$ and $s \geq \sum_{i=1}^n \sum q_i^e + cv^\mathbb{R}$.

\(^6\)This makes $\xi$ scalar. Note that $\xi$ may be either discrete or continuous so long as $\Xi$ is compact.
Table 2: Additional Notation.

<table>
<thead>
<tr>
<th>(t_{0,1}(\xi))</th>
<th>(t : \frac{dh}{dt} = -c_l)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(t_{0,2}(\xi))</td>
<td>(t : \frac{dh}{dt} = -c_D)</td>
</tr>
<tr>
<td>(t(0, \xi))</td>
<td>(t : \frac{dh}{dt} = 0)</td>
</tr>
<tr>
<td>(\hat{t}(\xi))</td>
<td>(t : \frac{h(t, \xi) - h(t_{0,2}, \xi)}{t - t_{0,2}})</td>
</tr>
<tr>
<td>(t_0^*(\xi))</td>
<td>Optimal completion time when (c_R = 0)</td>
</tr>
<tr>
<td>(\eta(\tau))</td>
<td>(\frac{\partial E[\Pi(b, t^B s, \sigma)]}{\partial t} \big</td>
</tr>
<tr>
<td>(\lambda_i)</td>
<td>(q_i/q_i^e), for each (i)</td>
</tr>
</tbody>
</table>

\(t_{0,1}\) and \(t_{0,2}\) obtained by equating the marginal cost of expediting with the incentive or disincentive rate. 
\(t(0, \xi)\) is the completion time that occurs when the contractor spends little or no extra effort.

Because \(h\) is decreasing convex in \(t\), it is easy to confirm that \(t_{0,2}(\xi) \leq t_{0,1}(\xi)\).

Additional details about \(\hat{t}\) are presented in the proof of Proposition 1.

**Assumption 2.** \(\underline{t} < t(0, \xi) + t_I\) for every \(\xi\).

Assumption 1 specifies acceptable ranges of unit and time bids as well as overall bid scores (see Section 3 for details). If Assumption 1 were not true, then the contractor’s bid would be disqualified. Assumption 2 states that \(\underline{t}\) should not be so large that the contractor is prevented from choosing zero effort level in any realization of \(\xi\). This assumption is trivially satisfied in 30 out of 38 MnDOT projects’ data because in those projects \(\underline{t}\) is not specified (i.e. \(\underline{t}\) equals 0) and \(0 \leq t(0, \xi)\) holds for any \(\xi\). When \(\underline{t}\) is specified, it is typically set equal to a small value relative to \(t^e\), which supports Assumption 2 because engineers’ estimate \(t^e\) assumes no extra effort, i.e. \(t^e \approx E(t(0, \xi))\). When \(\underline{t}\) is much smaller than \(t^e\), it is unlikely for a contractor to complete the project within \(\underline{t}\) without extra effort.

Although the contractor chooses expediting effort after knowing \(\xi\), which is equivalent to choosing actual completion time as explained earlier, it must factor its anticipated expediting cost into its optimal bid parameters \((b^*, t^B, t^e)\). Therefore, we begin by characterizing the contractor’s choice of completion time given \(b, t^B, s, \sigma\) and \(\xi\). The presence of a kink and a discontinuity in (1) requires us to consider the choice of \(t\) in different regions. First, taking the derivative of \(\Pi(b, t^B, t \mid s, \sigma, \xi)\), we obtain:

\[
\frac{\partial \Pi(b, t^B, t \mid s, \sigma, \xi)}{\partial t} = \begin{cases} 
-h'(t, \xi) < 0 & \text{if } t < t^B - t_I \\
-h'(t, \xi) - c_I & \text{if } t^B - t_I \leq t \leq t^B \\
-h'(t, \xi) - c_D & \text{if } t > t^B 
\end{cases}
\]

From (2), we observe that \(\Pi(b, t^B, t \mid s, \sigma, \xi)\) is concave in \(t\) in each region. These arguments imply that if \(c_R = 0\), then an optimal choice of completion time, \(t_0^*(\xi)\), would be one of the following four possible values. We also identify the range of \(t^B\) values under which each possibility exists. In the remainder of this paper, we often do not show the dependence of \(t_0^*, t_{0,1}, t_{0,2}\) and \(h\) on \(\xi\) for
notational convenience.

\[ t_0^* = \begin{cases} 
  t_{0,2} & \text{if } t^B < t_{0,2}, \\
  t^B & \text{if } t_{0,2} \leq t^B < t_{0,1}, \\
  t_{0,1} & \text{if } t_{0,1} \leq t^B \leq t_{0,1} + t_I, \\
  t^B - t_I & \text{if } t^B > t_{0,1} + t_I,
\end{cases} \tag{3} \]

We are now ready to present our first result concerning optimal completion time function \( t^* \) with reputation cost.

**Proposition 1.** Given \( b, t^B, s, \sigma \) and \( \xi \), the optimal completion time \( t^* \) can be calculated as follows:

\[ t^* = \begin{cases} 
  t_{0,2} & \text{if } t^B < \hat{t}, \\
  t^B & \text{if } \hat{t} \leq t^B < t_{0,2}, \\
  t_{0,1} & \text{if } t_{0,1} \leq t^B \leq t_{0,1} + t_I, \\
  t^B - t_I & \text{if } t^B > t_{0,1} + t_I,
\end{cases} \tag{4} \]

**Proof:** Because the contractor incurs a fixed cost \( c_R \) when actual completion times exceeds \( t^B \), \( t^* \) may be less than \( t_{0,2} \) when \( t^B < t_{0,2} \). This is because by completing at \( t^B \), the contractor can avoid fixed costs \( c_R \). Given \( \xi \) the optimal completion time would be \( t_{0,2} \) if

\[ h(t^B, \xi) > h(t_{0,2}, \xi) + c_D(t_{0,2} - t^B) + c_R, \tag{5} \]

which is equivalent to

\[ \frac{h(t^B, \xi) - h(t_{0,2}, \xi)}{t^B - t_{0,2}} < -c_D + \frac{c_R}{t^B - t_{0,2}}. \tag{6} \]

Because for every \( \xi \), \( h(t, \xi) \) is convex in \( t \), there exists a \( \hat{t} \leq t_{0,2} \) such that \( \frac{h(\hat{t}, \xi) - h(t_{0,2}, \xi)}{t - t_{0,2}} = -c_D + \frac{c_R}{t - t_{0,2}} \). Therefore, from (3) and above arguments, the optimal completion time is obtained as shown in (4). Hence proved. #

Proposition 1 shows that depending on the realization of \( \xi \), a contractor may be either early, on time, or late with respect to the time bid. It is not guaranteed to complete at \( t^B \), as Lewis and Bajari (2011a) suggests. We illustrate optimal completion time calculations with the help of a numerical example in Figure 1. In each panel, the open circle shows \( t^B \), whereas the asterisk shows the ex-post optimal completion time. In the bottom two panels, the contractor finishes earlier than bid. In the top left panel, it completes late and in the top right panel, it finishes on time.
Figure 1: Optimal Completion Time $t^*$ and $t^B$.

