Speed and Efficiency in Transportation Construction
Procurement Auctions

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June 1, 2009
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Abstract

Speedier contract execution is increasingly important in the procurement of transportation construction services. However, academic literature has paid little attention to speed as a factor in contract mechanism design. The focus of this article is on mechanisms that use proposed cost (A component) and proposed time (B component) to score contractors’ bids. These mechanisms, also called A+B bidding, have been adopted by several state transportation agencies. We develop mathematical models to study the A+B mechanism, and show that it does not lead to truthful revelation of contractors’ unit costs and completion times. Contractors’ incentives are to bid a shorter completion time than expected. Payments based on adjustments in quantities after construction is completed and time-based incentives generate windfall profits for contractors. These are incorporated strategically into their bids, with the result that the lowest-cost bidder does not necessarily win. However, contractors do exert optimal effort to achieve lower completion time. Inaccurate revelation complicates scheduling of project engineers’ time with oversight on such projects and scheduling of interdependent activities. It could also have adverse impacts on transportation agencies if contractors start bidding according to the equilibrium bidding strategy.
1 Introduction

Federal and State agencies spend billions to procure goods and services — this activity is estimated to be approximately 10% of the GDP in developed economies (McAfee and McMillan 1989). Government procurement is subject to a variety of regulations; see, for example, Subchapters B, C and F of the Federal Acquisition Regulation (2006). The focus of this paper is on the procurement of transportation construction services by state transportation agencies (STAs), although its managerial insights could be more widely applicable to other similar procurement scenarios. Transportation construction is a significant economic activity. According to the US Census Bureau (2009), the January 2009 estimate of seasonally adjusted annual value of state and local dollars committed to highway and bridge construction was $50,989 million and $24,256 million, respectively. Nearly all such construction work is performed by contractors and the vast majority of state agencies use first-price sealed-bid procurement auctions to award construction contracts.

In recent years, the need for speedier contract execution has become a prominent issue in the procurement of construction services for the military, in response to natural disasters, and in the rebuilding of the transportation infrastructure. The criticality of speed when dealing with wartime troop support and disaster response services is obvious. For building highways and bridges, speed has become important because of the negative public response to the duration of road closures, which increase commute times and cause traffic jams, and because of the escalating fuel and raw material prices, which can lead to significantly higher costs when projects take longer to complete.

Speed pertains to two different but interdependent ideas: speed of contract letting, and speed of work completion. Each affects contract mechanism design in different ways. In this paper, we are concerned with the speed of work completion. In spite of a demonstrable need to shine light on contract mechanism design in such environments, academic literature has paid little attention to speed as a factor. This is not entirely surprising because the design of multi-attribute procurement auctions is a difficult problem. STAs have responded by experimenting with and developing project letting mechanisms that have not been studied in detail in the academic literature. One such mechanism is referred to as the A+B bidding in transportation construction industry; see Innovative Contracting in Minnesota 2000–2005 for examples of projects let using the A+B mechanism. We focus on that in particular, and on multi-attribute procurement auctions in general, in this paper.

A state agency that uses A+B mechanism selects the winner based on two components of each contractor’s bid, called A and B. The request for proposals includes a detailed product design, a list of all pieces of work, and engineers’ estimates of materials required to complete each piece of work. Each contractor submits unit bids (material and labor costs) for each piece of work. Unit bids are multiplied by quantity estimates announced by STA engineers and added to obtain the A component of each contractor’s bid. The B component is the completion-time budget which is obtained by multiplying a state agency specified daily road user cost ($c_{U}$) with the number of calendar days in which the contractor proposes to complete the project. The winning bid has the
The contractor who performs the work is paid an A component which equals the sum of its unit bids multiplied by actual quantities and any approved changes in work scope. Changes in scope of work usually arise as a result of unanticipated utility work or discrepancies between the assumptions underlying the engineers’ estimate and actual geological properties of the work site. The contractor is also paid a B component, which consists of either an incentive or a penalty, depending on under- or over-runs relative to the completion-time budget. Viewed in this light, A+B bidding can be described as multi-attribute procurement auctions with quantity, work scope, and completion-time based payment updates. In contrast to this, standard contracts, which are called A-only contracts in this paper, have no B component. STAs specify a maximum completion time after which contractors may be required to pay for extra supervisory effort by project engineers, but there are no incentives for earlier completion. Note, quantity and work-scope related adjustments apply to A-only contracts as well.

In this paper, we analyze the effectiveness of the A+B bidding mechanism and address the following questions. Do A+B contracts lead to faster project completion? How much do the state agencies pay for shorter completion times? Given the winner selection and payment mechanisms described above, how should a contractor bid? Is the A+B mechanism efficient, i.e. does the lowest-cost contractor win? Assuming that state agencies can estimate the true cost of disrupting traffic during construction periods, how should they set road user costs and incentive/disincentive rates?

We use mathematical models to determine the contractors’ equilibrium bidding strategies as a function of STAs’ choices of parameters such as incentive/disincentive rates, daily road user costs, and the maximum number of incentive days. Because final payments are different from bid amounts as a result of quantity and scope-related adjustments, contractors have an opportunity to earn a windfall. The windfall calculations are factored into equilibrium bids in both A-only and A+B mechanisms. We show that depending on the relative magnitude of the windfall, the lowest-cost bidder may not win. We also show that the maximum number of incentive days is irrelevant for the purpose of determining equilibrium bids and that an equilibrium bidding strategy for contractors is not to reveal their true unit costs or their truthful estimate of project completion time. Strategic bids for unit prices stem from the differences in quantity estimates by contractors and STA engineers.

Inaccurate time bids present a challenge to STAs who may find it difficult to budget project engineers’ time for project oversight and management activities. A+B bidding does, however, provide incentives for the winning bidders to expend an optimal level of effort on expediting project completion. Our results are based on sophisticated mathematical arguments that are difficult to anticipate entirely by intuition. However, we do provide intuitive explanations for our results throughout the paper.

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1The incentive is usually capped by specifying a maximum number of incentive days for which the daily incentive is paid.
STAs typically set daily road user costs equal to the daily disincentive rate, both of which are set greater than the daily incentive rate. Under these conditions, equilibrium bidding strategies have contractors placing bids with early completion dates and completing later than the bid dates. Analysis of data reveals that contractors are not as yet bidding according to this strategy and recent successes of large-scale A+B construction projects (Landers 2005, Foti 2008) have generated praise for STAs. In the future, bids can be expected to be more similar to the equilibrium identified here. Frequent completion delays using A+B contracts could have unfavorable effects on the perception of STAs by the general public and elected officials. We provide a comprehensive analysis of the strategic behavior induced by A+B auction design, which is essential for developing policy in the area.

We turn next to a review of economics and construction management literature dealing with procurement. Competitive bidding typically involves a single buyer and several sellers (monopsony). This contrasts with an auction that typically has a single seller and several buyers (monopoly). However, auctions and bids are mathematically the same, with the reversal of signs of certain variables, and there is a vast economics and management science literature on this topic. Excellent surveys on auctions, bids, and the use of incentives in procurement can be found in Engelbrecht-Wiggans et al. (1983), McAfee and McMillan (1987), Laffont and Tirole (1993), and Krishna (2002). Therefore, we shall present an abbreviated review and focus on how this paper differs from previous work.

Within the context of procurement auctions, we emphasize sealed-bid first-price auctions, as that is the methodology exclusively employed by STAs. Moreover, the winner of the auction is awarded the contract, eliminating the need to analyze post-auction negotiations (Engelbrecht-Wiggans and Katok 2006). In construction auctions bidders receive a multi-attribute signal regarding their costs. If these costs are aggregated to a single price, we have a traditional sealed-bid auction (Snir and Hitt 2003). However, STAs frequently adjust final payment based on realized quantities, in effect converting the auction to the unit-price auction studied in Ewerhart and Fieseler (2003). In their highly stylized model they assume that bidders’ information is private-value and find that ex-post quantity adjustments impact bidding behavior. The gap between a bidder’s quantity requirement and the buyer’s estimate determine bidding strategies leading to imbalanced bids, meaning that some inputs are priced below cost while other are priced above cost. Adjusting quantities at the end of construction emphasizes the multi-attribute nature of the bidder’s signal.

Maskin (1992) shows that multi-attribute signals result in inefficient allocations, where the low cost contractor is not awarded the contract. Ewerhart and Fieseler (2003) similarly find that unit-price auctions are also inefficient. In addition, they argue that these auctions can reduce buyer’s cost, since they subsidize high-cost contractors, requiring more aggressive bids by low-cost providers. Multi-attribute auctions also outperform single-attribute auctions in an experimental setting (Chen-Ritzo et al. 2005).

Che (1993) studies multi-dimensional auctions using a scoring rule, to compare different auction
mechanisms, assuming bidders costs are private-values. Similar to our study, Che finds that scoring rules generate strategic bidding, where bids on sub-components are chosen to minimize the score evaluated by the buyer. If the buyer can commit to a scoring rule, he maximizes profit by choosing a lower score for quality than is socially optimal. We find a similar result where STAs may choose to score completion time at a rate that is less than the actual user cost for the roadway. Bajari and Lewis (2009) study A+B construction contracts assuming independent private values and similar to our paper, they find that time-based incentives produce the desired effect on completion times. However, Bajari and Lewis’s model does not account for the windfall created by the bidding mechanism, the contractor’s choice of completion effort, and the fact that the lowest-cost bidder may not win. Our approach is also different from theirs. We build a theoretical model and relate some of its predictions with the data. In contrast, Bajari and Lewis (2009) carry out largely an empirical study.

Contractors can arrive at their costs independently, i.e., their production technologies and raw-material sources may be sufficiently different to warrant an assumption of independence. In the auctions literature, this is called independent-private-values’ assumption. On the other hand, contractor costs can be correlated, for example, when costs depend on a common technology or purchases are from similar suppliers of raw materials. In these cases, it is appropriate to assume that the marginal distributions of contractors’ costs are positively associated (Müller and Stoyan 2002). These models are referred to as common-values’ models.

