A+B BIDDING: CONTRACTOR AND AGENCY

PERSPECTIVES

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ABSTRACT

A+B contracting is one of many innovative methods used in letting transportation construction projects. In this scheme, contractors’ bids are evaluated on both construction cost and completion time, with the objective of shortening project durations. Although there is significant literature on state transportation agencies’ experience with the use of the A+B mechanism, previous studies fail to provide guidance to agencies on setting key parameters used to score bids and determine incentives or penalties upon project completion. Also not included are recommendations for contractors on what time bids to offer and how much expense to incur on expediting. This paper provides recommendations for both contractors and agencies. It shows that by setting all time-based parameters equal, an agency can ensure that (1) contractors exert more effort on expediting, and (2) competitive forces are the sole drivers of contractors’ profits. When these parameters are unequal, as is frequently the case, contractors have an incentive to offer unrealistically short completion times, anticipating penalties.

Keywords: A+B bidding, Procurement auctions, Incentives, Transportation construction management, Bids, Contractors, Construction cost/time, Delay time.

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1 November 22, 2011
Faster completion of highway and bridge rehabilitation projects has become important to state transportation agencies (STAs) because it reduces complaints by road users. STAs, also referred to as agencies in this paper, have started to pay for speedier project completion in the form of early completion incentives. For example, construction on the interchange between US 75 and Interstate 635, commonly called “Dallas High Five,” included a $32,000 daily incentive for early completion. The $261 million project was completed 13 months before schedule. This project received many accolades, including the Public Works Project of the Year award by the American Public Works Association. The award explicitly recognized superior management of the project and its early completion.

A+B bidding is a project letting mechanism that favors contractors who bid short completion times. Under this mechanism, each bid is evaluated based on the sum of two components. The A component consists of labor, material and mobilization costs. The B component is obtained by multiplying the number of days each contractor bids by a daily road user cost. The contractor with the smallest total bid, i.e. smallest value of A+B, is awarded the contract (see, e.g. Innovative Contracting in Minnesota 2005).

The use of incentives to encourage contractors to bid early completion times and to expend effort to complete projects early raises a number of questions from both contractors’ and agencies’ perspectives. Consider the contractors’ perspectives first. Empirical studies show that in A+B bids, contractors bid short completion times (relative to engineers’ estimates) and complete the projects even earlier. However, is that how contractors ought to bid? That is, do contractors maximize their expected profit by bidding short completion times and trying to earn an incentive by completing early? Contractors could bid such that they receive either an incentive or a penalty for the same overall A+B score. Which of these two strategies results in greater expected profit?

Answers to these questions usually cannot be arrived at simply by examining data from
projects that were let using the A+B mechanism because optimal bids depend on the competitive environment. That is, a rational approach toward bidding requires that contractors take into account how their competitors might bid in addition to estimating the relationship between their own costs and completion times. In this paper, we present a model of strategic bidding decisions that helps explain how rational contractors ought to choose their bid parameters.

Next, consider the agencies’ perspectives. Agencies use economic models (see Minnesota Department of Transportation 2007 for an example) to estimate the true per-day cost of road closures, which is denoted by $c_T$ in this paper. Then, they choose the following time-based incentive parameters, which affect contractor bids and expediting effort:

1. Daily road user cost (RUC), denoted by $c_U$, 
2. Daily incentive rate, denoted by $c_I$, 
3. Daily disincentive rate (or penalty), denoted by $c_D$, and 
4. Maximum number of incentive days, denoted by $t_I$.

Due to a court case involving Alabama Department of Transportation (DOT), agencies recognize that $c_D > c_U$ cannot be enforced\(^1\). Therefore, by and large, agencies choose $c_D \leq c_U$. Frequently, STAs also set $c_U < c_T$ to reduce total expenditures on the project. However, upon examining Minnesota DOT data from projects that have been let using the A+B mechanism, we found wide variation in the choice of the remaining time-based incentive parameters, which indicates a lack of clear policy on how to set time-based incentives. Moreover, such issues have not been addressed in previous papers. We use our model to derive specific recommendations for choosing time-based incentives.