Upon substituting $t^*$ in the expression for $\Pi$ we obtain,

$$
\Pi(b, t^B, t^* | s, \sigma, \xi) =
\begin{cases}
  b \cdot q - c \cdot q - h(t_{0,2}, \xi) - c_D(t_{0,2} - t^B) - c_R & \text{if } t^B < \hat{t}, \\
  b \cdot q - c \cdot q - h(t^B, \xi) & \text{if } \hat{t} \leq t^B < t_{0,1}, \\
  b \cdot q - c \cdot q - h(t_{0,1}, \xi) + c_I(t^B - t_{0,1}) & \text{if } t_{0,1} \leq t^B \leq t_{0,1} + t_I, \\
  b \cdot q - c \cdot q - h(t^B - t_{I}, \xi) + c_I t_I & \text{if } t^B > t_{0,1} + t_I.
\end{cases}
$$

(7)

Let $\frac{\partial h}{\partial t}(y)$ denote the derivative of $h(t, \xi)$ at $t = y$. Next, taking the derivative with respect to $t^B$, we obtain

$$
\frac{\partial \Pi(b, t^B, t^* | s, \sigma, \xi)}{\partial t^B} =
\begin{cases}
  c_D & \text{if } t^B < \hat{t}, \\
  -\frac{\partial h}{\partial t}(t^B) & \text{if } \hat{t} \leq t^B < t_{0,1}, \\
  c_I & \text{if } t_{0,1} \leq t^B \leq t_{0,1} + t_I, \\
  -\frac{\partial h}{\partial t}(t^B - t_I) & \text{if } t^B > t_{0,1} + t_I.
\end{cases}
$$

(8)

It can be verified that $\frac{\partial \Pi(b, t^B, t^* | s, \sigma, \xi)}{\partial t^B}$ is decreasing in $t^B \geq \hat{t}$. However, because $-\frac{\partial h}{\partial t}(\hat{t}) \geq c_D$, $\frac{\partial \Pi}{\partial t^B}$ is not necessarily monotone decreasing in $t^B$. That is, $\Pi(b, t^B, t^* | s, \sigma, \xi)$ is not necessarily concave in $t^B$, which complicates the analysis. Note that if $c_R = 0$, then $\Pi$ is indeed concave in $t^B$.

Parameters $\hat{t}, t_{0,1}$ and $t_{0,2}$ do not depend on $t^B$ and their distributions depend on the distribution of $\xi$ and the expediting cost function. Therefore, taking expectation over all values of $\xi$, we obtain
the following expression for the contractor’s expected profit:

\[
E[\Pi(b, t^B|s, \sigma)] = b \cdot q - c \cdot q - E(h(t^*)) + c_I E \left( E[t^B - t^*|t_{0,1} \leq t^B \leq t_{0,1} + t_I] \right) \\
+ c_I t_I P(t^B \geq t_{0,1} + t_I) - c_D E \left( E[t_{0,2} - t^B|t^B < \hat{t}] \right) - c_R P(t^B < \hat{t}).
\] (9)

In the above expression, double expectation notation signifies inner conditional expectation given the conditioning inequality and the outer expectation over the conditioning inequality. Note that \(E[\Pi(b, t^B|s, \sigma)]\) is separable in \(b\) and \(t^B\) and that it is linear in components of the vector \(b\). However, \(E[\Pi(b, t^B|s, \sigma)]\) is not necessarily concave in \(t^B\). This requires us to identify all possible bidding strategies that satisfy first order optimality equations and choose the best solution overall. We proceed to do so next.

The contractor obtains optimal bid parameters, as a function of a desired score, by solving the following nonlinear program:

\[
\begin{align*}
\max_{b, t^B} & \quad E[\Pi(b, t^B|s, \sigma)] \\
\text{s.t.} & \quad c_U t^B + b \cdot q^e = s, \quad (10) \\
& \quad b_i \leq \overline{c}_i, \quad i = 1, 2, \ldots, n, \quad (11) \\
& \quad -b_i \leq -\underline{c}_i, \quad i = 1, 2, \ldots, n, \quad (12) \\
& \quad -b_0 \leq 0, \quad (13) \\
& \quad -t^B \leq -\underline{t}, \quad (14) \\
& \quad t^B \leq \overline{t}. \quad (15)
\end{align*}
\]

Reorder item indices such that \(q_1/q^e_1 \geq q_2/q^e_2 \geq \cdots \geq q_n/q^e_n\) or equivalently, \(\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n\). Define \(m\) as the largest index for which \(\lambda_i \geq 1\). Denote \(\eta(\tau) = \frac{\partial E[\Pi(b, t^B|s, \sigma)]}{\partial t^B}\) \(|_{t^B=\tau}\). With these notation in hand, we obtain the contractor’s optimal bid parameters as follows.

**Theorem 1.** Under assumptions 1 – 2, the contractor’s optimal bid parameters are as follows. There exists a maximum \(m_0\) (if \(\{\lambda_1, \ldots, \lambda_m\} = \emptyset\), set \(m_0 = 0\)) within the set of indices for which \(q_i \geq q^e_i\) such that \(b^*_i = \overline{c}_i\) for all \(i > m_0\), \(b^*_i = \overline{c}_i\) for \(i < m_0\), and then \(b^*_m\) and the optimal \(t^B\), denoted by \(t^{B*}\), are one of the following possible values:

1. \(t^{B*} = \underline{t}\) and \(b^*_m = \min\{\overline{c}_{m_0}, [s - c_U \underline{t} - \sum_{i=m_0+1}^n \overline{c}_i q^e_i - \sum_{i=1}^{m_0-1} \overline{c}_i q^e_i]/q^e_{m_0}\}\) if \(\eta(\underline{t}) \leq \lambda_{m_0} c_U\).

2. \(\underline{t} < t^{B*} < \overline{t}\) such that either

   (a) \(\eta(t^{B*}) = \lambda_{m_0} c_U\), and \(b^*_m = \min\{\overline{c}_{m_0}, [s - c_U \underline{t} - \sum_{i=m_0+1}^n \overline{c}_i q^e_i - \sum_{i=1}^{m_0-1} \overline{c}_i q^e_i]/q^e_{m_0}\}\),

   or

   (b) \(t^{B*} = [s - \sum_{i=m_0+1}^n \overline{c}_i q^e_i - \sum_{i=1}^{m_0} \overline{c}_i q^e_i]/c_U\) and \(b^*_m = \overline{c}_{m_0}\) if \(\eta(t^{B*})/c_U \in (\lambda_{m_0+1}, \lambda_{m_0})\).

Alternatively, when \(m_0 = 1\), \(t^{B*} = [s - \sum_{i=1}^n \overline{c}_i q^e_i]/c_U\) and \(b^*_m = \overline{c}_{m_0}\) if \(\eta(t^{B*})/c_U \in (\lambda_1, \infty)\).
3. $t^B_* = \bar{t}$ and $b_{m_0}^* = \min\{\bar{c}_{m_0}, [s - c_U \bar{t} - \sum_{i=m_0+1}^{n} q_i c_i - \sum_{i=1}^{m_0-1} c_i q_i]/q_{m_0}^U\}$ if $\eta(\bar{t}) \geq \lambda_{m_0} c_U$.