When contractors’ costs depend on the price of a key common input (common-values’ model), the winner can suffer from the winner’s curse – since the winner’s ex ante lower cost estimate may signal that it will likely incur a higher cost ex post and lose money on the contract. Milgrom and Weber (1982) show that if the buyer has independent information about costs, with which the contractors’ cost estimates are correlated, then the buyer can increase its welfare by publicizing this information. However, bidders value the privacy of their information. In general, it is more difficult to solve for equilibrium bidding strategies for common values’ procurement auctions and equilibrium bidding strategies may not exist (Jackson 1999). Goeree and Offerman (2003) develop a model and derive equilibrium bidding strategies when a bidder’s signal involves both a private-value component and a common-value component. Under the assumptions of their model information dissemination by the buyer reduces winner’s curse and increases buyer profit. The importance of dissemination is greatest when the common-value component is large.

The Operations Management (OM) literature has paid little attention to the problem of contract mechanism design for the procurement of transportation construction services, which has certain domain-specific features that make the adoption of general-purpose techniques inappropriate (details in Section 2). Related practitioner literature can be found on the web and in reports published by the Federal Highway Administration (FHWA) and the American Association of State Highway and Transportation Officials (AASHTO) (fhwa.dot.gov and transportation1.org/aashtonew/default.aspxaashto.gov). Reports posted on these web pages describe various experimental con-
tracting schemes and the experience of early adopters. No model-based evaluation is carried out and there are no guidelines for setting important contract parameters. Moreover, it is not clear that success of early adopters can be sustained in the future. There is a significant amount of work reported in OM literature on procurement issues arising in supply chains. Papers that deal with such issues typically model supplier-buyer interactions as bilateral negotiations with one of the two players acting as the principal and the other as the agent (Elmaghraby 2000). These models do not carry over to the procurement problems faced by STAs, which explains the need for this paper.

The remainder of this paper is organized as follows. In Section 2, we present an analysis of data we obtained from the Minnesota Department of Transportation (MnDOT), which serves as the basis for the mathematical models of the decision problems faced by contractors and STAs (referred to as owners in the transportation industry). These models are also included in Section 2. Analyses of these models can be found in Section 3. We summarize findings, insights and model implications in Section 4 and conclude the paper in Section 5.

2 Model Formulation

This section has three parts. In the first part, we present a preliminary analysis of MnDOT data. This analysis serves as the basis for developing formal models from the viewpoints of the owner and the contractor separately in parts two and three. We formulate the contractors’ problem assuming independent private values. This leads to a tractable model. We justify this assumption in Section 2.3 and discuss extensions of our models to common values framework in Section 5.

2.1 The Data

We obtained a convenience sample of 414 standard (i.e. A-only) projects’ data from MnDOT and an exhaustive sample of 25 projects that used A+B mechanism. Among the A-only projects, the smallest winning bid was $70,723, the largest was $86,396,096, the mean bid amount was $3,203,130 and the standard deviation of bid amounts was $7,384,337. The total value of all bids was $1.33 billion dollars. Similarly, among the 25 A+B projects, the mean winning bid was $8,127,099, and the minimum and maximum winning bids were $911,335 and $35.2 million. Because the A+B mechanism is relatively new, fewer projects have been completed that were let using the A+B mechanism. Among the A-only projects, median number of bidders was 5 with 25% of the projects having 4 or less bidders and another 25% having more than 7 bidders. Similarly, among the A+B contracts, median number of bidders was 4 with 25% of the projects having 3 or less bidders and 25% having more than 5 bidders.

For all projects, MnDOT develops an engineers’ estimate of project cost. For A+B bids, the engineers’ estimate also includes the B component. Engineers’ quantity and time estimates are included in the project description, but cost estimates are reveled to the contractors only after the
winning bid is determined and bid abstracts are posted. As noted earlier, the actual amount paid to the contractor is subject to updates. Thus, each project can be studied in terms of three key numbers — the winning bid, the total amount paid to the contractor, and the engineers’ estimate. To summarize projects in terms of these quantities, we calculated two financial ratios: Ratio 1 = (total amount paid)/(the A-component of the winning bid), and Ratio 2 = (total amount paid)/(the A-component of the engineers’ estimate). Table 1 below presents summary statistics for both these ratios for both types of projects. We dropped three A-only projects for these calculations and from all subsequent analyses because in these projects either Ratio 1 was less than 0.5 or Ratio 2 was greater than 2. Very small or very large ratios indicated that those projects were either not completed or ran into serious technical problems during execution.

Table 1: Summary of financial ratios for A-only and A+B contracts.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Obs</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A-only Projects</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ratio 1</td>
<td>411</td>
<td>1.059894</td>
<td>.1213029</td>
<td>.6341097</td>
<td>1.710829</td>
</tr>
<tr>
<td>Ratio 2</td>
<td>411</td>
<td>.9918718</td>
<td>.2098435</td>
<td>.4043764</td>
<td>1.843921</td>
</tr>
<tr>
<td><strong>A+B Projects</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ratio 1</td>
<td>25</td>
<td>1.067715</td>
<td>.070143</td>
<td>.9492269</td>
<td>1.238371</td>
</tr>
<tr>
<td>Ratio 2</td>
<td>25</td>
<td>1.055219</td>
<td>.1346407</td>
<td>.8172323</td>
<td>1.415998</td>
</tr>
</tbody>
</table>

Note that A+B mechanism is designed such that the winning bidders are not meant to be paid the B component. Instead, they receive either an incentive or a disincentive if they complete the project early or late. Therefore, it makes sense to take the ratio of total amount paid to the A-component of the winning bid or the A-component of the engineers’ estimate for the A+B projects. Table 1 shows that contractors were paid 5-7% more, on average, as compared to the amount they bid. More precisely, all ratios except Ratio 2 for A-only projects are statistically larger than 1 (at 0.05 significance level). A t-test (unequal variances) of comparison of the means of Ratio 1 for A-only and A+B projects was not significant (p-value = 0.3057 for one-sided test). However, a similar test for Ratio 2 was significant (p-value = 0.0178 for one-sided test).

The above comparison of financial ratios shows that for both types of projects, the contractors are able to extract a greater payment than their bid amounts. As we explain later in this paper, the extra payment comes from a windfall that arises from the contract design. The opportunity to earn a windfall is similar in both A-only and A+B projects with the result that the amount of this windfall relative to the winning bid is not significantly different under the two mechanisms. However, the winner earns significantly more in A+B projects relative to the engineers’ estimate, thereby suggesting that they do cost more. We revisit this issue in Section 4.

Next, we investigated the amount by which the contractors under- or over-bid relative to the actual completion time. The mean, standard deviation, minimum and maximum difference between the actual completion time and bid time were -1.149891, 9.491529, -32, and 17.05273. That is,
contractors, on average, completed the projects faster than the bid time. However, this is not a statistically significant result. A hypothesis test in which the hypothesized mean of the difference between actual completion and bid time was set to zero showed that the null hypothesis could not be rejected ($p$-value = 0.55). Figure 1 below shows the distribution of the difference between actual completion and bid time. It confirms the lack of a pattern. In particular, of the 25 projects, the actual completion time was smaller than the bid time in 12 projects, equal to bid time in 6 and greater than bid time in 7 projects.

![Figure 1: Winning bidders’ completion time behavior.](image)

We examined the ratios of the dollar amounts of non-winning bids to the winning bids to better understand the bid spread. The mean bid spread was found to generally increase with the bidder’s rank among those bidding, though this pattern was not uniform. The mean bid spread ranged from 1.07 to 1.96 among A-only projects and from 1.06 to 1.21 among A+B projects. This suggests that there exist substantial differences in contractor’s estimates of overall project costs.

### 2.2 The Owner’s Problem

We use upper case letters to denote random variables and sets, except when they are used as subscripts. The letters $c$ and $t$, with various subscripts and superscripts, denote costs and time, respectively. Vector dimensions are not marked; they should be clear from the context. The owner (i.e. STA) posts a request for bids for a particular project with the following information.
1. Project description, design, and scope of work.

2. The fact that A+B method will be used to determine the winning bid.

3. Estimated quantities of materials required to complete each piece of work in the project scope. We denote this by the vector $q^e$, where $q^e = (q^e_0, q^e_1, \ldots, q^e_n)$, $q^e_0 \equiv 1$, $q^e_i$ is the estimated quantity of materials needed for the $i$th piece of work and there are $n$ distinct pieces of work. The 0th item not a consumable. It is a place holder for fixed costs. This becomes clearer later in this section.

The units of measurement vary by item. For example, units may be tons for items like Asphalt Concrete, linear feet for items such as Thermoplastic Traffic Stripes, or an integer number for items such as Sewer Manhole Covers. Many pieces of work are also bid on a lump sum basis, e.g. Traffic Control Systems and Construction Area Signs. For these items, the corresponding quantity is set to 1 and it is not subject to updates.

4. Cost of time – daily road user cost $c_U$ used in calculating the A+B score, incentive rate $c_I$ and disincentive rate $c_D$. Note that due to a court case involving Alabama DOT (Milton Construction Company v. Alabama DOT, US 11th Circuit Court of Appeals 1990), $c_D \leq c_U$ must hold.

5. A maximum time by which the project must be completed, $t$. A maximum incentive in terms of the number of days, $t_I$.

An example that illustrates daily road user cost calculations can be found at http://www.dot.state.mn.us/i35wbridge/rebuild/pdfs/road-user-cost-computations.pdf. It is not clear to what extent such detailed calculations are done to compute $c_U$ for each road or bridge construction project. Furthermore, the appropriateness of each particular method that STAs could use to calculate the true cost of traffic delays can be debated. We do not deal with such issues in this paper. Instead, we assume that the owner may use a road user cost that is different from the true cost, $c_T$, and that the true cost is known to the owner. We have anecdotal evidence that when estimated $c_T$ is very large, e.g. for interstate highway pavement rehabilitation projects, $c_U$ is usually set much lower than $c_T$ because STAs believe that a very high $c_U$ will result in higher bids. We address this issue in Section 3.2. We require that $c_U \leq c_T$ to ensure that no legal challenges, similar to the Alabama case cited above, arise as a result of approximating the daily road user cost.

The owner develops an estimate of the project completion time, denoted by $t^e$. Estimates are based on engineers’ experience and historical data. For example, if the project involves pavement rehabilitation and the estimated quantity of bituminous asphalt needed is 24 thousand tons, the engineer may estimate eight working day completion time based on an average rate of three thousand tons per day. These calculations may be adjusted if traffic conditions make it difficult to haul materials to the project site or if the site is far away from known asphalt plant locations.
The owner also develops its own estimate of labor and material costs. Let $c^e = (c_{e0}^e, \cdots, c_{en}^e)$ denote the owner’s cost estimate. Components of the cost vector are $c_{e0}^e$, the fixed cost, and $c_{ei}^e$, the unit cost of the $i$th item. A variety of methods are used to obtain these estimates. Two examples are as follows: (1) using unit prices from recent winning bids on projects requiring similar quantities, and (2) obtaining quotes from materials’ suppliers. The owner’s estimate of material and labor costs of the project is $c_{e0}^e + \sum_{i=1}^{n} c_{ei}^e q_i^e$, which we write as $c^e \cdot q^e$. This amount is used as the basis for preparing budgets and for securing funding for the project from state and federal sources.