Budget considerations can affect the choice of time-based incentive parameters. For

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\(^1\)The Alabama DOT set the disincentive rate higher than the RUC in a contract and attempted to impose penalties when the contractor was late (Milton Construction Company v. Alabama DOT, US 11th Circuit Court of Appeals 1990). The contractor sued and won, setting a precedent that $c_D > c_U$ cannot be enforced.
example, we have anecdotal evidence that when budgets are tight, agencies are more likely
to set $c_I = 0$, but still include a non-zero penalty for late completion relative to bid time.
Does this practice actually reduce agencies’ total project costs? More generally, how are
agencies’ total project costs affected by their choice of time incentives? By total project
costs we mean the sum of the $A$ component of the winning bid, the true cost of traffic delay
($c_T$ times the expected project completion time of the winning bid), and the cost of incentives
paid to the winning bidder.

Highway and bridge rehabilitation projects often have complementary and interdependent
activities. If STAs rely on bid times to schedule such activities, and bid times are skewed,
early completion on the project may not deliver higher overall value. In fact, inaccurate
estimates of completion times, driven by strategic bidding, may lead to scheduling difficulties.

For example, the Dallas High Five was only one part of a larger plan for improving the LBJ
(US 635) corridor. If Texas DOT had known that the project would be completed before
2006, then perhaps other parts of the LBJ plan could have been implemented early as well.
As such, it may be difficult to take full advantage of early completion when completion times
are not accurately stated by contractors.

In this paper, we use a combination of models, examples, and insights from data to address
the issues identified in the preceding discussion and to test whether the $A+B$ mechanism
selects the “best” bidder in terms of total cost. When a mechanism fails to select the best
bidder, some contractors may be able to earn unreasonable profits. This is also referred to as
incorrect bid sorting. Thus, one of our goals is to investigate whether the $A+B$ mechanism
gives rise to unreasonable profits for some contractors. We emphasize that the best bidder
is not necessarily the one with the lowest $A+B$ score, but rather the one for which the total
$cost$ — i.e. the sum of labor, materials, and expediting costs, plus the cost of traffic delays
— is the smallest among all bidders.

The results shown in this paper indicate that when contractors bid strategically, they bid
short completion times and frequently pay penalties for being late with respect to the bid time. Surprisingly, this is also aligned with the agency’s goal of providing stronger incentives for expediting completion time. We recommend setting $c_I = c_D = c_U$ to avoid incorrectly sorted bids, i.e. the contractor with the lowest total cost may not win. When time based parameters are equal, contractors’ profits depend solely on competitive factors like cost and the number of bidders. This also eliminates incentives to misrepresent completion time. The results indicate that setting $c_I = c_D = c_U < c_T$ reduces agency expenditure, and may be justified because the cost of public inconvenience is not charged to the agency’s budget.

This paper makes a contribution to the construction engineering management literature assisting both agencies and contractors. Using the A+B mechanism STAs can improve the chances of realizing their objectives by anticipating strategic bidding and choosing incentive parameter values appropriately. Our analysis also provides guidance for contractors. We identify contractors’ optimal bidding strategy in response to incentives, both in preparing bids and in expediting. These strategies increase the probability of being awarded a contract and maximize profit. Throughout, we assume that contractors are rational and risk-neutral decision makers, who choose their bids to maximize expected profits.

The remainder of this paper is organized as follows. In Section 2, we provide a review of related literature and institutional background. This section helps to position the contribution of this paper with respect to previous studies. It also contains a basic formulation of the contractor’s problem of setting bid parameters for a given set of time-based incentives. Analysis of the model and examples are presented in Section 3. Section 4 deals with insights from analysis of data from the Minnesota DOT (Mn/DOT). Section 5 concludes the paper.

LITERATURE, BACKGROUND, AND NOTATION

A number of papers discuss the impact of incentives and disincentives on contractors. Early work in this area concentrated on modeling expediting cost from a project management perspective, and providing insights into the internal managerial implications of time-
based incentives (El-Rayes 2001; Senouci and El-Rayes 2009). These and other papers that deal with project expediting provide quantitative methods for estimating contractors’ cost of expediting. However, none of these papers addresses the direct effect of incentives on competitive bidding.

From an agency’s perspective, a key question that arises in the context of A+B bidding is how to set the incentive and disincentive rates. While there are methodologies available for calculating $c_T$, the true per-day cost of road closure (Minnesota Department of Transportation 2007), less is known about setting parameters such as $c_U$, $c_I$ and $c_D$. Moreover, choosing the appropriate incentive scheme from a plethora of choices including I/D and A+bonus, is not a simple task. A survey of current contracting practices and suitability of different schemes to different project settings can be found in Anderson and Damnjanovic (2008). In A+bonus and I/D projects, the target completion time is specified by the STA. The difference is that in the former, a lump sum bonus is paid if the project is completed on time, whereas in the latter there is a daily incentive and disincentive rate. A+bonus is also called “no excuse bonus.”