A proof of Theorem 1 is presented in the Appendix. On an intuitive level, Theorem 1 allocates a fixed resource $s$ among $b_i$’s and $t^B$ to maximize expected profit under constraints (12)–(16). Because profit increases at rate 1 for allocations made to $b_0$, only those $b_i$ would be candidates for values $> \underline{c}_i$ for which $\lambda_i \geq 1$. Budget allocation to item $i$ increases profit at constant rate $\lambda_i$, but it takes budget away from $t^B$ changing profit at a nonlinear rate $\eta(t^B)$. The rate of change of $E[\Pi(b, t^B|s, \sigma)]$ in $(b, t^B)$ has multiple kinks because of constraints (12)–(16) and reputation cost. Therefore, which items should be bid at their maximum and which at their minimum thresholds depends on $s$, contractor’s private information, and unresolved uncertainty at the time of bidding, requiring evaluation of all cases mentioned in Theorem 1.

One of the conclusions from Theorem 1 is that some bids should be at their extreme values, either at $\underline{c}_i$ or at $\bar{c}_i$. We evaluated whether this result is consistent with bids tendered in the MnDOT dataset. In practice contractors do not compare their estimates with engineers’ estimates for every line item. Rather they concentrate on items for which they believe there may be a significant discrepancy between the two, and bid strategically for those items. We could evaluate the implications from Theorem 1 by comparing actual quantities used in completed projects with engineer’s estimates and corresponding bids. There were 1562 relevant line items. For the 29 line items that had $q_i/c_i^U \leq 0.2$ and 130 line items that had $q_i/c_i^U \geq 1.5$, we compared their $b_i/c_i^U$ to $b_i/c_i^U$ across all items. We refer to the ratio $b_i/c_i^U$ as the normalized line-item bid. We found that when engineers estimates were highly inflated (respectively deflated), contractors submitted lower (resp. higher) normalized line-item bids with $p$-value $\approx 0$ (resp. 0.042).

Agencies often choose $c_I < c_D = c_U$ or $c_I = c_D = c_U$. They typically do not specify $\underline{t}$ (this happens 30 out of 38 times in MnDOT data) or choose a very small value of $\underline{t}$ relative to $\bar{t}$, where the latter is an estimate of average completion time under normal operations. This implies that $\underline{t}$ is either zero or very small. We hereafter assume that $-\frac{\partial h(t)}{\partial t} > \max\{c_U, c_D + c_R\}$. This ensures that $t^*$ lies in $[\underline{t}, \bar{t}]$ for every $\xi$.

Consider the case when reputation costs are negligible ($c_R = 0$) and $c_I < c_D = c_U$. From (8) we observe that the derivative of $\Pi$ is never more than $c_D$, which is itself bounded by above $c_U$. Moreover, $\Pi$ is concave in $t^B$. Therefore, $c_U \geq \eta(t^B)$ for every $t^B \geq \underline{t}$ and $t^B_* = \underline{t}$ is an optimal bid. Upon doing so, the contractor will finish at $t_{0.2}(\xi)$ for each realization of $\xi$. That is, it will expedite just enough so that the marginal cost of expediting is $c_D$ and it will finish late with respect to $t^B$.

In contrast, when $c_R = 0$ and $c_I = c_D = c_U$, then $t_{0.2} = t_{0.1}$ for every $\xi$ and $\Pi$ is invariant in $t^B$ so long as $t^B \leq t_{0.1} + t_I$ in every realization $\xi$. Let $\underline{t}_{0.1} = \min_{\xi}\{t_{0.1}(\xi)\}$ be the minimum value of $t_{0.1}$. Then, any $t^B \in [\underline{t}_{0.1}, \bar{t}_{0.1} + t_I]$ is an optimal $t^B$. Furthermore, after $\xi$ is realized, either $t^B < t_{0.1}$ and $t^*_U = t_{0.1}$, which implies the contractor finishes late with respect to $t^B$, or $t_{0.1} < t^B \leq t_{0.1} + t_I$ and $t^*_U = t_{0.1}$, which implies the contractor finishes early. It is also possible to have $t_{0.1} = t^B$ and in that case, the contractor will finish at $t^B$. That is all three realizations are feasible - early, on time, and tardy completion.

Next, consider how positive reputation cost ($c_R > 0$) affects the contractor’s response. If
If \( c_I < c_D = c_U \) and the contractor bids \( \underline{t} \), it is guaranteed to be late because \( \underline{t} \leq t_{0.2} \leq \bar{t} \). It can reduce the probability of incurring \( c_R \) by bidding \( t^B > \underline{t} \). If \( t^{B*} > \underline{t} \) is optimal, it is possible in particular realizations of \( \xi \) for the contractor to either complete late, on time, or early relative to \( t^{B*} \) (see Equation 4). Therefore, when \( c_R > 0 \), the contractor may choose a \( t^{B*} \) such that it ends up completing on-time or early. The same arguments also apply to the case when \( c_I = c_D = c_U \), except now the contractor would choose a \( t^B \) that has a relative low risk of being tardy. We find that in less than 5% of A+B projects, contractors bid \( \underline{t} \) and in only 12% of the projects, winners complete late. This is consistent with the existence of positive reputation costs (\( c_R > 0 \)).

We illustrate the above concepts via an example. In this example, we set \( c_I < c_D = c_U \) and vary \( c_R \). Other parameters used in this example are summarized in the Appendix. The contractor’s choice of bid score is fixed such that \( m_0 = 2 \) and there are three different realizations of \( \xi \). In Figure 2, we show two panels. The panel on the left shows the probability of completing early (solid line) and late (dotted line) for a fixed \( s \) when bid parameters are chosen optimally. The panel on the right shows the optimal values of \( t^B \) and \( b_2 \) as functions of \( c_R \). As \( c_R \) increases, the contractor chooses a larger value of \( t^B \). Whereas, the probability of being late is 1 when \( c_R = 0 \) and decreases as \( c_R \) increases, the contractor chooses expediting effort such that it is more likely to finish on time in the middle range. For large \( c_R \), the contractor chooses a large time bid to lower the probability of being late. After expediting cost function is revealed, it may find it economical to finish early and earn incentives. Note that the value of \( b_2^* \) decreases because the contractor uses more of its bid score \( s \) towards its completion time budget (i.e. \( c_U t^B \)). Figure 2 serves to exemplify that \( c_I < c_D \) and large reputation costs interact to lengthen bids, increasing the likelihood that the contractor would finish on time.

Figure 2: Effect of Reputation cost \( c_R \).