Estimated costs are not revealed to the contractors. Time and unit cost estimates are used by the owner to determine which bids are imbalanced. If a contractor bids very high unit prices on some items and very low unit prices on others, then the bid is declared imbalanced. Such bids are disqualified. We use $[\underline{t}, \overline{t}]$ and $[\underline{c}_i, \overline{c}_i]$, for each $i$, to denote acceptable ranges of bid time and unit prices. The owner typically lets contractors know the ranges of values of unit bids that would be considered acceptable and imbalanced bids are not common. Our data set did not include any disqualified bids.

The point of the above discussion is that there are standard engineering procedures for determining $q^e$, $c_T$, $c^e$, $t^e$, $[\underline{t}, \overline{t}]$, and $[\underline{c}_i, \overline{c}_i]$. Where the owner needs help is in setting $c_U$, $c_D$, $c_I$ and $t_I$ and in understanding what contractor behavior to expect in response to the value of these variables. We assume that the owner prefers a bid-letting mechanism that is (1) truth revealing (i.e. bidders reveal their private estimates of costs and completion time), (2) efficient (i.e. the lowest-cost bidder is awarded the contract), and (3) able to induce an optimal amount of effort on expediting project completion.

### 2.3 The Contractor’s Problem

We now describe the problem from an arbitrary contractor’s point of view (hereafter referred to as the contractor). For ease of exposition, we do not use a contractor index in this section. Each contractor contemplating a bid is risk neutral and knows the scope of work, estimated quantities $q^e$, permissible ranges of unit bids $[\underline{c}_i, \overline{c}_i]$, and time related parameters $[\underline{t}, \overline{t}]$, $c_U$, $c_D$, $c_I$ and $t_I$. The contractor chooses $b = (b_0, \cdots, b_n)$ unit prices and the bid time $t^b$ such that $t^b \in [\underline{t}, \overline{t}]$ and $b_i \in [\underline{c}_i, \overline{c}_i]$ for each $i = 0, \cdots, n$. It receives a score of $s(b, t^b) = b_0 + c_U t^b + \sum_{i=1}^{n} b_i q_i^e = c_U t^b + b \cdot q^e$ for its bid. Note that the same score can be obtained by numerous combinations of unit prices and bid time.

It is convenient to decompose the contractor’s decision into two components: (1) its total bid $s$, and (2) unit bid prices and time $(b, t^b)$. Note that the winner-determination problem depends only on the total score and not on individual unit prices and bid time so long as they are chosen within reason. The first problem requires the contractor to have a model of the behavior of other contractors, but in the setting we describe below and given a score $s$, the second problem can be solved independently of what other contractors might do. We focus first on the second problem.
The actual time it takes to complete a project has two components: a project-scope related component over which the contractor has little or no control, and a component that depends on the amount of effort that a contractor puts into expediting. Examples of contractor effort include working longer hours, working weekends, using multiple construction crews, and making use of faster automated equipment. We denote the contractor effort by a non-negative parameter $\gamma$ and the actual project completion time by $T(\gamma) = \xi + g(\gamma)$, where $\xi \geq 0$ is the random component and $g(\gamma)$ depends on contractor effort. The cost of effort level $\gamma$ is $h(\gamma)$. Functions $g(\gamma)$ and $h(\gamma)$ are known to the contractor before it bids. Both are non-negative and convex, and $g(\gamma)$ is decreasing in $\gamma$ whereas $h(\gamma)$ is increasing. Other relationships between effort and completion time are possible. Our choice of a relatively simple model is driven by the need to have a tractable model.

Two types of models can be developed with respect to the timing of when the contract learns its true completion time. In one instance, the contractor may observe its personal value of $\xi$ (denoted as $t$ hereafter) before submitting its bid. This is the contractor’s private information. It is not known to other bidders and to the owner. In the second instance, the contractor bids based on its distributional assumptions about $\xi$ without knowing its realized value. Once again, two types of models are possible at this stage—in one case, the contractor observes the random component of completion time before committing effort, whereas in the second case, the random component of completion time is not known until after effort commitments are made. Each of these models is slightly different. We analyze in detail the model in which the contractor observes $\xi$ before bidding. We briefly discuss the effect of other variants in Section 4. A complete analysis of the possible variants of our model is a topic of ongoing research effort by the authors.

The contractor’s unit costs and estimated quantities are denoted by $C = (C_0, \ldots, C_n)$ and $Q = (Q_0, \ldots, Q_n)$. Recall that $Q_0 = 1$ with probability 1. Given $(c, t, q, g, h)$, a contractor’s private cost, $\phi$, of submitting a bid with score $s(b, t^b)$ does not depend on $(b, t^b)$. It does depend on $\gamma$ and parameters $(c, t, q, g, h)$ according to the relationship below.

$$\phi = c \cdot q + c_U[t + g(\gamma)] + h(\gamma).$$ (1)

In Section 3, we compute the optimal bid parameters $(b, t^b)$ and optimal effort $\gamma$ for each $s$. We also compute the equilibrium bid $s^*$ and define an index $s_{\text{min}}$, which we refer to as the contractor type, that depends on parameters $\phi$ and $q$.

A key assumption in our model is that prior to submitting its bid, each contractor receives a signal that reveals its type — i.e. parameters $\phi$ and $q$. These parameters are also assumed to be private and independent of other contractors’ parameters. This is a potentially contentious assumption because contractors may anchor their distributional assumptions on $q^e$s, which is known at the time of bidding. We provide several arguments below that suggest that private-independent-values may be a reasonable first-pass assumption in our context\textsuperscript{2}. We also discuss possible extension

\textsuperscript{2}Note that previous literature in the area of multi-attribute auctions has also made this assumption; see for example, Che (1993) and Ewerhart and Fieseler (2003).
of our model involving common values in Section 5. Our model is best viewed as the first formal model of an emerging contracting practice by STAs.

Suppose contractors viewed parameters \((\phi, q)\) as common values. An immediate consequence of that assumption would be that the greater the number of bidders, the less a bidder would believe its own draw of each common values’ quantity. This would lead to greater shading and a higher bid in expectation (see, for example, Krishna 2002, p. 85). In contrast, under private independent values’ assumption, bids should decrease in the number of bidders. In order to test how bids change with the number of bidders, we regressed bid price on the number of bidders and the engineers’ estimate. We included engineers’ estimates to account for different scope of work in different projects. The results, shown in a table below, are based on 1908 observations, the \(R^2\) and adjusted-\(R^2\) are both 0.9878, and the model is highly significant.

| Bid Amount | Coef.  | Std. Err. | \(t\)  | \(P > |t|\) | [95% Conf. Interval] |
|------------|--------|-----------|-------|-----------|---------------------|
| No. of bidders | -8620.021 | 7251.148 | -1.19 | 0.235     | -22841.04 5601.003  |
| Eng Estimate  | 1.024939  | .0026174 | 391.59 | 0.000     | 1.019806 1.030073  |
| Constant     | 14576.4  | 48057.28 | 0.30  | 0.762     | -79674.03 108826.8 |

Note that the coefficient of the number of bidders is negative and that it is not significantly different from zero (\(p\)-value = 0.235). This lends some support to the assumption that contractors view \((\phi, q)\) as independent private values. Other arguments in favor of our assumption are as follows. Private independent values’ assumption leads to a simpler model that provides meaningful insights for STAs (see Section 4). During interactions with project engineers we have discovered that indeed different contractors can face significantly different costs for some materials. Factors such as scale economies, production technologies, ownership of materials’ manufacturing plants, and location of plants relative to project site can affect a contractor’s costs significantly. We have also discovered that contractors may have different ex ante estimates of their chances of gaining approval for project scope changes that are under the discretion of project engineers. This happens either because they have greater experience of completing projects in a geographical area, or because of their previous interactions with a particular project engineer. Finally, labor contracts and experience/expertise of the construction crew can significantly affect base-case completion time and the cost of expediting, both of which vary from one contractor to another.

Faced with an option to bid on a particular request for bids, a contractor needs to choose the building blocks of its bid for each fixed \(s\), and then determine the optimal total bid. We address these issues in the next section.

3 Results

We analyze the A+B mechanism from both the contractor’s and the owner’s perspectives. To improve exposition, and to keep the model tractable, we make three key assumptions. Relaxing
these assumptions is not expected to change our results in a qualitative fashion, but that will require a more complicated analysis. First, we assume that there is a single piece of work that is bid on a lump-sum basis (including mobilization) and a single piece of work for which the required quantity does not equal 1 for all bidders. In terms of our formulation in Section 2, this means \( n = 1 \) and \( q = (q_0, q_1) \). In reality, bid abstracts contain dozens of items (i.e. \( n > 1 \) is the norm). However, all pieces of work mentioned in the abstracts belong to one of the two categories that we consider in our analysis. Second, we assume a private independent values’ model with symmetric information. Specifically, quantities \((\phi, q_1)\) are revealed to each contractor before it places its bids and these quantities are independent draws from a common distribution for all contractors. Note, because \( n = 1 \) and \( q_0 = 1 \) for all contractors, \( q_1 \) is the only quantity that is a private value for each contractor. Third, based on commonly occurring cases in the MnDOT data we consider \( c_I \leq c_D = c_U \). Contractor behavior when \((c_I, c_D, c_U)\) are not related in this fashion is discussed in Section 3.2.

3.1 Analysis of The Contractor’s Problem

Let \( \pi(b, t^b, \gamma \mid s) \) denote the expected profit of a contractor as a function of its bid prices, time, and effort. Then, we obtain

\[
\pi(b, t^b, \gamma) = (b - c) \cdot q - h(\gamma) + c_I \min\{t_I, (t^b - t - g(\gamma))^+\} - c_D(t + g(\gamma) - t^b)^+.
\] (2)

In the above expression, \((b - c) \cdot q - h(\gamma)\) is the expected profit of the contractor if it completes the project in exactly the number of days bid. The next two terms are the expected incentive and penalty, respectively, if the project completes early or gets delayed with respect to the bid time.