Shr and Chen (2004) argue that STAs need to understand the production cost curve for expediting in order to correctly set I/D parameters. They develop a formal model for a contractor’s expediting cost and empirically estimate it using data from previous projects (see also Shr et al. 2004). Pyeon and Park (2010) develop a decision support system to aid STAs in choosing the appropriate type of incentive contract based on project parameters. The alternatives considered are A+B, I/D, and A+bonus. For their analysis, the authors collected a large sample of incentive contracts from the Florida DOT and utilized statistical methods to evaluate which parameters drive cost and time performance. This statistical analysis is the foundations for a simulation model to predict the cost and completion time of future projects, including a distribution of possible values around the mean. Similarly, Sillars and Riedl (2007) developed a decision support system using data from Oregon DOT.
This literature provides a foundation for our model. Based on results reported in these papers, our expediting cost function has the following features: a cost is incurred for each day that project completion time is shortened, and this cost increases as the contractor expedites more. This literature also provides a methodology that STAs and contractors can use to estimate similar cost functions based on historical data.

Ellis et al. (2007) analyzes a comprehensive dataset of projects administered by Florida DOT. This paper provides a variety of summary statistics on the performance of various contracting schemes in terms of time and cost. One of the observations from this data analysis is that in a sample of 115 A+B and A+B+bonus projects, the average awarded completion time was shorter than the average engineer’s estimate. However, actual completion time was longer than the bid time, on average. This is consistent with the results we derive from our model.

In developing our models of bidding in transportation construction auctions we draw heavily on the economics literature. The majority of this literature, developed over the past 30 years, deals with strategic bidding – how auction participants respond to incentives generated by the chosen auction mechanism; see Krishna (2002) for a review. The A+B mechanism is an auction with a scoring rule, where bids are evaluated on both cost and completion time. In an early article on scoring auctions Maskin (1992) finds that the project may not be awarded to the low-cost provider. Ewerhart and Fieseler (2003) study generalized scoring auctions and find that the nature of competition has an impact on bidding. Negative correlation across attributes lowers bidder profit, but positive correlation increases the winner’s profit. There is also research in the economics domain studying construction auctions. The majority of this research is empirical, trying to validate theoretical results using data from STA auctions (see, Hong and Shum 2002, and Krasnokutskaya 2009).
Background and Notation

When using the A+B project-letting mechanism, one of the first tasks for an agency is to determine the true value of time $c_T$. The STA may adopt a different daily road user cost $c_U$ to evaluate bids. To be specific, when a contractor submits a bid of $t_b$ days, the B component of that bid is $B = c_U \cdot t_b$. STAs also specify an allowable range for bid time, no less than $t_{\text{min}}$, and no more than $t_{\text{max}}$.

Suppose the project is completed in $t_a$ actual number of days. It is not uncommon for STAs to allow some adjustments, i.e. not count some days as part of actual completion time if delays are caused by factors outside a contractor’s control. We ignore such adjustments throughout this paper. The project is considered early when $t_a < t_b$. In this case the contractor earns incentive pay at a rate of $c_I$, which generates positive windfall. However, incentive payments are capped at an upper limit for early completion. Denote the maximum number of days that incentives are paid by $t_I$. Total incentive payment to the contractor is $c_I \cdot \min\{(t_b - t_a)^+, t_I\}$, where $(\cdot)^+$ denotes the maximum between 0 and the quantity within parentheses. In contrast, when $t_a > t_b$ the project is late and the contractor pays a penalty of $c_D$ for every day it is late relative to $t_b$, for a total penalty of $c_D \cdot (t_a - t_b)^+$. The time-based parameters $c_U$, $c_I$, and $c_D$ are decision variables for STAs. In this paper we assume that $c_I \leq c_D \leq c_U \leq c_T$, which is consistent with practice and legal precedent.