The previous analysis raises questions about the relative magnitude of reputation costs that contractors face. We performed two statistical tests, which suggest that contractors face moderate to high reputation costs. First, the average bid time among 106 bids tendered in 25 projects was 20 days shorter than the average engineers’ estimate (one-sided p-value was nearly zero). Second, the winning contractors completed projects, on average, 4.5 days early (one-sided p-value 0.023). Both results are statistically significant. Together they suggest that contractors face some reputation costs (because they generally avoid tardiness penalties) but that these costs are not too extremely
Equilibrium Bids

Upon substituting the value of \((b^*, t^{B*})\) from Theorem 1 and optimal completion time from Proposition 1 into Equation (9), we obtain the contractor’s optimal expected profit if it chooses bid score \(s\) upon observing \(\sigma\) and wins. We write this expression below.

\[
\pi(s \mid \sigma) = E[\Pi(b^*, t^{B*} \mid s, \sigma)] = b^* \cdot q - c \cdot q - E(h(t^*)) + c_I E\left( E\left[ t^{B*} - t^* \mid t_{0,1} \leq t^{B*} \leq t_{0,1} + t_I \right] \right) \\
+ c_I t_I P(t^{B*} \geq t_{0,1} + t_I) - c_D E\left( E[t_{0,2} - t^{B*} \mid t^{B*} < \hat{t}] \right) - c_R P(t^{B*} < \hat{t}).
\]  

(17)

From (17), we obtain certain properties of the contractor’s expected profit as a function of bid score \(s\). First, the conditional expected profit is monotone increasing in \(s\). This follows from the fact that if \(s_1 < s_2\), then the optimal bid parameters under \(s_1\) are feasible under \(s_2\) after the contractor increases \(b_0\) by \(s_2 - s_1\). Each extra dollar budgeted in \(b_0\) increases the contractor’s profit by the same amount. Second, we observe that the contractor’s expected profit may increase more than linearly when \(s\) is small. This happens because if bid score is small, the contractor does not have sufficient budget to benefit fully from discrepancies in material quantities and time incentives. However, when \(s\) is sufficiently large, the contractor’s profit increases linearly in \(s\) because larger \(s\) causes \(b_0^*\) to increase, while \(b_i^*, i \geq 1\) and \(t^{B*}\) remain invariant. Also, in such cases \(b_0\) is strictly greater than 0.

Upon examining the MnDOT data, we found that \(b_0\) is never zero in all bids tendered. This suggests that \(c_0\) is sufficiently large that discrepancies in material quantities and time incentives do not cover contractors’ mobilization costs. Therefore, they choose \(s\) in a region where \(b_0 > 0\) after taking into account discrepancies in estimates of material quantities and time incentives. In this range of values of \(s\), the contractor’s profit is linear in \(s\). That \(\pi(s \mid \sigma)\) is linearly increasing in \(s\) is implicitly assumed in all previous transportation procurement auction papers although this assumption is not explicitly stated because previous papers do not model multi-attribute private information and multidimensional bids. Our analysis helps clarify that although such an assumption is justified in practice, it is tantamount to assuming that contractors face large mobilization costs. We also make this assumption for tractability.

Using (17) and upon solving the equation \(\pi(s \mid \sigma) = 0\), we obtain a zero-profit score for an arbitrary contractor. We denote this quantity by \(x(\sigma)\). That is, for each signal that the contractor draws there is a corresponding zero-profit score \(x(\sigma)\). Note that no rational bidder will bid less than \(x\). For the purpose of determining equilibrium bid \(s\), we can interpret the draw of the signal \(\sigma\) as a draw of the zero-profit score \(x\), where the latter is obtained after applying the results of Theorem 1. Although a particular \(x(\sigma)\) may be obtained by many different combinations of bid parameters, it is sufficient for the contractor to know its zero-profit score. In effect, we demonstrate how multi-attribute private values are converted into an equivalent scalar parameter under optimal
bidding, and the requisite conditions.

We use \( F(x_1, x_2, \ldots, x_\nu) \) to denote the joint cumulative distribution function (CDF) of signals received by the \( \nu \) contractors, with support \([0, \bar{x}]^\nu\), and density function (PDF) \( f(\cdot) \). We assume that \( x_i \)'s may be affiliated, which means that a smaller realized zero-profit score for one contractor makes lower zero-profit scores for other contractors more likely (see Milgrom and Weber 1982). After the contractor receives its private signal \( x \), its conditional profit function for a bid \( s \), if it were to win, can be written as \( \Pi(s, x) = s - x + \epsilon \) and its conditional expected profit as \( \pi(s, x) = s - x \). Put differently,

\[
\epsilon = \left\{ E(h(t^*)) - h(t^*) \right\} - c_I \{ E[ E[t^{B*} - t^*|t_{0,1} \leq t^{B*} \leq t_{0,1} + t_I]] - E[t^{B*} - t^*|t_{0,1} \leq t^{B*} \leq t_{0,1} + t_I]\}
- c_{II} \{ P(t^{B*} \geq t_{0,1} + t_I) - 1_{\{t^{B*} \geq t_{0,1} + t_I\}} \} + c_D \{ E[ E[t_{0,2} - t^{B*}|t^{B*} < \hat{t}]] - E[t_{0,2} - t^{B*}|t^{B*} < \hat{t}] \}
+ c_R \{ P(t^{B*} < \hat{t}) - 1_{\{t^{B*} < \hat{t}\}} \}
\tag{18}
\]

has five terms, where each term accounts for the difference between an expected and a realized value of either a cost term or a windfall term. We call \( \epsilon \) the random component of a contractor’s conditional profit function and use \( D(\epsilon) \) to denote its distribution function. Note that \( E(\epsilon) = 0 \).

The random component of the conditional profit stems from the uncertainty about \( \xi \), which affects \( t_{0,1}, t_{0,2}, \hat{t} \) and \( t^* \). However, the effect of these parameters is additive to the conditional expected profit because \( \xi \) is independent of bid parameters \( b^* \) and \( t^{B*} \), and \( b^*_i, i \geq 1 \), and \( t^{B*} \) remain invariant in \( s \geq x \).

With risk-neutral contractors an additive random component is incorporated in a straightforward manner into the bid, as we show below via informal arguments (see Milgrom and Weber 1982 for complete details). Also consistent with the auction literature, we restrict attention to symmetric, monotone bidding strategies, characterized by the bidding function (or strategy) \( s(x) \).

In terms of our earlier discussion, \( s(x) \) is the function that returns the bid score that a bidder will target given that its zero-profit score is \( x \). This also means that a bidder with signal \( x \) bids \( s(x) \) and upon winning earns a conditional expected profit equal to \( s(x) - x \). From Milgrom and Weber (1982) there exists a unique Nash equilibrium strategy — see Esö and White (2004) and Lu and Perrigne (2008) for the case of stochastic outcomes.

Suppose the contractor of interest is indexed 1. Denote by \( Y_1 \) the lowest zero-profit score among the \((\nu - 1)\) contractors indexed \( 2, \cdots, \nu \). Suppose that each of these bidders submits a bid of \( s(y) \) upon observing its private cost signal to be \( y \). Because \( Y_1 \) and \( x \) are affiliated, we use \( G_\nu(\cdot | x) \) and \( g_\nu(\cdot | x) \) to denote the CDF and the PDF of \( Y_1 \), conditional upon Contractor 1’s signal being equal to \( x \). We also define

\[
L(a | x) = e^{-\int_a^x \frac{g_\nu(u | x)}{1 - G_\nu(u | x)} \, du}
\tag{19}
\]

because it helps explain how contractors choose their A+B scores in Proposition 2.