We solve the contractor’s problem in three steps. In all steps, we fix \((\phi, q_1)\) as the contractor’s private values. The first step solves for \((b, t^b)\) given \((s, \gamma)\). The next step obtains the optimal \(\gamma\) and the third step calculates the equilibrium bid value \(s^*\). The constrained optimization problem of the first step is stated formally below and its solution is presented in Proposition 1.

\[
\max_{(b, t^b)} \left\{ \pi(b, t^b \mid s, \gamma) = bq - \phi + (c_U - c_I)(t + g(\gamma)) - (c_D - c_I)(t + g(\gamma) - t^b)^+ \right\}
\] (3)

Subject to:

\[
s = b \cdot q^e + c_U t^b
\] (4)

\[
c_i \leq b_i \leq \bar{c}_i \quad \text{for each } i = 0, 1
\] (5)

\[
t \leq t^b \leq \bar{t}
\] (6)

**Proposition 1.** If \( c_I \leq c_D = c_U \), and parameters \((\phi, q_1)\), effort \(\gamma\), and bid score \(s\) are fixed, then \(t^b \leq t + g(\gamma)\) and the contractor’s optimal bid parameters are as shown in Table 2 below.
Define \( z(\gamma) = \frac{h'(\gamma)}{g'(\gamma)} \) and let \( \gamma_0 \) be a value of \( \gamma \) such that \( z(\gamma_0) = 1 \). Note that \( z(\gamma) \) is a non-negative and increasing function of \( \gamma \) and that \( \gamma_0 \) is the unconstrained optimal effort level. The former follows from the facts that \( g(\cdot) \) and \( h(\cdot) \) are, respectively, decreasing and increasing convex functions. Similarly, define \( \gamma_1 \) such that \( z(\gamma_1) = \frac{4}{q_1} \), \( \gamma_2 \) such that \( g(\gamma_2) = \bar{t} - t \), and \( \gamma_3 \) such that

\[
\begin{array}{|c|c|c|}
\hline
q_1 \geq q_1^c & q_1 \leq q_1^c \\
\hline
\overline{c}_1 \leq \frac{s-(b_0+c_Ut^b)}{q_1^c} & \overline{c}_1 \geq \frac{s-(b_0+c_U\min\{\bar{t}, t+g(\gamma)\})}{q_1^c} \\
\hline
b_1 = \overline{c}_1 & b_1 = c_1 \\
(b_0, t^b) \text{ such that } b_0 + c_Ut^b = s - \overline{c}_1 q_1^c & (b_0, t^b) \text{ such that } b_0 + c_Ut^b = s - c_1 q_1^c \\
\hline
\end{array}
\]

Table 2: Optimal Bid Parameters for fixed \( s \) and \( \gamma \).

A proof of Proposition 1 can be found in the Appendix. We explain our findings on an intuitive level in the ensuing discussion.

First, it is easy to see that no contractor will bid \( t^b > t + g(\gamma) \) because \( c_I \leq c_D = c_U \). For each extra day in its bid, the contractor’s score goes up by \( c_U \), but if it completes the project as bid, it earns a reward of \( c_I \). Therefore, the reward is no more than the cost and the contractor is better off bidding a \( t^b \) smaller than its true completion time \( t + g(\gamma) \) and extracting greater rent by increasing either \( b_0 \) or \( b_1 \). Second, because of updates that are applied ex post, the contractor receives a windfall. If \( q_1 > q_1^c \), then the windfall is \( b_1(q_1 - q_1^c) \). In this case, the contractor benefits from picking as large a value of \( b_1 \) as possible. Similarly, if \( q_1 < q_1^c \), the contractor minimizes its shortfall of \( b_1(q_1^c - q_1) \) by picking as small a value of \( b_1 \) as possible. In this case, contractor’s windfall comes from choosing a correspondingly high value of \( b_0 \). These arguments give rise to the two broad categories of results shown in Table 2.

The minimum and maximum values of \( b_1 \) are \( \underline{c}_1 \) and \( \overline{c}_1 \), respectively. Therefore, when the contractor can choose these values without violating other constraints — see cases listed in column 1 and 3 of Table 2 — the contractor picks those unit bids and it is indifferent among a range of possible values of \( t^b \). There are two cases in which either the lower or the upper limit on bid times becomes a binding constraint earlier than the bounds on possible values of \( b_1 \). Those cases are shown in columns 2 and 4 of Table 2. In those cases, the contractor picks the highest (lowest) possible value of \( b_1 \) while avoiding disqualification. The corresponding value of bid times are either \( \bar{t} \) or \( \min\{\bar{t}, t + g(\gamma)\} \). This explains all four cases in Table 2.

Proposition 1 shows that because of the windfall, contractors should choose extreme values of unit bids. It is easy to check that this behavior is optimal in A-only contracts as well. That is, common procurement mechanisms used by STAs are not truth revealing. Next, we turn to the determination of the optimal effort.
\[ g(\gamma_3) = \frac{s - \tau_0 - g_1}{c_U} - t, \text{ and } \gamma_0 = \max\{\gamma_1, \gamma_2, \gamma_3\}. \]

With these notation in hand, we can characterize the contractor's choice of effort as stated in Proposition 2. A proof of this proposition is also provided in the Appendix.

**Proposition 2.** The contractor chooses the effort level \( \hat{\gamma}_0 \) when \( q_1 < q_1^* \), \( t + g(\gamma_0) \leq \bar{t} \), and

\[ \bar{t} \leq \frac{s - (c_0 + c_U(t + g(\gamma_0)))}{q_1^*}. \]

In all other instances, it chooses the optimal effort level \( \gamma_0 \).

Effort level \( \gamma_0 \) is what the owner would prefer the winning contractor to pick. The fact that in virtually all cases, a contractor would choose \( \gamma_0 \) is desirable. This result can be explained on an intuitive basis as follows.

When \( q_1 \geq q_1^* \), from the first two columns of Table 2, note that the contractor is either indifferent among all values of bid time between \( \underline{t} \) and its true completion time, or it bids \( \underline{t} \), the minimum allowed bid time. That is, in this instance, the bid time is independent of the effort level. Therefore, when \( q_1 \geq q_1^* \), the contractor chooses \( \gamma_0 \) because that minimizes its private cost. When \( q_1 \leq q_1^* \) and \( \bar{t} \leq t + g(\gamma_0) \), the contractor's time bid depends on the relative magnitudes of \( \omega_1 \) and \( \frac{s - (c_0 + c_U(t + g(\gamma_0)))}{q_1^*} \). The optimal bid time is either not unique, or \( \tilde{t} \), but in each instance it is independent of the effort level, with the result that the contractor picks \( \gamma_0 \). This leaves only one case, described in the statement of Proposition 2, for which \( t^b \) depends on the effort level and the contractor does not choose \( \gamma_0 \).

In the first three cases discussed above, the contractor picks an optimal effort level \( \gamma_0 \) but we observe that it may not bid its true completion time. In contrast, when the contractor picks effort level \( \gamma_0 \), it does bid its true completion time. Our analysis suggests that the presence of limits on bid parameters can lead to suboptimal effort on part of the contractor. Because this is undesirable, we relax the bounds on the permissible values of \( c_0 \). In particular, this means that \( \omega_1 = 0 \) and we allow \( \tau_0 \) to be arbitrarily large. With this relaxation, the case described in column 4 of Table 2 does not occur and the contractor chooses \( \gamma_0 \) in all cases. We have anecdotal evidence that STAs typically do not limit the amount that a contractor may include in its bid as fixed (called mobilization) costs. Therefore, it is not unreasonable to make \( c_0 \) unbounded from above from this point forward.

What is the contractor’s expected profit in each case? We now work out the details using results from Propositions 1 and 2. If \( q_1 \geq q_1^* \), then

\[ \pi(s) = \max_{(b, t^b, \gamma)} \pi(b, t^b, \gamma \mid s) \]

\[ = \begin{cases} 
  s - (c_0 + c_1 q_1) - h(\gamma_0) - c_U(g(\gamma_0) + t) + \tau_1(q_1 - q_1^*) & \text{if } \tau_1 \leq \frac{s - c_U q_1^*}{q_1^*} \\
  s + (s - c_U q_1^*)/(q_1^*/q_1^*) - \phi & \text{otherwise}
\end{cases} \]  

\[ (7) \]

Note that in (7) and hereafter \( \phi \) is the contractor’s private cost when it exerts an optimal effort level, although this is not explicitly indicated in order to keep the notation simpler. Similarly, when


$q_1 \leq q_1^e$, and there is no upper bound on the choice of fixed costs, then

$$
\pi(s) = s + c_1(q_1 - q_1^e) - \phi
$$

(8)

The contractor receives a windfall in both (7) and (8) because of ex post updates.

A rational contractor will not place a bid that loses money. Therefore, the minimum bid of a contractor depends on its draw of $(\phi, q_1)$. The minimum bid (i.e. the smallest $s$ for which $\pi(s) = 0$) can be written as follows.

$$
s_{\min}(\phi, q_1) = \begin{cases} 
\phi - \tau_1(q_1 - q_1^e) & \text{if } q_1 \geq q_1^e \text{ and } \tau_1 \leq (\phi - c_U L)/q_1 \\
(\frac{q_1}{q_1^e})[\phi + c_U L(q_1 - q_1^e)/q_1^e] & \text{if } q_1 \geq q_1^e \text{ and } \tau_1 \geq (\phi - c_U L)/q_1 \\
\phi - \tau_1(q_1 - q_1^e) & \text{otherwise}
\end{cases}
$$

(9)

From this point forward, we refer to $s_{\min}(\phi, q_1)$ as an index that defines a contractor’s type. When we say that a contractor observes its own type before bidding, then that is equivalent to the statement that the contractor knows its minimum bid before bidding.