From the contractor’s perspective time-based parameters are fixed. The contractor’s decisions are resource investments in shortening the project duration and bid parameters. Specifically, a contractor’s bid parameters are its A component, denoted by $A_b$, and bid time, denoted by $t_b$. These parameters result in a score $s_b$, which can be calculated as follows:

$$s_b = A_b + c_U t_b. \quad (1)$$

Note that because contractors bid strategically, $A_b$ and $t_b$ need not equal true estimates of their A component cost and project completion time.
We assume that for each contractor there is a nominal completion time $t_0$, which would occur in absence of explicit time-based incentives. Note, there may be implicit benefits to contractors from completing early, such as receiving payment early. Implicit benefits are included in $t_0$. A contractor has numerous ways to shorten completion time relative to $t_0$, including working additional shifts, hiring more experienced labor, leasing additional equipment, and outsourcing. Each of these induces additional costs. Choosing which methods to use to shorten project duration depends on the resource availability and incentives and disincentives facing the contractor. For simplicity we use the term effort to represent all techniques that a contractor may employ to shorten project duration. The level of effort is denoted by $e$, where $e$ represents the number of days by which the project is shortened relative to $t_0$. The expected actual project completion time is $t_e = t_0 - e$.

A cost of expediting is modeled by a quadratic function of the form $he^2$. This means that it is relatively inexpensive to exert a small amount of effort in expediting, since initially the contractor chooses simple activities. When simple solutions are exhausted, the contractor invests in more costly alternatives, implying that the rate of change in the cost of effort is increasing. Previous research has addressed the implications of incentives and costs on project management in detail (Shr and Chen 2004; Shr et al. 2004). Results in these papers support our choice of a quadratic expediting cost function.

In the next section, we develop a model to describe how a contractor may choose its bid parameters. For this purpose, we need to define several other parameters. We define a break-even score $s_0$, which generates zero expected profit for the contractor. Finally, we define a parameter $c_X$, which represents the marginal revenue from expediting. Its value can be one of $c_D$, $c_I$, or 0. Specifically, when the project is expedited more than the maximal

2A project’s actual completion time may be random, i.e. $T_a = t_e + \xi$, where $\xi$ is the random component. One may view $\xi$ as the uncertainty in the contractor’s mind about the precise values of $t_0$ and $e$. In all models presented in this paper, we assume that $\xi$ is negligibly small, and hence it may be ignored. Our results carry over to the case in which completion times are random so long as each contractor’s estimate of its $\xi$ is independent of other contractors’ estimates (see Gupta and Snir 2010 for details).

3Note that we use the quadratic cost function to illustrate the concepts introduced in this paper. Our results hold so long as the expediting cost is increasing each day project completion is shortened.
incentive days, the marginal revenue is zero. When the project is finished early (but not
shorter than the maximal incentive time) the marginal revenue is the daily incentive rate.
When the project is finished late the marginal revenue is the daily penalty. More formally,
c_X is defined as follows:

\[
c_X = \begin{cases} 
0 & \text{if } t_e < t_b - t_I, \\
c_I & \text{if } t_b - t_I \leq t_e \leq t_b, \text{ and} \\
c_D & \text{otherwise.}
\end{cases}
\]  

MODEL FORMULATION & ANALYSIS

In this section we focus attention on a single representative contractor, which we refer
to as the contractor. The contractor’s bid gives rise to its A+B score, which determines
contract award because the lowest score wins the contract. The score is also an important
determinant of the contractor’s profit. Note that the contractor’s profit is the difference
between what the contractor is paid and its costs, but the amount the contractor is paid is
not equal to its score. However, payment received by the contractor is increasing in score.
Any bid with a score higher than the break-even score \(s_0\) generates profit.

We focus primarily on the B component of the bid and view the A component as a lump
sum. The A component typically consists of a mobilization cost plus a unit bid on each
line item that constitutes the scope of work for the project. When contractors submit unit
bids, they can also impact the magnitude of payments for time and material relative to the
A component included in the bid (Gupta and Snir 2010). Such differences are referred to
as windfall in this paper. The windfall that comes from the A component is also present
in standard contracts that do not require a time bid. Therefore, in order to keep our dis-
cussion focused on the role of time incentives, we do not consider the windfall due to the
A component. The A component of the contractor’s bid, denoted by \(A_b\), may be different
from its actual costs. This gives flexibility to the contractor to account for the effect of
windfall related to time bid. Such windfall may be positive or negative, depending on how
the contractor bids. Positive windfall is expected when the contractor anticipates completing
the project before the bid date \( t_e < t_b \). Windfall is important because ignoring a positive
windfall suggests the contractor bid too high and may lose a contract that it could have won.
Similarly, ignoring a negative windfall may lead to aggressive bidding and subsequent losses,
as illustrated in the ensuing example.