**Proposition 2.** An equilibrium bidding strategy, given contractor 1’s signal equals \( x \), can be ob-
\( s^* = x + \int_{\bar{x}}^{x} L(a \mid x)da. \)  

(20)

A proof of Proposition 2 is included in the Appendix. The first term in (20) is the contractor’s break-even bid score and \( \bar{x} \) is the maximum zero-profit score among all bidders. The second term shows how much the contractor *shades* its bid, given auction parameters and the level of competition from other bidders, which depends on both the number of bidders and the affiliation among their signals. The term \( L \) captures the effect of signal affiliation and competition. If the cost signals were not affiliated, then the second term above would simplify to the conditional expected zero-profit score of the bidder with the second lowest score minus \( x \) given that the tagged contractor’s score \( x \) is the lowest score. In that case (20) would give the optimal bidding strategy in the IPV setting. Affiliation among contractor signals causes each bidder to submit less aggressive bids (higher values of \( s \)) because when \( x \) is high, the tagged contractor would assume higher values of zero-profit scores of other contractors as well.

Equilibrium bidding strategies in the auction environment imply that competitive pressures, not time incentives, determine contractor profit. Anticipated windfall is incorporated into bids and \( x(\sigma) \). For example, setting \( c_I < c_D \) may impact expediting, sorting of contractors, and agency cost, but it does not substantially impact profit for the winning contractor. The latter are determined primarily by the number of bidders. Similarly, inflated estimates by engineers to provide budget for contingencies generate anticipated windfall for contractors, but do not increase project cost.

### 7 Policy Implications

Next, we discuss impact of the agency’s choice of time incentives on bid parameters. For this purpose, we define three concepts: social welfare, constrained social welfare, and lowest-cost bidder. We say that a contractor exerts *social-welfare maximizing* effort when it chooses completion time \( t^{**}(\xi) \) for each \( \xi \), where

\[
t^{**}(\xi) = t : \frac{\partial h}{\partial t} = -c_T.
\]

(21)

This is the amount of effort that the agency should expend to maximize social welfare if its expediting cost were given by the function \( h(t, \xi) \). Agencies often set \( c_U < c_T \) to lower project cost. We refer to the best effort with \( c_U < c_T \) as the *constrained social welfare*, which is realized when \( t^* = t_{0.2} \) for all realizations of \( \xi \). Put differently, constrained social welfare is the maximum social welfare if true RUC were \( c_U \). We observed in Section 5 that contractors internalize \( c_T \) via expediting effort when \( c_I = c_D = c_U \). We define the *lowest-cost contractor* under \( c_U \) to be that contractor among those that place bids that has the overall lowest labor, material, and expediting costs (i.e. \( c \cdot q + E(h(t_{0.2})) \)).

In the first corollary to Propositions 1 and 2, and Theorem 1, we claim that the agency can maximize social welfare by setting \( c_I = c_D = c_U = c_T \) and not capping incentives days.
Corollary 1. By setting \( c_I = c_D = c_U = c_T \), without limiting \( t_I \), the agency ensures that the contractor’s choice of expediting effort maximizes social welfare.

Proof: When \( c_I = c_D = c_U = c_T \) and \( t_I \) is not capped, \( t_{0,1} = t_{0,2} = t^{**} \). Consider a contractor whose private reputation cost is zero. In that case, from Equation (3), the contractor finishes at \( t_{0,1} = t_{0,2} = t^{**} \) regardless of \( t^B \). Therefore, it expends socially optimal effort.

Next, consider the case in which \( c_R > 0 \). Recall that each contractor’s private value of \( c_0 \) is assumed to be high enough that for every \( s \geq x \), \( t^{B*} \) is invariant in \( s \). This can occur only in cases 1, 2(a), and 3 shown in Theorem 1 (because in Case 2(b), \( t^{B*} \) increases in \( s \)). If either Case 1 or 3 occurs, it is straightforward to see from Equation (4) that \( t^* = t_{0,1} = t_{0,2} = t^{**} \) for every \( \xi \). If Case 2(a) occurs, then the expected profit increases either at rate \( c_T \) or \( \lambda_{m_0}c_T \geq c_T \) in \( t^{B*} \) (because \( m_0 \geq 1 \); see Proof of Theorem 1). But at \( t^B = t_{0,1} = t_{0,2} = t^{**} \), the rate at which expediting cost decreases in \( t^B \) is \( c_T \). Because \( h \) is a decreasing function of \( t \), it means that \( t^{B*} \geq t_{0,1} = t_{0,2} = t^{**} \) for every \( \xi \) and the contractor chooses effort to complete at \( t_{0,1} = t_{0,2} = t^{**} \). The contractor is assured to complete early and therefore avoid \( c_R \) so long as \( t^{B*} \in [\max_{\xi} \{t_{0,1}(\xi)\}, \mathcal{T}] \). Hence proved.

Intuitively, when \( c_R > 0 \) the contractor sets completion time by internalizing the RUC – i.e. by equating commuters’ cost of delays with its expediting cost. Although contractors with higher \( c_R \) lengthen their time bids, each contractor completes at its respective \( t_{0,1} = t_{0,2} = t^{**} \), ignoring \( c_R \), and incorporating incentives and expediting costs into its bid by adjusting \( b_0 \). Because \( t^* \) is determined solely by the tradeoff between the expediting cost and the RUC, the contractor’s expediting effort maximizes social welfare. Also, in this situation, the auction mechanism selects the lowest-cost contractor because the contractor with the smallest draw of \( x \) is also the lowest-cost contractor.

We argue next that if \( c_R > 0 \) and either \( c_I < c_D = c_U \) or \( t_I \) is limited, then there exists a possibility that the lowest-cost contractor under \( c_U \) will not be awarded the contract. Consider two contractors such that contractor 1 has higher \( (c \cdot q) \) but lower expediting cost. Suppose \( c_R \) is significant for both contractors, with the result that they select time bids to avoid being late. Their expediting effort in this case will be determined by equating marginal expediting cost with \( c_I \) (see Equation (4) and Figure 2). In this case, a lower value of \( c_I \) puts Contractor 1 at a disadvantage. In fact, it is possible that Contractor 1 would lose even though if the agency were to select \( c_I = c_U \), then Contractor 1 would have been the lowest-cost contractor.

While using the RUC as the daily incentive rate maximizes social welfare, it causes contractors’ bids to be higher because they include the cost of expediting and incentive payments in the A component of their bids. The resulting increase in cost may be prohibitive for the agency (Lewis and Bajari 2011a). As a result, it may be justified to set \( c_U < c_T \). Budget constraint (i.e. \( c_U < c_T \)) reduces social welfare and introduces the possibility that the contract may not be awarded to the lowest-cost contractor. However, budget-constrained solution gives rise to a useful benchmark, which we call constrained social welfare.