How does a contractor select its equilibrium bid? We answer this question in Proposition 3 below. A proof of this proposition is included in the Appendix. In order to present the result, we need additional notation that we introduce next. Let $s_{\min}^{i}(\phi^{i}(i), q_1^{i}(i))$ denote the type of the $i$-th contractor and let there be $v$ contractors. The $i$-th contractor assumes that each $s_{\min}^{j}$, $j = 1, \ldots, v$, is a random draw from a common distribution. We define random variable $S_{\min}$ to be such that its distribution is identical to the common distribution of contractor types $s_{\min}^{j}$ for all $j$. In order to estimate the distribution of $S_{\min}$, we will start with the joint distribution of $(\phi, q_1)$ and calculate the probability that $q_1 \geq q_1^e$. Then, we will find a mixture of two conditional distributions where a separate conditional distribution of $S_{\min}$ corresponds to the case when $q_1 \geq q_1^e$ and when $q_1 \leq q_1^e$ [see Equation 9]. We use $S_{k,\min}^{(v)}$ to denote the $k$-th smallest order statistics of $S_{\min}$, when there are $v$ bidders.

**Proposition 3.** Given a contractor type $s_{\min}^{i}(\phi^{i}(i), q_1^{i}(i))$, the contractor chooses effort level $\gamma_0$ and bid parameters $(b^{(i)}, t^{b^{(i)}})$ such that

$$
(b^{(i)}, t^{b^{(i)}}) = \arg \max_{(b, t^b)} \pi(b, t^b | s^*, \gamma_0),
$$

(10)

where $s^* = E[S_{k,\min}^{(v)} | S_{1,\min}^{(v)} = s_{\min}^{i}(\phi^{i}(i), q_1^{i}(i))]$ is the equilibrium bid. The contractor’s expected profit is $s^* - s_{\min}^{i}(\phi^{i}(i), q_1^{i}(i))$.

Contractors’ equilibrium bidding strategies when the owner uses A-only mechanism can be obtained from the analysis presented above by setting $c_U = c_D = c_I = 0$. In that case, the contractor does not expend any effort on expediting, but factors its windfall into its equilibrium bidding strategy in a manner similar to that explained in Proposition 3. Because these arguments are straightforward, we do not present the details here.
3.2 Analysis of The Owner’s Problem

The A+B mechanism provides incentives for the contractors to choose their optimal effort for reducing completion times. In this sense, it achieves a key objective of STAs. This effort is also socially optimal when \( c_U = c_T \). For the purpose of choosing the optimal effort level, given \( c_I \leq c_U \), \( t_I \) does not matter and the same outcome will be realized (in expectation) if there were no incentive for early completion. Because \( \phi \) is increasing in \( c_U \), from \( (9) \), the bids are also increasing in the road user costs (via \( s_{\min} \)). This matches with the owners’ intuition. For each fixed \( s \), the contractor’s profit is also decreasing in \( c_U \) (see \( 7 \) and \( 8 \)). Therefore, contractors are likely to tender higher bids when \( c_U \) is larger and STAs may not choose \( c_U = c_T \) to keep project costs within their budgets.

For a given set of parameters, our model can help state agencies calculate the impact of setting different values of \( c_U \) in terms of expected costs and completion times. We explore a few cases below.

In the previous section, we assumed \( c_I \leq c_D = c_U \leq c_T \). A number of variations of these assumptions can be explored. If we set \( c_I = c_D = c_U \), i.e. equal incentive and penalty rates, then the contractors’ strategic choice of bid time becomes irrelevant. The economic impact of any difference between bid time and completion time (positive or negative) is incorporated into the fixed component of the bid. Contractors may voluntarily choose to reveal their true completion time because the incentives from offering a short completion time are eliminated. This is however not the case. Among the 25 A+B projects, we found \( c_I = c_D \) occurred in 15 (60\%) cases. However, bidders appear to prefer incentives to penalties by bidding \( t^b \geq t + g(\gamma_0) \) — of the 15 projects in which \( c_I = c_D \), 11 winning bidders had \( t^b \geq t + g(\gamma_0) \). Also, 4 out of the 15 bids were at \( \bar{t} \).

Another possible variant of incentives is \( c_I < c_D < c_U \). From a strategic bidding perspective this is similar to the case we explore. It provides incentives for offering an early completion date, since the contractor prefers penalties over incentives. With \( c_D < c_U \), the contractor’s incentives for completion are skewed. The contractor uses marginal daily cost of \( c_D \) when calculating its effort and chooses \( \tilde{\gamma} \) such that \( h'(\tilde{\gamma})/[-c_Dg'(\tilde{\gamma})] = 1 \). This leads to a lower investment in effort. There is also an impact on bidding. For each day of reducing \( t^b \), the contractor gains \( c_U \), but only pays a penalty of \( c_D \) for being late. This generates a windfall profit upon contract completion of \( c_U - c_D > 0 \) per day. This would induce bidding \( t^b \) as small as possible, and subtracting the windfall \( (c_U - c_D)(t + g(\gamma) - t^b) \) from the fixed component of the bid (see Proposition 1 for a similar impact of unit bids \( b_1 \)).

The last case we discuss is the one in which the owner sets \( c_U \) strategically (with \( c_D = c_U \)). Although, \( c_U = c_T \) maximizes social welfare, this strategy transfers all benefits of early completion to the contractor, inducing optimal effort. From a budget perspective there are difficulties with such an approach. Much of the cost of road closures is not borne by STAs, but by the commuting public. There is currently no efficient method of monetizing the benefits of faster construction. Alternatively, if one were to view STAs as profit-maximizing buyers who face a private cost of \( c_T \).
per day of construction, then the buyers can benefit by setting $c_U < c_T$. For example, if the lowest-construction-cost is also the most efficient in terms of completion time, then this contractor will benefit from both cost-based and time-based efficiencies, relative to the other bidders. To reduce the profits accrued by such a bidder, the buyer may prefer to score contractors on $c_U < c_T$. This is similar to results in Che (1993), Laffont and Tirole (1987), and McAfee and McMillan (1987).

Both A-only and A+B mechanisms also produce a variety of undesirable outcomes, some of which can pose serious problems for the owners. We discuss these issues next. Under both mechanisms, contractors generally do not bid their true costs and their true completion times. Upon examining data from MnDOT, we found that the ratio of unit bid to winning unit bid is highly variable with a mean of 6.17, a minimum of the order of $10^{-6}$, and a maximum of 32000. These observations confirm that unit bids are not reliable estimates of contractors’ true costs. The former can lead to problems in preparing engineers’ estimates for future projects (because unit bid prices are distorted) and in deciding how much of the state agency’s funds to encumber for the project. However, this problem can be overcome in part because owners also obtain material cost estimates directly from materials’ suppliers. Owners also budget for cost overruns based on past experience and on average, engineers’ estimates are close to the actual amounts paid to contractors (see Table 1).

Inaccurate time bids can be a serious problem because the owner is unable to realistically budget supervising engineers’ time devoted to project management activities. Inaccurate bids also complicate scheduling of complementary activities by other contractors and construction projects whose start times depend on the completion of the project in question. STAs can eliminate one reason for bidding less than the true completion time by setting $c_I = c_D = c_U$. However, this will still not result in truthful bidding because as observed in Proposition 1, the contractor’s profit is invariant in $t^b$ over some range of values. An even more serious problem is that the lowest-cost bidder does not always win. We illustrate this with a simple example.

Suppose there are two bidders and $\phi^{(1)} > \phi^{(2)}$. An efficient mechanism should pick contractor 2 because it has the lower overall cost. Next, suppose $q^{(1)}_1 > q^{(2)}_1 = q^e_1$. Then, contractor 1 can win in spite of being the higher cost contractor because of the windfall. STAs can eliminate the windfall by contracting on $q^e_1$, i.e. eliminating quantity updates and paying contractors a firm price for materials irrespective of the amounts actually used. Some agencies have started moving in this direction, but they allow updates if actual quantities deviate by more than 25% (or a similar threshold) from the estimated amounts. This is also a topic of ongoing research by the authors.

4 Insights

Our model predicts that contractors ought to bid less than the actual completion times and end up being in the disincentive range. This does not match well with the outcome observed in Figure
1, although it is also not contradicted by the data. There could be a variety of explanations why completion times are not consistently more than bid times. Some examples are as follows — (1) completion times are in fact random with significant estimation errors; (2) effort can be changed in discrete steps only because of work rules (e.g. 4-hour increments for overtime); (3) contractors have not figured out that they can do better by bidding even shorter completion times; and (4) contractors worry about their reputation, which is not considered in the model. Contractors who bid relatively low completion time and then complete on or before time do get attention from the press. Conversely, contractors who complete late, relative to their bid, attract negative press. For example, Flatiron Corporation that rebuilt the I-35W bridge in Minneapolis, MN, earned a bonus of approximately $25 million by completing the project earlier than bid. This led to significant attention from the press (see, for example, Foti 2008), which may help this contractor in future bids where performance-based contractor pre-qualification is a factor. Similarly, an example involving a $700,000 penalty and negative press can be found at http://www.naplesnews.com/news/2006/jul/30/its_payback_timen/?local_news. In what follows, we consider the effect of random completion times.

How is the contractor behavior affected if $\xi$ is realized after bidding is completed? That is, each contractor prepares its bid based on random $T(\gamma)$. Consider first the situation where $c_I < c_D = c_U$, effort is exerted before knowing $\xi$, and that the effort level does not change with the realization of $\xi$. Then for a fixed score $s$, the contractor prefers penalties over incentives. Under the assumption that the choice of $b_0$ is unbounded, the contractor’s choice of effort will be $\gamma_0$, and the contractor is assured to be late when $t \leq t^b \leq g(\gamma_0)$. That is, the contractor’s optimal bid in columns 1 and 3 of Proposition 1 is $t \leq t^b \leq g(\gamma_0)$. If $c_D < c_U$ and all other parameters remain as before, then the optimal effort will depend on $c_D$ and it will be lower than $\gamma_0$ previously identified. If effort is exerted after knowing $\xi$, the bidding behavior does not change because by choosing $t \leq t^b \leq g(\gamma_0)$, the contractor ensures being in the disincentive range. In summary, the timing of the revelation of $\xi$ makes actual completion time more variable relative to the bid time, but it does not affect contractors’ effort or bid time strategy.

A different way to understand the implications of the equilibrium bidding strategy obtained in Section 3 is to ask the following questions. Do winning bidders submit shorter completion times? If some of the contractors were to use the equilibrium strategy (while others did not), would this impact the outcome of the procurement auction? Among the 25 A+B projects, the winning bidders had the smallest bid times in 6 cases. So winning bidders may be slightly better at understanding equilibrium bidding than others, although its difficult to validate that with a small sample. In several cases where the winning bidder bid lowest completion time, it would not have won the contract if it had bid the next highest bid time. That is, bid time may very well be the difference between winning or not winning a contract. The second question is harder to answer because we only know completion times for the winning bidder. However, we can use the data to consider some hypothetical scenarios. For example, if all contractors, other than the winning contractor, were to bid at 80% of the actual completion time of the winning bidder, then the winner would change.
in four auctions. These comparisons clearly underscore the importance of considering equilibrium strategy when preparing bids. They also strengthen our argument that contract decisions may be based on bidding strategies not competency.