**Example 1:** Consider the following hypothetical but typical scenario for a highway resurfacing project. The engineer’s estimate for the project cost is an A component of $3 million
and a completion time of 250 days. This is also the maximum allowed time bid. The minimum time bid is set at \( t_{min} = 175 \) days. Time based daily incentive parameters are as follows: \( c_I = $3,000 \), \( c_D = $4,000 \), \( c_U = $5,000 \) and \( c_T = $7,000 \). Incentives are paid for up to 30 days, i.e. \( t_I = 30 \). Assume that the engineer’s estimate is accurate. That is, for the contractor of interest, the mobilization and line-item components cost $3 million and the base completion time, without exerting effort to expedite, is \( t_0 = 250 \) days. The contractor has an expediting cost parameter of \( h = $50 \).

Consider two possibilities. In the first case, the contractor bids \( t_b = 250 \) days, although it plans to expedite by 30 days and complete the project in \( t_e = 220 \) days. Expediting cost is $45,000. If the contractor were to place a bid consisting of \( A_b = $3.045 \text{ million} \) and \( t_b = 250 \) days, its A+B score would be $4.295 million. Note that \( A_b \) covers mobilization, line-item components, and expediting costs. However, this bid does not account for a windfall earning of $90,000, which could reduce the contractor’s chances of winning the contract. In the second case, suppose the contractor bids \( t_b = 185 \) days with an intention to complete the project in 210 days. In this case, the contractor’s expediting cost is $80,000 and its expected penalty is $100,000. If the contractor were to ignore the penalty and bid its mobilization, line-item components, and expediting cost of \( A_b = $3.08 \text{ million} \), its A+B score would be $4.005 million. The lower score may result in the contractor winning the project, but this would lead to monetary losses. To cover the cost of penalties, the contractor needs to increase the mobilization component by $100,000.
Suppose the contractor bids $s_b$. Then, its profit is based on the following components:

1. the difference between $A_b$ and actual costs $A$, which may be positive or negative;
2. cost of expediting; and
3. incentive payments or penalties.

There are three cases to consider, as discussed below.

**Case 1:** $t_e < t_b - t_I \Leftrightarrow t_0 < t_b - t_I + e$

$$\pi_b(e, t_b) = A_b - A - he^2 + c_I t_I$$

**Case 2:** $t_b - t_I \leq t_e \leq t_b$

$$\pi_b(e, t_b) = A_b - A - he^2 + c_I (t_b - t_e)$$
$$= A_b - A - he^2 + c_I (t_b - t_0 + e)$$

**Case 3:** $t_b < t_e$

$$\pi_b(e, t_b) = A_b - A - he^2 - c_D (t_e - t_b)$$
$$= A_b - A - he^2 - c_D (t_0 - e - t_b)$$

Note that in all three cases $A_b = s_b - c_U t_b$, $s_b$ is fixed, and the contractor’s profit is separable in $e$ and $t_b$. It is also linear in $t_b$, and concave in $e$. This means that the optimal values of these parameters can be found by equating the first derivative of $\pi_b$ to zero or else by making a defining constraint binding. For example, in case 1, if the defining constraint is binding, then $e = t_0 - t_b + t_I$. 
By taking the derivative with respect to $e$, and utilizing the resulting first-order condition, we find the best unconstrained effort level, $e^*$ as follows

$$e^*(c_X) = c_X / 2h$$

(3)

where $c_X$ is defined in (2). The above expression relates expediting effort, measured in number of days, to the incentives offered to the contractor. It shows that stronger incentives lead to greater effort, which in turn generates shorter projects. Equation (3) also provides a key relationship for several important results, which we discuss next.

**Proposition 1.** When $c_I \leq c_D$, projects in which $t_b \leq t_e$ are completed earlier.

When a contractor anticipates penalties, its marginal incentive rate is $c_D$. When it expects to complete the project early, its marginal incentive rate is $c_I$. Since $c_I \leq c_D$, effort is higher when penalties are expected, and the expected completion date, $t_e = t_0 - e$ is earlier relative to $t_0$. There are important implications of Proposition 1 on the bid time $t_b$. If the contractor bids an unrealistic short completion date ($t_b$), penalties are likely, and the expected completion time is earlier. This suggests that the STA may want the contractor to offer an unrealistic short lead time, if its objective were to encourage greater expediting effort. Later, we will show that a contractor also prefers $t_b \leq t_e$. Throughout the paper, we use the term “prefer” to mean weak preference.