Corollary 2. When \( c_R = 0 \), the agency can realize outcomes consistent with constrained social
welfare by choosing \( c_I \leq c_D = c_U \). The optimal time bid \( t^{B^*} = t_1 \) and the contractor completes at \( t_{0,2} > t^{B^*} \) incurring tardiness penalties.

**Proof:** If a contractor has zero reputation cost \( (c_R = 0) \), then it maximizes its time-based incentives by setting \( t^{B^*} = t_1 \) because \( c_D = c_U \) (see Theorem 1 for formal arguments). After construction begins, the contractor faces marginal daily cost of \( c_U \). Hence, it chooses \( t^* = t_{0,2} \leq t_{0,1} \). Knowing its ex-post optimal action, the contractor incorporates the expected expediting cost and penalties in its bid via \( b_0 \). Weaker incentives from earlier completion are irrelevant and the auction parameters listed in the statement of the corollary achieve constrained social welfare. Hence proved. 

State transportation departments frequently set \( c_I < c_D = c_U \), with or without capping incentive payments. At times, no incentives are offered, i.e. \( c_I = 0 \). Corollary 2 shows that this should result in short time bids and late completion in the absence of reputation costs.

We consider non-negligible reputation costs and asymmetric incentives next, i.e. \( c_R > 0 \) and either \( c_I < c_D \) or a cap on \( t_I \). In this case, it is not possible to recommend the optimal time bid for the contractor without working out all cases listed in Theorem 1. However, the following general observations are possible. A longer time bid reduces the likelihood of tardiness, but weakens expediting incentives and lengthens completion time. Uncertain expediting cost implies that \( t^* \) may differ from \( t_{0,2} \) with the implication that constrained social welfare may not be attained. As the difference between \( c_D \) and \( c_I \) increases, so does the difference between \( t_{0,1} \) and \( t_{0,2} \). All other parameters remaining invariant, this increases the likelihood that the contractor will complete exactly at \( t^B \) (see Proposition 1).

Consider the implications of maximizing the constrained social welfare. If \( c_I = c_D = c_U = c_T \), contractors internalize the cost of delays to the commuting public, and equate the marginal cost of expediting with \( c_T \), maximizing social welfare. This gives a competitive advantage to contractors with lower expediting costs. However, if daily incentive and disincentive rates are equal to \( c_U < c_T \), then upon placing optimal time bids, contractors expedite less with the following implications. First, completion is delayed at the expense of the commuting public. Second, a contractor with higher construction cost and lower expediting costs is less competitive when facing \( c_I = c_D = c_U < c_T \), potentially changing the sorting of contractors relative to daily rates of \( c_T \). This may cause the lowest-cost contractor (under \( c_T \)) to lose the contract, when facing rates equal to \( c_U \). As implied by Proposition 2 misspecification of incentive rates does not increase profit for the winning contractor.

The analysis of line-item bids in Theorem 1 indicates that when contractors’ quantity estimates differ from engineer’s estimates, line item bids are extreme. This raises the possibility of modifying the A component of bids to lump-sum and not allowing quantity adjustments based on actual usage. While this change eliminates windfall payments associated with biased bids, differences between engineer’s estimates and actual usage remain, with contractors bearing this risk. To limit possible losses contractors would raise initial bids for the A component. Moreover, substantial changes in line-items would lead to frequent renegotiation, lengthening projects. For these reasons, both agency engineers and contractors favor unit bids over lump-sum.
8 Concluding Remarks

The need for substantial investments in transportation infrastructure enhancements and commuters’ inconvenience from road closures necessitate the use of innovative mechanisms to affect faster rehabilitation. This paper analyzes in depth the A+B mechanism, which provides time-based incentives and has been demonstrated to shorten project duration. This is the first paper to comprehensively analyze A+B bidding strategies in a normative model with multi-attribute private information and multidimensional bids, residual uncertainty at the time of bidding, and reputation costs. Our detailed analysis provides guidance for agencies — agencies should equate all time-based parameters and not cap incentives. This is a departure from current practice, where incentives are weaker than penalties and incentives are often capped.

Analyzing contractor response to auction parameters clarifies the implications from current practices. First, $c_I < c_D$ increases the likelihood that the project is not awarded to the lowest-cost contractor. Second, substantial private reputation costs lead to contractors submitting long bids and frequently completing the project either on time or early. In the absence of reputation costs, uncertainty at the time of bidding would lead to short bids and late completion. Third, line-item bidding induces contractors to submit extreme bids. Data on A+B contracts is consistent with the above-mentioned theoretical findings. Finally, reputation costs are, to some extent, dependent on environmental factors and agency policies, which may change over time. For example, if A+B were to become a more prevalent contracting mechanism, bonding companies might not attach as much weight to tardiness in A+B projects when assessing a company’s credit-worthiness. Conversely, agencies may increase the use of Design-Build or incorporate past tardiness into A+B scores. The former may lower reputation costs, whereas the latter may serve to increase them.

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References


A Summary of MnDOT A+B Bids’ Data

We analyzed data from MnDOT for all 38 projects that were let using the A+B mechanism during April 2000 and August 2008. We had different levels of details for different projects. Basic information on all projects included the number of bidders, the A and B components, the total bid scores of each bidder, and the agency specified road user cost and time parameters. The smallest winning bid was $911,335 and the largest was $128 million. Additional information that could be obtained for subsets of projects was as follows. We had actual completion times for 27 projects, line-item unit bids for 22 projects, and quantities consumed, change orders, and schedule of payments made to the contractors for 15 projects. This happened because different types of data are stored in different databases and MnDOT’s central office from which we obtained the data does not have complete records of all projects, particularly those handled by district offices. A summary of project parameters is presented in Table 3. Projects for which $c_I, t_I,$ and $t$ were zero are not included in the summary statistics for those variables. For example, a non-zero incentive rate $c_I$ was specified in only 21 projects. The statistics for $c_I$ in Table 3 pertain to those 21 projects.

Among the 27 projects for which we knew the actual number of days it took to complete, the maximum bid days ranged from 15 to 227 with a mean of 95.63 whereas winning bidders’ time
Table 3: Summary of Relevant Project Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Sample Size</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu$</td>
<td>38</td>
<td>2</td>
<td>11</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>$c_U$</td>
<td>38</td>
<td>$3,000$</td>
<td>$28,000$</td>
<td>$9,845$</td>
<td>$10,000$</td>
</tr>
<tr>
<td>$c_D$</td>
<td>38</td>
<td>$3,000$</td>
<td>$30,000$</td>
<td>$9,779$</td>
<td>$10,000$</td>
</tr>
<tr>
<td>$c_I$</td>
<td>21</td>
<td>$2,000$</td>
<td>$10,000$</td>
<td>$7,362$</td>
<td>$7,500$</td>
</tr>
<tr>
<td>$t_e$</td>
<td>38</td>
<td>15</td>
<td>1067</td>
<td>134.7</td>
<td>82.5</td>
</tr>
<tr>
<td>$t$</td>
<td>38</td>
<td>15</td>
<td>1067</td>
<td>135.1</td>
<td>82.5</td>
</tr>
<tr>
<td>$t_I$</td>
<td>8</td>
<td>15</td>
<td>987</td>
<td>266.4</td>
<td>125</td>
</tr>
<tr>
<td>$t_{II}$</td>
<td>19</td>
<td>5</td>
<td>40</td>
<td>16.1</td>
<td>10</td>
</tr>
</tbody>
</table>

bids ranged from 6 to 216 days with a mean of 80.3 days. The actual number of days it took to complete these projects ranged from 5.5 to 194 with a mean of 75.2 days.