Because contractors have not bid according to the equilibrium strategy, several large-scale A+B construction projects have been successful (Foti 2008, Landers 2005). This has led to the impression that time-based incentives lead to faster completion relative to the bid time, generating praise for STAs. The analysis presented in this paper shows that this is not an equilibrium outcome. Equilibrium bidding strategies have contractors placing bids with early completion dates and experiencing delays. In the future, bids can be expected to be more similar to the equilibrium identified here. Frequent completion delays using A+B contracts could be a cause for concern to STAs in the future. Therefore it helps obtain answers to the following questions. How much more do A+B projects cost? Are these extra expenditures justified by reduction in faster project completion? We provide additional insights on these topics next.

We showed two financial ratios in Table 1 in Section 2. We return to these ratios to shine light on the first question raised above. Recall that Ratio 1 is the ratio of the total amount paid and the A-component of the winning bid, and Ratio 2 is the ratio of the total amount paid and the A-component of the engineer’s estimate. Ratio 2 in Table 1 shows that A+B projects cost on average 5% more than the engineers’ estimate whereas the average cost of an A-only project is approximately the same as the engineers’ estimate. When we examine the amount actually paid and the amount bid (the difference can be attributed to windfall), we observe no statistical difference between A-only and A+B projects. This is consistent with the analysis presented in the previous section — both mechanisms allow the contractor to earn a windfall profit through quantity adjustments.

Because A-only projects do not have an a priori completion time, we can only compare the actual completion times under A-only and A+B contracts. The mean, minimum and maximum actual completion times under the two regimes were (106.53, 10, 1400) and (85.3, 5.5, 534), which suggests that contractors finish A+B projects sooner. To further study how completion time varies with project letting mechanism, we regressed the natural log of completion time (Ta) with the natural log of engineers’ estimate (EngEst) and an index $I_{AB}$ that equalled 1 if project was let using A+B mechanism and 0 otherwise. We used log-transformed variables Ta and EngEst because there are a few large projects that otherwise tend to dominate statistical results. The results are shown in a table below.

<table>
<thead>
<tr>
<th></th>
<th>ln(Ta)</th>
<th>ln(EngEst)</th>
<th>$I_{AB}$</th>
<th>Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coef.</td>
<td>.5105605</td>
<td>.5105605</td>
<td>-.8771002</td>
<td>4.048317</td>
</tr>
<tr>
<td>Std. Err.</td>
<td>.0223319</td>
<td>.1207634</td>
<td>.1207634</td>
<td>.0293039</td>
</tr>
<tr>
<td>$t$</td>
<td>22.86</td>
<td>-7.26</td>
<td>-7.26</td>
<td>138.15</td>
</tr>
<tr>
<td>$P &gt;</td>
<td>t</td>
<td>$</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>[95% Conf. Interval]</td>
<td>[.4666681, .5544529]</td>
<td>[-1.114456, -.6397447]</td>
<td>[-1.114456, -.6397447]</td>
<td>[3.990721, 4.105912]</td>
</tr>
</tbody>
</table>
With $R^2$ and adjusted-$R^2$ values of approximately 0.55 (436 observations), the model

$$\ln(Ta) = b_0 + b_1 \ln(\text{EngEst}) + b_2 I_{AB}$$

explains slightly more than half of the variability in the dependent variable. The model is significant and each of the coefficient is significantly different from zero. The model can be rewritten as

$$Ta = e^{b_0} \cdot (\text{EngEst})^{b_1} \cdot e^{b_2 I_{AB}}.$$ 

This means that A+B projects had an average completion time that was $e^{-0.877} = 0.42$ times the average completion time of A-only project after accounting for project-scope related effects.

The A+B projects cost more, although this does not appear to be due to additional windfall opportunity resulting from time bids. The excess cost seems to come from higher unit bids. The A+B projects also finish earlier. Data shows that they are completed in slightly less than half the time of standard A-only projects after accounting for scope-related effects. However, we caution against attaching too much significance to this observation for the following reasons — (1) the size of A+B projects’ sample is small, and (2) there exists a strong selection bias because owners carefully select projects for A+B letting. Projects let using the A+B mechanism tend to be those that have a well-defined scope of work and that do not require utility work (Anderson and Damnjanovic 2008). Projects involving utility work are widely known to experience delays in project completion. However, with this caveat, A+B mechanism does appear to have the desired effect (i.e. significantly shorter completion times) at a price that is about 5% of the engineers’ estimate.

5 Concluding Remarks

State transportation agencies commit billions each year in new construction contracts. STAs are innovating procurement mechanisms to achieve best value — faster completion, higher quality and lower cost. In this article, we presented mathematical models to analyze A+B contracts and showed that this mechanism does not lead to truthful revelation of contractors’ unit costs and completion times, and the lowest-cost bidder does not necessarily win. However, the winner exerts optimal effort to lower completion time.

Our analysis shows that A+B mechanism (as also the A-only mechanism) creates incentives for the contractor to bid extreme values of unit costs depending on whether it expects the actual quantities to be higher or lower than the estimated quantities. Our models also predict that contractors should bid low completion times, exert optimal effort, complete late, and pay a penalty. Data from MnDOT supports many of our conclusions. However, contractors do not consistently bid low completion times and finish late. In fact, in many cases, contractors finish early relative to their bid and earn an incentive. We offer a variety of explanations that explain this behavior.
There are a number of possible extensions of this research. Because many auction scenarios, including ours, exhibit both private and common value components (Laffont 1997), we discuss how our models would be affected if common value components were included. In our setting, private costs borne by the contractor stem from labor contracts, management know how, capital equipment, access to commodities, and technology, to name a few. Common-value costs exist as well due to the amount of materials needed and their future market prices. The amount of materials needed is estimated by all contractors without extensive engineering evaluation of the design, leading to a common-value setting and generating a potential winner’s curse. Similarly, the market price of these commodities has to be estimated well in advance of construction. Asphalt prices, for example, are highly correlated with oil prices, and may exhibit large fluctuations over even a short period of time. Their estimation, for which all contractors are likely to rely on the same inputs, generates a common-value model.

When the private-value component dominates, the models we developed in Sections 2–3 are appropriate. The general model incorporating both private and common values components is not widely studied in the literature, and an equilibrium may not exist in the general case (Jackson 1999). One exception is the article by Goeree and Offerman (2003) that develops equilibrium bidding strategies in such a context under certain assumptions. It may be possible to adapt this approach to our setting by adding a few assumptions. We do not present the adaptation, which is a topic of future research, but discuss the assumptions that would be needed and the implications of including common-value components.

Suppose a contractor’s \( b_0 \) and time-related components are primarily private-value whereas unit costs and quantities of inputs are driven by common-values. This is reasonable because time-related costs are influenced by labor contracts, capital equipment, and management know how, which are primarily fixed for each contractor. Furthermore, suppose that the true cost of the aggregate common-value component is the average of individual signals. This assumption has been made previously in the literature (see Goeree and Offerman 2003 and references therein). Finally, for both the private-value and common-values components the distribution of signals is log-concave across contractors. This assumption is consistent with many probability distributions. These assumptions allow the application of Goeree and Offerman’s approach to our model.

The analysis in Goeree and Offerman (2003) also provides some insight into the implications of including a common-value component in our model. Contractors will still have an incentive to represent their type strategically and adjust their bid based on their projected windfall. The inefficiency problem we identify in the private-values case is exacerbated when a common-value component is incorporated because both strategic bidding of unit-prices and the common value uncertainty may contribute to scenarios where the low-cost contractor is not awarded the contract. In addition, uncertainty over commodity cost and quantities, the common-value components, increases the magnitude of potential winner’s curse, increasing contractors’ bids. This raises the winning contractor’s profit at the expense of the owner. To mitigate these effects, owners would
like to reduce uncertainty. This can be achieved by introducing more bidders into the auction. In this setting, competition dampens the importance of the common-value component, increasing owner revenue. Similarly, revenue is increased by providing engineers’ estimates for commodity requirements, as recently initiated by Oklahoma Department of Transportation (De Silvaaa et al. 2008)

The issue of completion times is important in many procurement auctions, including construction projects in the public or private sector. The inability to use roadways or unavailability of real-estate are burdensome and expensive. We show, however, that simple methods of incorporating time into the auction have both positive and negative outcomes. They are effective at providing incentives for early completion. However, contractors’ bids do not accurately reflect private estimates for completion time, which complicates scheduling of owner’s complementary technical-support and managerial activities. We identify the magnitude of these effects in this paper, providing a foundation for improving auction mechanisms and managerial decisions.

Acknowledgments

This material is based upon work supported, in part, by the National Science Foundation under Grant No. CMMI-0653451, and a matching grant by the Center for Transportation Studies, to Diwakar Gupta. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation or the Center for Transportation Studies. This research was also supported by a small grant from the Graduate School of the University of Minnesota (joint with Professor Patrick Bajari of Department of Economics). Research assistants supported by that grant helped in assembling some of the data used in this study.

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Appendix

Proof of Proposition 1

Recall that the contractor’s profit is

\[
\pi(b, t^b, \gamma | s) = (b - c) \cdot q - h(\gamma) + c_1 t_1 \cdot 1_{\{t^b > t + g(\gamma) + t_1\}} + c_1 (t^b - g(\gamma) - t) \cdot 1_{\{t^b \leq t + g(\gamma)\}}.
\]

(11)

In the above expression, the function \(1_{\{\}\} \) returns value 1 when the logical expression within the curly braces is true, and the value 0 otherwise. We proceed to solve the above problem in two steps. First, we fix \(\gamma\) (in addition to \(s\)) and find the best combinations of \(b_0, b_1\), and \(t^b\) for each fixed \(\gamma\). Then, we choose the profit maximizing \(\gamma\) as a function of \(s\).