**Proposition 2.** Capping incentive days reduces incentives to expedite, lengthening projects.

When the contractor expects to earn the maximal incentive pay ($t_e < t_b - t_I$), the marginal incentive rate becomes 0. The contractor does not have an incentive to shorten the project below $t_b - t_I$. There are two important implications from this. First, as before, the STA may prefer an unrealistic short bid time to assure that the contractor has greater incentives to continuously shorten the project. Second, capping incentive payments lengthens projects unnecessarily. The implications on STA expenditures that come from removing incentive
Recommendations: Based solely on the analysis of expediting effort, we can identify a number of recommendations for parameter choices by STAs. These are (1) remove incentive caps, (2) set \( c_I = c_D \), and (3) provide stronger incentives by setting the I/D rates equal to the road user cost, i.e. \( c_I = c_D = c_U \). Our analysis of contractors’ bids in the next section further strengthens these recommendations.

Contractor bids

We notice in the previous section that relative to its bid, a rational contractor should not expedite the project by an amount greater than the maximum incentive time. When incentives are too powerful (i.e. \( t_0 - c_I/2h < t_b - t_I \)), the contractor should expedite to assure completion exactly at \( t_b - t_I \), with \( e^* = t_0 - t_b + t_I \). Upon taking into account the effect of incentives on expediting effort in the remaining cases (i.e. upon substituting for \( e^* \) into the profit function), we obtain the contractor profit function as follows.

\[
\pi_b(t_b) = \begin{cases} 
A_b - A - h(t_0 - t_b + t_I)^2 + c_I t_I & \text{if } c_I > 2h(t_0 - t_b + t_I), \\
A_b - A - (c_I^2/4h) + c_I(t_b - t_0 + c_I/2h) & \text{if } t_b - t_I \leq t_e \leq t_b \text{ and } c_I \leq 2h(t_0 - t_b + t_I) \\
A_b - A - (c_D^2/4h) - c_D(t_0 - t_b - c_D/2h) & \text{otherwise.} 
\end{cases}
\]

The second and third expressions above can be re-arranged to yield, respectively, \( A_b - A + (c_I^2/4h) + c_I(t_b - t_0) \) and \( A_b - A + (c_D^2/4h) + c_D(t_b - t_0) \), which has the following common form.

\[
\pi_b(t_b) = A_b - A + (c_X^2/4h) + c_X(t_b - t_0) \\
= s_b - (c_U - c_X)t_b - A + (c_X^2/4h) - c_X t_0, \tag{5}
\]
where $c_X$ is either $c_I$ or $c_D$, depending on the value of $t_b$ relative to $t_e$. The second equality above is obtained after substituting $A_b = s_b - c_I t_b$, which allows us to think about the contractor’s choice as consisting of two components – the bid score $s_b$ and the offered completion time $t_b$.

The above simplifications also allow us to conclude that the contractor will not voluntarily choose $t_b$ such that $c_I > 2h(t_0 - t_b + t_I)$. To see why this is true, we successively simplify the contractor’s profit in case 1 above.

Given $c_I > 2h(t_0 - t_b + t_I)$,

\[
\pi_b(t_b) = A_b - A - h(t_0 - t_b + t_I)^2 + c_I t_I
\]

\[
< A_b - A - (c_I/2)(t_0 - t_b + t_I) + c_I t_I
\]

\[
= A_b - A + (c_I/2)(t_b - t_0 + t_I).
\]

Clearly, the profit of the contractor calculated above is smaller than $A_b - A + (c_I^2/4h) + c_I (t_b - t_0)$, its profit in the second case described earlier\footnote{Upon taking the difference between the contractor profit in the two cases, we obtain $A_b - A + (c_I/2)(t_b - t_0 - t_I) - [A_b - A + (c_I^2/4h) + c_I(t_b - t_0)] = (c_I/2)[(t_b - t_0 + t_I) - c_I/2h]] - c_I(t_0 - t_I) < 0$. The preceding inequality comes from the assumption that $c_I > 2h(t_0 - t_b + t_I)$.}. From here onward, we assume that in relation to $c_I$, $t_{min}$ specified in the project is not so high that it prevents the contractor from choosing $t_b \leq (t_0 + t_I) - c_I/2h$. Following this reasoning, there is no need to consider Case 1 in the remainder of the analysis.