MnDOT specified a maximum completion time (in days) for all 38 projects. In all but one instance, the maximum completion time was set equal to the engineers’ estimate of completion time, i.e. $t^* = t_e$. This means that in the vast majority of cases, contractors must bid completion time that is equal to or shorter than MnDOT’s estimate of the time it would take to complete the project without extra completion effort. In contrast, a minimum time was specified in 8 projects.

Of the 37 projects that met the criterion that $c_D \leq c_U$, 17 projects had $c_I = c_D = c_U$, 16 had $c_I = 0$ and $c_D = c_U$, 2 had $0 < c_I < c_D = c_U$, 1 had $c_D < c_I = c_U$, and 1 project had $c_I = c_D < c_U$. Furthermore, both $t_I$ and $t$ were specified in only 2 out of the 37 projects and the value of $t_I$ ranged from 5 to 40 days. These statistics show a significant variation in auction parameters, supporting our claim that agency engineers lack guidance on how to set these parameters.

Among the 17 projects in which $c_I = c_D = c_U$ was specified, we have actual completion times in 14 cases. Within these 14 projects, the winning contractor finished on time or slightly earlier than bid in 11 cases. This supports our inference that contractors face moderate reputation costs.

## B Proof of Theorem 1

The KKT necessary conditions are as follows:
\[\eta(t^B) - \lambda c_U + \gamma_1 - \gamma_2 = 0.\] (22)
\[q_0 - \lambda q_0^e + \mu_3 = 0.\] (23)
\[\forall i \geq 1, \quad q_i - \lambda q_i^e - \mu_{1,i} + \mu_{2,i} = 0.\] (24)
\[\lambda(s - (c_U t^B + b \cdot q^e)) = 0.\] (25)
\[\forall i \geq 1 \quad \mu_i,1(b_i - \eta_i) = 0.\] (26)
\[\forall i \geq 1 \quad \mu_{i,2}(-b_i + \omega_i) = 0.\] (27)
\[\mu_3 b_0 = 0.\] (28)
\[\gamma_1(-t^B + t) = 0.\] (29)
\[\gamma_2(t^B - 7) = 0.\] (30)

From (23) and the fact that \(q_0 = q_0^e = 1\), we get \(\lambda = 1 + \mu_3 \geq 1\) because \(\mu_3 \geq 0\). Similarly, from (22), we get that \(\gamma_1 - \gamma_2 = \lambda c_U - \eta(t^B)\). Because \(\gamma_1\) and \(\gamma_2\) cannot both be positive, either \(t^B = t\) or \(t^B = \tilde{t}\), or \(t < t^B < \tilde{t}\). We consider each of these three cases separately.

**Case 1:** \(t^B = t\)

If \(t^B = t\), then there must exist a \(\lambda \geq 1\) such that \(\lambda c_U - \eta(t) \geq 0\). From (24) we find that \(\mu_{i,1} = q_i^e (q_i^e - \lambda)\) for all \(i = 1, \ldots, n\). This implies that: (1) If \(q_i^e > \lambda\), then \(b_i^* = \eta_i\) because \(\mu_{i,1} = 0\), \(\mu_{i,j}\) are non-negative and only one of the \(\mu_{i,j}\)'s can be strictly positive. (2) And, for reasons similar to those mentioned above, if \(q_i^e < \lambda\), then \(b_i^* = \omega_i\).

In addition to requiring \(\lambda \geq 1\), we observe that value of \(b_i^*\) changes only when \(\lambda\) belongs to the set \(\{\lambda_1, \ldots, \lambda_m\}\), where \(m \leq n\) is the number of items for which \(q_i/q_i^e \geq 1\). That is, for the purpose of choosing optimal bid parameters, we need to consider at most \(n\) possible values of \(\lambda\). Suppose we pick \(\lambda = \lambda_{m_0} \geq 1\). If \(q_i^e < 1\) for all \(i\), then set \(m_0 = 0\). Then, we claim that \(m_0\) must be such that \(b_i^* = \omega_i\) for all \(i < m_0\), \(b_i^* = \omega_i\) for all \(i > m_0\) and

\[b_{m_0}^* = \min\{\omega_{m_0}, [s - c_U t^B - \sum_{i=m_0+1}^n \omega_i q_i^e - \sum_{i=1}^{m_0-1} \omega_i q_i^e]/q_{m_0}\}.\] (31)

If \(m_0 = 0\), then (31) gives the value of \(b_0^*\). Otherwise, if the value of \(b_{m_0}^*\) calculated in (31) is such that \(\omega_{m_0} \leq b_{m_0}^* < \omega_{m_0}\), then \(b_0^* = 0\). If \(b_{m_0}^* = \omega_{m_0}\), then \(b_0^* \geq 0\), where the latter can be calculated from the fact that \(s = b_0 + \sum_{i=1}^n b_i^* q_i^e + c_U t^B\). Finally, if \(b_{m_0} < \omega_{m_0}\), then the current choice of \(t^B\) is infeasible. We are now ready to present an algorithm that can be used to find an optimal set of bid parameters for each fixed \(s\). The algorithm searches over possible values of \(m_0\).

**Step 1** Choose \(t^B = t\), reorder item indices such that \(q_1/q_1^e \geq q_2/q_2^e \geq \cdots \geq q_m/q_m^e\), and define \(m\) as the number of items for which \(q_i/q_i^e \geq 1\). This means that \(q_i/q_i^e < 1\) for items indexed \(m + 1, \ldots, n\).

**Step 2** For all items for which \(q_i/q_i^e < 1\), set \(b_i^* = \omega_i\). Set current iteration index \(m_0\) equal to \(m\).
Step 3 Attempt to set $b_i^* = \bar{c}_i$ for all items indexed 1 through $m_0 - 1$. If upon doing so, it is possible to set $b_{m_0}^*$ according to Equation (31), $b_i^* = \underline{c}_i$ for each $i > m_0$, and $b_0^* \geq 0$, then this is an optimal set of parameters. Record the solution and stop. Otherwise, go to Step 4.

Step 4 If the current iteration index is greater than 1, set current iteration index equal to previous index minus 1 and return to Step 3 above. Otherwise stop.

If $\bar{\ell}$ is a potential solution, then the above algorithm will return a solution. Otherwise, we conclude that $t^{B*}$ cannot equal $\bar{\ell}$. Note that this occurs when either $\lambda_1 \geq 1$ and $\lambda_1 c_U < \eta(1)$, or $\lambda_1 < 1$ and $c_U < \eta(\bar{\ell})$.