For a fixed \(\gamma\), the contractor’s problem consists of three subproblems corresponding to the regions where (I) \(\frac{t}{g} \leq t^b \leq \min\{\bar{t}, t + g(\gamma)\}\), (II) \(\max\{\bar{t}, t + g(\gamma)\} \leq t^b \leq \min\{t + g(\gamma) + t_1, \bar{t}\}\) and (III) \(\max\{\bar{t}, t + g(\gamma) + t_1\} \leq t^b \leq \bar{t}\). Within each subproblem, we solve for an optimum \((b_0, b_1, t^b)\) and obtain the corresponding contractor profit. We then compare the three solutions to find the range in which the optimal profit is the highest. Ties can be broken arbitrarily.

**Region I:** In this case, \(t^b\) is chosen such that \(\frac{t}{g} \leq t^b \leq \min\{\bar{t}, t + g(\gamma)\}\). The contractor’s expected profit is derived from Equation (11) as follows. Note, superscript \(I\) denotes region I.

\[
\pi^I(b, t^b | s, \gamma) = b_0 + b_1 q_1 - (c_0 + c_1 q_1) - h(\gamma) - c_U(g(\gamma) + t - t^b)
\]

(12)

However, since \(s = b_0 + b_1 q_1 + c_U t^b\), the contractor can fix only two out of the three bid parameters. Suppose the contractor fixes \(b_1\) and \(t^b\), then the above profit function can be rewritten as follows.

\[
\pi^I(b_1, t^b | s, \gamma) = s - c_U t^b + b_1 (q_1 - q_1^*) - (c_0 + c_1 q_1) - h(\gamma) - c_U(g(\gamma) + t - t^b)
\]

\[
= s + b_1 (q_1 - q_1^*) - (c_0 + c_1 q_1) - h(\gamma) - c_U(g(\gamma) + t).
\]

(13)

The range of feasible values of \(t^b\) is obtained from the defining inequalities of Region I. The bounds on the feasible values of \(b_1\) can be obtained as follows.

\[
b_1 \leq 1_{1} = \min [c_1, \frac{s - \min(b_0 + c_U t^b)}{q_1^*}] = \min [c_1, \frac{s - (c_0 + c_U t^b)}{q_1^*}].
\]

(14)

\[
b_1 \geq 2_{1} = \max [c_1, \frac{s - \max(b_0 + c_U t^b)}{q_1^*}] = \max [c_1, \frac{s - (c_0 + c_U \min\{\bar{t}, t + g(\gamma)\})}{q_1^*}].
\]

(15)
Note from Equation (13) that $\pi^I$ is increasing in $b_1$ if $q_1 \geq q_1^e$ and decreasing if the opposite inequality holds. Therefore, in the former instance, the contractor would choose $b_1^I$ and in the latter instance, it would choose $b_1^1$. When $q_1 = q_1^e$, the contractor is indifferent among the feasible values of $b_1$. This gives rise to the four cases reported in Table 2. It turns out that the optimal bid parameters in Region I are also overall optimal because the contractor’s profit in Region I dominates its profit in other regions, as shown in the remainder of this proof.

Region II: In this case, the range of $t^b$ values is $\max\{s, t + g(\gamma)\} \leq t^b \leq \min\{t + g(\gamma) + t_1, \overline{t}\}$. The contractor’s expected profit is derived from Equation (11) as follows.

$$\pi^{II}(b_1, t^b | s, \gamma) = s - (c_U - c_I)t^b + b_1(q_1 - q_1^e) - (c_0 + c_1q_1) - h(\gamma) - c_I(g(\gamma) + t).$$

(16)

The feasible values of $t^b$ are obtained from the defining inequalities of Region II. The bounds on $b_1$ values are similar to the bounds we obtained for Region I. They can be written as

$$b_1 \leq b_1^{II} = \min [\overline{b}_1, \frac{s - (\overline{c}_1 + c_U \max\{t + g(\gamma)\})}{q_1^{II}}].$$

(17)

$$b_1 \geq b_1^{I} = \max [\underline{b}_1, \frac{s - (\overline{c}_0 + c_U \min\{t + g(\gamma) + t_1, \overline{t}\})}{q_1^{II}}].$$

(18)

From Equation (16), we observe that $\pi^{II}$ is decreasing in $t^b$ (since $c_I < c_U$). Moreover, $\pi^{II}$ is increasing in $b_1$ if $q_1 \geq q_1^e$ and decreasing otherwise. We are now ready to compare the contractor’s profit functions in Region I and Region II.

If $q_1 \geq q_1^e$, then the contractor’s profit is increasing in $b_1$ in both regions. However, because $\overline{b}_1^{II} \leq \overline{b}_1^{I}$, it means that for each feasible value of $b_1$ in Region II, there is an equal or larger feasible value in Region I. Moreover, for each fixed $b_1$, $\pi^{II} \leq \pi^I$ irrespective of the values of $t^b$ in the two regions (this follows from a straightforward comparison from (13) and (16)). This means that when $q_1 \geq q_1^e$, the contractor prefers $t^b$ in Region I. For this reason, we do not need to find optimal bid parameters in Region II when $q_1 \geq q_1^e$.

If $q_1 < q_1^e$ and $b_1^I = b_1^{II} = \underline{c}_1$, the range of feasible values of $b_1$ in Region II is a subset of the range in Region I. This means that every feasible $b_1$ in Region II is also feasible in Region I. But for each fixed $b_1$, $\pi^{II} \leq \pi^I$ irrespective of the values of $t^b$ in the two regions. Therefore, under the conditions listed above, the contractor once again prefers values of $t^b$ in Region I.

The remaining scenarios have $q_1 < q_1^e$ and $b_1^I > b_1^{II}$. In particular, these scenarios arise when

$$b_1^{I} = \frac{s - (\overline{c}_0 + c_U \max\{t + g(\gamma)\})}{q_1^{I}} > \underline{c}_1$$

and

$$b_1^{II} = \frac{s - (\overline{c}_0 + c_U \min\{t + g(\gamma) + t_1, \overline{t}\})}{q_1^{II}}$$

both of which are

\footnote{In this paper, we use increasing (resp. decreasing) to mean non-decreasing (resp. non-increasing).}
Suppose \( t^* = \frac{q_1^c}{q_1^e} \). Substituting \( b_1 = \frac{q_1^c}{q_1^e} \) in Equation (13), we obtain

\[
\pi^I(s, \gamma) = \bar{c}_0 - c_U(t + g(\gamma) - \bar{t}) + \left( \frac{q_1^c}{q_1^e} \right) (s - (\bar{c}_0 + c_U \min\{t, t + g(\gamma)\})) - (c_0 + c_1 q_1) - h(\gamma)
\]

\[
= \bar{c}_0 + \left( \frac{q_1^c}{q_1^e} \right) (s - (\bar{c}_0 + c_U(t + g(\gamma)))) - (c_0 + c_1 q_1) - h(\gamma).
\]

The simplification in the second equality above comes from the fact that according to the defining inequality of Region II, \( t + g(\gamma) \leq \bar{t} \).

How should the contractor choose its bid parameters when \( q_1 < q_1^c \) and \( \frac{q_1^c}{q_1^e} > \frac{q_1^r}{q_1^e} \)? The question can be answered by solving a constrained convex optimization problem. There is also a direct and more intuitive argument that is consistent with the formal argument. We have chosen the latter approach here because it is easier to explain. Recall that the contractor's total score \( s \) is fixed and it is comprised of three components – the fixed cost, the materials cost and the time cost. For each unit increase in the contractor's score due to materials cost, its profit decreases by \( \frac{q_1^c}{q_1^e} \); whereas for each unit increase in its score due to time cost, its profit increases by \( \frac{c_U}{c_U} \).

In the ensuing analysis, we will show that the contractor's profit is larger in Region I in each of the above two scenarios. Note that the contractor may not be able to choose a corner solution, i.e. the minimum \( b_1 \) or the minimum \( t^b \), due to feasibility constraints. In such cases, the contractor’s profit will be smaller than the two situations considered below. Therefore, Region I will continue to dominate in all such cases. In the interest of brevity, we do not consider such cases.

**Case a**: Suppose \( \frac{q_1^c}{q_1^e} \geq \frac{c_U}{c_U} \). Then, the contractor’s optimal corner solution is \( b_0 = \bar{c}_0 \), \( b_1 = c_1 \) and \( t^b = \frac{s - \bar{c}_0 - c_1 q_1^c}{c_U} \). From (16), the contractor’s profit is bounded as follows.

\[
\pi^{II}(s, \gamma) \leq \bar{c}_0 + c_1 q_1^c + \left( \frac{c_U}{c_U} \right) (s - (\bar{c}_0 + c_1 q_1^c + c_U(t + g(\gamma)))) - (c_0 + c_1 q_1) - h(\gamma)
\]

\[
= \bar{c}_0 + c_1 q_1^c (1 - \frac{c_U}{c_U}) - (c_0 + c_1 q_1) - h(\gamma) + \left( \frac{c_U}{c_U} \right) (s - (\bar{c}_0 + c_U(t + g(\gamma))))
\]

\[
\leq \bar{c}_0 + \left( \frac{q_1^c}{q_1^e} \right) (s - (\bar{c}_0 + c_U(t + g(\gamma)))) - (c_0 + c_1 q_1) - h(\gamma)
\]

\[
= \pi^I(s, \gamma)
\]

(20)

The second inequality above utilizes the fact that \( s - (\bar{c}_0 + c_U(t + g(\gamma))) \geq c_1 \). The final equality comes from comparing the previous expression with (19). Therefore, the contractor will choose \( t^b \) in
Region I.