**Example 2:** To illustrate how the choice of offered bid time affects profits, consider the contractor described in Example 1. Suppose this contractor bids mobilization, line-item components costs of $3 million and a completion time of 250 days. With $c_I = $3,000, the optimal value of $e$ is 30 days, which results in a net extra profit of $45,000. If the cap on incentive payment were $t_I = 20$ days the contractor would expedite only 20 days, with an extra profit of $40,000. Finally, given these tradeoffs, if the contractor wants to bid away a part of its incentive payment, or wishes to increase its profit further, it can do so by either increasing or decreasing the value of its $A_b$. 

\#
A first step in deriving a target bid score is identifying a bidding strategy that minimizes score $s_0$ associated with the break-even profit. This is true because any target bid can be obtained from $s_0$ by inflating the mobilization cost component of $A_b$, while keeping other bid parameters intact. No rational contractor will bid lower than $s_0$. The tradeoffs that a contractor faces when deciding how much to inflate its bid are as follows: greater markup over $s_0$ increases profit if the contractor wins, but it also reduces the probability of winning. An optimal target is the bid score that maximizes the contractor’s expected profit, which is the product of profit and probability of winning. It should be clear that these tradeoffs can be evaluated independently of the problem of choosing bid parameters that minimize $s_0$, and that regardless of the extent by which the contractor inflates $s_0$, it would want to choose bid parameters that minimize $s_0$. Therefore, we first propose the following optimization problem to obtain $s_0$.

$$s_0 = \min s_b$$

Subject to: $\pi_b = 0$

From (5), upon equating $\pi_b(t_b)$ to zero and replacing $s_b$ by $s_0$ in the resulting equation, we obtain the following expression for the break-even score as a function of $t_b$.

$$s_0(t_b) = A - c_X^2/4h + (c_U - c_X)t_b + c_X t_0.$$  \hspace{1cm} (7)

Because $c_I \leq c_D \leq c_U$, the break-even score is an increasing function of $t_b$. That is, a rational contractor would choose $t_b = t_{\text{min}}$, expend effort $e^*$ (see Equation (3) for how $e^*$ is determined), and expect to incur penalties. We encapsulate this result in a proposition below.

**Proposition 3.** When $c_I \leq c_D \leq c_U$, a rational contractor prefers to bid $t_b = t_{\text{min}}$, expend effort $e^*$ according to the relationship shown in Equation (3), and incur penalties for being late.
Example 3: Consider the contractor whose situation is described in part 1 of Example 1. Recall that this contractor bids a completion time of $t_b = 250$ days and its true mobilization and line-item components cost of $3$ million. It expedites the project by 30 days and its cost of mobilization, line-item components, and expediting are $3.045$ million, with expected windfall of $90,000$. Subtracting the windfall from the cost generates a break-even mobilization and line-item components cost of $2.955$ million. Next, we rewrite (7) in a more convenient form by using the fact that $c_X / 2h = e^*$. This yields

$$ s_0(t_b) = A - (c_X / 2)e^* + c_X (t_0 - t_b) + c_U t_b. $$

(8)

It should be clear that what we call break-even mobilization and line item component cost above is nothing but $A - (c_X / 2)e^* + c_X (t_0 - t_b)$. Note, in this case $c_X = c_I = 3,000$. The contractor’s minimum bid score is therefore $s_0(250) = 2.955 + .005 \times 250 = 4.205$ million, where $c_U = 0.005$ when expressed in units of millions of dollars.

If, as in part 2 of Example 1, the completion time bid is 185 days, then cost of mobilization, line item components, and expediting is $3.085$ million, with expected penalties of $100,000$. The break-even mobilization and line-item components bid in that case would be $3.185$ million, and the minimum bid score would be $s_0(185) = 4.105$ million. This exemplifies our more general result in Proposition 3 that bidding a shorter completion time reduces minimum bid score and thereby makes the contractor’s bid more attractive. #

Earlier, we mentioned the notion of windfall and presented an example to underscore its importance in determining a contractor’s bid. It is useful at this point to explicitly recognize the components of a contractor’s profit function that we call windfall. We reorganize (7) to obtain

$$ s_0(t_b) = A + c_U t_e - c^2 X / 4h + c_U (t_b - t_e) + c_X (t_0 - t_b). $$

(9)