Case 2: $\underline{\ell} < t^{B*} < \bar{\ell}$

In this case $\gamma_1 = \gamma_2 = 0$, $\lambda \geq 1$ and $t^{B*}$ is determined from the equality $\eta(t^{B*}) = \lambda c_U$.

Case (a): $\lambda \in \{\lambda_1, \ldots, \lambda_m\}$. For each value of $\lambda \geq 1$, there is a corresponding value of $t^{B*}$, which comes from the equality $\eta(t^{B*}) = \lambda c_U$. Using (24), we have that if $\lambda = \lambda_{m_0}$, then $q_i/q_i^e \geq \lambda$ for all $i < m_0$, $q_i/q_i^e \leq \lambda$ for all $i > m_0$. If $\lambda_1 < 1$, then the value of $t^{B*}$ is determined from $\eta(t^{B*}) = c_U$ and in this case $m_0 = 0$. We do not discuss this case in detail because its analysis is similar to the treatment presented in Case 1.] Therefore $b_i^* = \bar{c}_i$ for all $i < m_0$, $b_i^* = \underline{c}_i$ for all $i > m_0$ and

$$b_{m_0}^* = \min\{\bar{c}_{m_0}, [s - c_U t^{B*} - \sum_{i=m_0+1}^{n} \underline{c}_i q_i^e - \sum_{i=1}^{m_0-1} \bar{c}_i q_i^e]/q_i^e_{m_0}\}.$$  \hfill (32)

If the value of $b_{m_0}^*$ calculated in (32) is such that $\underline{c}_{m_0} \leq b_{m_0}^* \leq \bar{c}_{m_0}$, then $b_0^* = 0$. If $b_{m_0}^* = \bar{c}_{m_0}$, then $b_0^* \geq 0$ and the latter can be calculated from the fact that $s = b_0 + \sum_{i=1}^{n} b_i^* q_i^e + c_U t^{B*}$. Finally, if $b_{m_0} < \underline{c}_{m_0}$, then the current choice of $t^{B*}$ is infeasible.

Solving for potential bid parameters in Case 2(a) is an iterative process. Beginning with $\lambda = \lambda_m$, we iteratively choose the next higher value of $\lambda \in \{\lambda_1, \ldots, \lambda_m\}$ and identify all solutions that satisfy the first-order optimality equations. At the end of this process, multiple optimal values of $t^{B*}$ are possible and corresponding to each such value, there will be a different set of optimal $b^*$ values.

Case (b): $\lambda \notin \{\lambda_1, \ldots, \lambda_m\}$. In this case, Using (24), we have that if $\lambda_{m_0+1} < \lambda < \lambda_{m_0}$, then $q_i/q_i^e > \lambda$ for all $i \leq m_0$, $q_i/q_i^e < \lambda$ for all $i > m_0$. Therefore $b_i^* = \bar{c}_i$ for all $i \leq m_0$, $b_i^* = \underline{c}_i$ for all $i > m_0$. In addition, by (23) we also have that $b_0 = 0$. Therefore, $t^{B*}$ is calculated by $t^{B*} = [s - \sum_{i=m_0}^{n} \underline{c}_i q_i^e - \sum_{i=1}^{m_0-1} \bar{c}_i q_i^e]/c_U$. If $t^{B*} \notin [t_{\min}, \bar{t}]$, $t^{B*}$ is not feasible. If $t^{B*} \in [t_{\min}, \bar{t}]$, then $t^{B*}$ is feasible and (22) is satisfied. The case $m_0 = 1$ is the same.

Solving for the potential bid parameters in Case 2(b) is similar to the procedure in Case 1, except that we choose $t^{B*}$ after choosing all $b_i^*$'s.

Case 3: $t^{B*} = \bar{t}$

Given $t^{B*} = \bar{t}$, we can argue that there must exist a $\lambda \geq 1$ such that $\lambda c_U - \eta(\bar{t}) \leq 0$. The difference between this case and Case 1 is that here we have an upper bound on possible value of $\lambda$, whereas we had a lower bound in Case 1. This case is not feasible if either $\lambda_{m_0} c_U - \eta(\bar{t}) > 0$, or $\lambda_1 < 1$ and $c_U - \eta(\bar{t}) > 0$. In other respects, the procedure for finding optimal $b^*$ is identical to
Case 1. Therefore, we omit the details. After solving for optimal bid parameters in the three cases, the choices that give rise to maximum overall profit will be the optimal choice.

C Parameters for the Example in Figure 2

The parameters used to develop the example in Figure 2 are as follows:

\[
\begin{align*}
q^e &= [430, 10880, 83, 493, 11938, 2914, 850, 47667, 64, 22035] \\
q &= [516, 11968, 83, 493, 11938, 2914, 850, 47667, 58, 17628] \\
c &= [405, 45, 100, 35, 16, 2.5, 85, 3.6, 22035] \\
c &= [700, 56, 392, 47, 23, 4.2, 250, 7, 310, 40] \\
t &= 10 \\
\bar{t} &= 194 \\
t_I &= 20 \\
c_U &= 7500 \\
c_D &= 7500 \\
c_I &= 7125
\end{align*}
\]

Note that in this example, \( \lambda = 1.2, 1.1 \) and 1. The target value of A+B score is set at \( s = q^e \cdot c + 94 \times c_U \), where the first product is a dot product. The expediting cost function is set as follows:

\[
h(t, \xi) = 30 \times (t - 194)^2 I_{\xi = 1} + 30 \times (t - 210)^2 I_{\xi = 2} + 45 \times (t - 210)^2 I_{\xi = 3}
\]

where \( P(\xi = 1) = 1/3, P(\xi = 2) = 1/2, P(\xi = 3) = 1/6 \). Finally, \( c_R \) changes from 0 to \( 25 \times c_U \).

D Proof of Proposition 2

If Contractor 1 upon receiving signal \( x \) bids \( s(z) \), i.e. behaves as if its cost signal were \( z \), then it wins only if \( s(z) < s(Y_1) \). Therefore, its expected profit is:

\[
\pi_1(z, x) = \int_{z}^{x} \left[ \int_{-\infty}^{\infty} (s(z) - x + \epsilon)dD(\epsilon) \right] g_{\nu}(y \mid x) dy
\]

\[
\quad = s(z)[1 - G_{\nu}(z \mid x)] - \int_{z}^{x} xg_{\nu}(y \mid x) dy.
\]

To find the profit-maximizing bidding strategy we use the first-order-condition:

\[
\frac{\partial \pi_1(z, x)}{\partial z} = s'(z)[1 - G_{\nu}(z \mid x)] + [z - s(z)]g_{\nu}(z \mid x) = 0.
\]
In a symmetric equilibrium it must be that an optimal value for $z$ is $x$. Upon setting $z = x$ and rearranging, we obtain the following differential equation that determines the optimal bid:

$$s(x) = x + \frac{s'(x)[1 - G_\nu(x | x)]}{g_\nu(x | x)}.$$ 

Solving the above differential equation with the boundary condition that $s(\bar{x}) = \bar{x}$, leads to the equilibrium bidding strategy. Upon using the definition of $L(a|x)$ (see 19), we obtain the statement in Proposition 2.