Case b: Suppose \( \frac{q^b}{q^1} < \frac{c_U}{c_I} \). Then the contractor chooses \( b_0 = \overline{c}_0, \ t^b = \max\{\underline{c}, t + g(\gamma)\} \), and \( b_1 = \frac{s-(\overline{c}_0+c_U(\max\{\underline{c}, t + g(\gamma)\}))}{q^1} \). Substituting these values in (16), we obtain the following expressions for the contractor’s profit.

\[
\pi^{II}(s, \gamma) = s - c_U(\max\{\underline{c}, t + g(\gamma)\}) + (s - (\overline{c}_0 + c_U(\max\{\underline{c}, t + g(\gamma)\}))(\frac{q^1}{q^1}) - 1) + c_I(\max\{\underline{c}, t + g(\gamma)\} - (t + g(\gamma))) - (c_0 + c_1q_1) - h(\gamma)
\]

\[
= \overline{c}_0 + (s - (\overline{c}_0 + c_U(\max\{\underline{c}, t + g(\gamma)\})))\left(\frac{q^1}{q^1}\right) + c_I(\underline{c} - (t + g(\gamma))) + (c_0 + c_1q_1) - h(\gamma)
\]

\[
\leq \overline{c}_0 + (s - (\overline{c}_0 + c_U(t + g(\gamma))))\left(\frac{q^1}{q^1}\right) - (c_0 + c_1q_1) - h(\gamma)
\]

\[
= \pi^I(s, \gamma).
\]  

(21)

The following arguments are used to obtain the inequality in (21) above. First, \( \max\{\underline{c}, t + g(\gamma)\} = t + g(\gamma) + (\underline{c} - (t + g(\gamma)))^+ \), which when substituted in the preceding equality leads to

\[
-\left(\frac{q^1}{q^1}\right) c_U(\max\{\underline{c}, t + g(\gamma)\}) + c_I(\underline{c} - (t + g(\gamma)))^+ = -\left(\frac{q^1}{q^1}\right) c_U(t + g(\gamma)) + (c_I - \frac{q^1}{q^1}c_U)(\underline{c} - (t + g(\gamma)))^+.
\]

Finally, the inequality results from the fact that \( \frac{q^b}{q^1} < \frac{c_U}{c_I} \). An immediate consequence of (21) is that in case b as well, the contractor would prefer \( t^b \) in Region I.

Region III: In this case, the range of \( t^b \) values is \( \max\{\underline{c}, t + g(\gamma) + t_I\} \leq t^b \leq \overline{c} \). The contractor’s expected profit is derived from Equation (11) as follows.

\[
\pi^{III}(b_1, t^b \mid s, \gamma) = s - c_U t^b + b_1(q^1 - q^1) - (c_0 + c_1q_1) - h(\gamma) + c_I t_I
\]

\[
= s - (c_U - c_I)t^b + b_1(q^1 - q^1) - (c_0 + c_1q_1) - h(\gamma) + c_I(t_I - t^b)
\]

\[
\leq s - (c_U - c_I)t^b + b_1(q^1 - q^1) - (c_0 + c_1q_1) - h(\gamma)
\]

\[
-c_I(t + g(\gamma)).
\]  

(22)

The last inequality comes from the fact that \( t^b \geq t + g(\gamma) + t_I \).

The feasible values of \( t^b \) are obtained from the defining inequalities of Region III. The bounds on \( b_1 \) values are similar to the bounds we obtained for Regions I and II. They can be written as

\[
b_1 \leq \overline{b}_1^{III} = \min [\overline{c}_1, s - (\overline{c}_0 + c_U \max\{\underline{c}, t + g(\gamma) + t_I\})],
\]  

(23)

\[
b_1 \geq \underline{b}_1^{III} = \max [\underline{c}_1, s - (\overline{c}_0 + c_U \overline{c})],
\]  

(24)
From Equation (22), we observe that \( \pi^{III} \) is decreasing in \( t^b \). Moreover, \( \pi^{III} \) is increasing in \( b_1 \) if \( q_1 \geq q_1^c \) and decreasing otherwise.

If \( q_1 \geq q_1^c \), then the contractor’s profit is increasing in \( b_1 \) in regions II and III. Because \( \pi^{III}_1 \leq \pi^{II} \), for each feasible value of \( b_1 \) in Region III, there is an equal or larger feasible value in Region II. Moreover, the time bid in region III is at least as large as the time bid in region II. Therefore, \( \pi^{III} \leq \pi^{II} \). This follows from the fact that the right hand side of (22) is also the contractor’s expected profit in region II (see Equation (16)).

We are now ready to compare the contractor’s profit functions in Region II and Region III when \( q_1 \leq q_1^c \). If the contractor’s choice of \( b_1 \) in Region III is also feasible in Region II (i.e. \( b_1^{III} = b_1^{II} = c_1 \), then \( \pi^{III} \leq \pi^{II} \). This follows from (22) and the fact that time bid in Region III is at least as large as in Region II. Thus, the only case we need to evaluate is one where \( b_1^{II} = \frac{s-(\tau_0 + c_U(t + g(\gamma) + t_I))}{q_1^c} \) and either \( b_1^{III} = c_1 \) or \( b_1^{III} = \frac{s-(\tau_0 + c_U(t))}{q_1^c} \), both of which are smaller than \( b_1^{II} \).

For Regions II and III to be both non-empty, we must have \( t < t + g(\gamma) + t_I \leq \tau \). Therefore, upon substituting \( b_1 = \frac{s-(\tau_0 + c_U(t + g(\gamma) + t_I))}{q_1^c} \) in (16) and simplifying, we obtain

\[
\pi^{II} = \tau_0 + \frac{q_1}{q_1^c} (s - (\tau_0 + c_U(t + g(\gamma) + t_I))) - (c_0 + c_1 q_1) - h(\gamma) + c_I t_I
\]

(25)

as a benchmark profit in Region II for comparing profits in the two regions. Turning next to the profit function in Region III, recall that when \( q_1 \leq q_1^c \), the contractor’s profit is decreasing in both \( b_1 \) and \( t^b \). The contractor may have to choose between minimizing either \( b_1 \) first or \( t^b \) first. Suppose the contractor prefers to minimize \( t^b \) first. That is, it sets \( t^b = t + g(\gamma) + t_I \) and subsequently chooses the smallest possible \( b_1 \). Clearly, this value of \( b_1 \) cannot be smaller than \( \frac{s-(\tau_0 + c_U(t + g(\gamma) + t_I))}{q_1^c} \). Upon substituting this value of \( b_1 \) in the right hand side of the first equality of (22), and using the fact that \( b_0 = s - c_U t^b - b_1 q_1^c \), we obtain

\[
\pi^{III} \leq b_0 + \frac{q_1}{q_1^c} (s - (\tau_0 + c_U(t + g(\gamma) + t_I))) - (c_0 + c_1 q_1) - h(\gamma) + c_I t_I \leq \pi^{II}.
\]

(26)

Similarly, from arguments presented in the previous paragraph, if the contractor chooses to minimize \( b_1 \) first, then its choice of \( b_1 \) cannot be smaller than either \( c_1 \) or \( \frac{s-(\tau_0 + c_U(t))}{q_1^c} \), whichever is larger. We substitute each of these values in the right hand side of the first equality of (22) and simplify to obtain

\[
\pi^{III} \leq \begin{cases} 
\tau_0 + c_1 q_1 - (c_0 + c_1 q_1) - h(\gamma) + c_I t_I & \text{if } c_1 \geq \frac{s-(\tau_0 + c_U(t))}{q_1^c}, \\
\tau_0 + \frac{q_1}{q_1^c} (s - (\tau_0 + c_U(t))) - (c_0 + c_1 q_1) - h(\gamma) + c_I t_I & \text{otherwise}.
\end{cases}
\]

(27)

Upon comparing (27) with (25), and the fact that \( \max \left[ c_1, \frac{s-(\tau_0 + c_U(t))}{q_1^c} \right] \leq \frac{s-(\tau_0 + c_U(t + g(\gamma) + t_I))}{q_1^c} \), it
follows that $\pi^{III} \leq \pi^{II}$. That is, the Region II benchmark profit is greater than an upper bound on the contractor’s profit in Region III. Clearly, the optimal profit in Region II is at least as good as the optimal profit in Region III.

The above arguments prove that the contractor will choose its bid parameters to lie in Region I. Hence proved.

### Proof of Proposition 2

The arguments presented below are based on the fact that $\pi'$ is a concave function of $\gamma$ and therefore the first order optimality equations are necessary and sufficient. From Table 2, note that when either (1) $q_1 \geq q_e^e$, or (2) $q_1 < q_e^e$ and $\bar{t} \leq t + g(\gamma_0)$, or else (3) $q_1 < q_e^e$, $t + g(\gamma_0) \leq \bar{t}$, and $\underline{\gamma} \geq \frac{s-(\tau_0+c_U(t+g(\gamma_0)))}{q_1^e}$, the bid parameters do not depend on $\gamma$. Therefore, in these situations, it is easy to confirm that $\gamma_0$ is an unconstrained optimum and a feasible effort level. Hence, this effort level is also optimal for the constrained problem. Effort level $\gamma_0$ is also the most preferred effort level for the Owner.

The remaining scenario occurs when $q_1 < q_e^e$, $t + g(\gamma_0) \leq \bar{t}$, and $\underline{\gamma} \leq \frac{s-(\tau_0+c_U(t+g(\gamma_0)))}{q_1^e}$. Now $b_1$ does depend on $\gamma$ (see column 4 in Table 2) and substituting the value of $b_1$ in (13), we get

$$\pi' = \tau_0 + \frac{q_1}{q_1^e} (s - (\tau_0 + c_U(t + g(\gamma)))) - (c_0 + c_1 q_1) - h(\gamma),$$

subject to the constraints mentioned above. An unconstrained optimum of (28) is $\gamma_1$. Since $\frac{q_1}{q_1^e} < 1$ and $z(\cdot)$ is an increasing function, $\gamma_1 \leq \gamma_0$ and furthermore $g(\gamma_1) \geq g(\gamma_0)$. If $\gamma_1$ satisfies the two constraints above, then it is also the overall optimum in this scenario.

If $\gamma_1$ violates one or more constraints, two cases are possible. If the first constraint is tight, then the optimal $\gamma$ is $\gamma_2$. Similarly, if the second constraint is tight, then the optimal $\gamma$ is $\gamma_3$. Since either $\gamma_2$ or $\gamma_3$ becomes a contender only when the corresponding constraint is violated with $\gamma = \gamma_1$, these solutions need to be considered only when they are at least as large as $\gamma_1$. Similarly, the larger of $\gamma_2$ and $\gamma_3$ will determine the constraint that becomes binding first. In summary, this means that the contractor would choose $\hat{\gamma}_0 = \max\{\gamma_1, \gamma_2, \gamma_3\}$, the largest candidate solution. Moreover, it will bid its true completion time in this instance. Hence proved.

### Proof of Proposition 3

Given the definition of $s_{\min}^{(i)}(\phi^{(i)}, q_1^{(i)})$ as an independent draw from $S_{\min}$, and similar draws by other contractors, the equilibrium bidding strategies can be constructed using standard arguments in auction theory — see, e.g., Krishna (2002) for details. Furthermore, once $s^*$ is determined, the contractor can determine its optimal bid parameters from Proposition 1.