The right-hand side of (9) can be thought of as consisting of 3 parts: (1) the contractor’s score if it were to bid actual value of the A and B components, i.e. $A + c_U t_e$, (2) the expediting
cost $c_X^2/4h$, and (3) the windfall consisting of $c_U(t_b - t_e) + c_X(t_0 - t_b)$. The windfall itself has two components: the first component comes from the difference between the bid time and the expected completion time, and the second from the difference between the nominal completion time and the bid time. This latter component can be loosely interpreted as the payoff from expediting. If $t_b = t_e$, then the second component equals the I/D payment (positive or negative) from expediting. It is usually positive, as the nominal completion time is a reasonable upper bound for bid time. Similarly, the first component may be either positive or negative. For example, if the offered bid time is equal to or longer than the expected completion time, then the first component is positive. If however, $t_b = t_{\text{min}}$, as suggested by our analysis above, then the first component is negative. Coupled with the fact that $c_I \leq c_D \leq c_U$, the first component may easily dominate the second, making the overall windfall negative. Note that a naïve bidder would include only the first two components, which leads to suboptimal bidding.

Before closing this section, we present arguments that describe how a contractor would choose $s_b$. Each contractor can estimate its own $s_0$ after a careful consideration of the consequences of its bid parameters, similar to the analysis performed above. It can also estimate a probability distribution from which other contractors’ values of $s_0$ will be determined, although each bidder does not know the precise value of other bidders’ $s_0$. Because each bidder, after observing its own $s_0$, will remain a contender only if it believes that its own $s_0$ is the smallest among all other bidders’ $s_0$ values, it will want to bid an amount that allows it to maximize its profit while still winning. This is done by choosing a value of $s_b$ that equals the expected value of the second highest $s_0$ among all contractors, given that the contractor’s own $s_0$ is the smallest value. Let $S_0^{(j)}$ denotes an ordered sequence of random realizations of break-even bids of $j = 1, \cdots, n$ contractors\(^5\). The contractor who observes its own break-even cost to be $s_0$ will bid an amount equal to $E[S_0^{(2)} | S_0^{(1)} = s_0]$. Details of this approach can be found in Krishna (2002), among other sources. To help provide additional

\(^5\)This means that $S_0^{(1)} \leq S_0^{(2)} \leq \cdots, S_0^{(n)}$. 

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intuition behind a contractor’s choice of bidding strategy, we present a stylized example.

**Example 4:** Suppose there are two contractors, labeled 1 and 2. For each contractor, the break-even score may be either low or high. This may happen when, for example, expediting costs depend on the number of other projects the contractor might be awarded. When there are few other projects, costs are low, whereas when there are many other projects, the break-even score is high.

Suppose that for Contractor 1 the low break-even score is $s_{1,L} = $3 million and the high break-even score is $s_{1,H} = $4 million. For Contractor 2 these values are $s_{2,L} = $3.25 million and $s_{2,H} = $3.5 million respectively. We assume that each contractor knows its own break-even score at the time of bidding, and also knows the possible scores the other contractor may have. However, each contractor does not know which score the other contractor realized for this project. That is, from the perspective of Contractor 1, the score for Contractor 2 is a random variable, and vice versa. In practice, contractors may learn about the range of possible outcomes for other contractors from previous interactions in this market.

These assumptions allow us to develop a bidding strategy for each contractor. The strategy depends on the contractor’s realized score. For Contractor 1, when the realized score is low, any bid below 3.25 is assured to win the project, hence it would bid $3.249 million. When the realized score is high, any bid offered will not win the project, so the optimal bid is exactly the break-even score of $4 million.

For Contractor 2, the bid does not depend on the realized score for the following reasons. If Contractor 1 realizes a low score, Contractor 2 cannot place a bid that would win the project and earn a profit. If, however, the realized cost for Contractor 1 is high, then Contractor 2 can offer any bid just below $4 million and be awarded the project. Thus, the optimal bid (in equilibrium) is to bid $3.999 million, and be awarded the project whenever Contractor 1 realizes high costs. The bid placed by Contractor 2 does not depend on its realized break-even score.

#
For the general case, with multiple contractors and a more complex distribution of scores, the calculations are more complicated, but the information requirements and the logic underlying the calculation of optimal bids remain the same.

**Bid sorting**

We now turn to the question of whether the A+B mechanism awards contracts to the lowest-cost bidder, which would be an ideal outcome. The cost of each contractor comprises of the sum of its A component, its expediting cost, and its expected completion time multiplied by $c_T$. Note that delay penalty $c_D(t_e - t_{\text{min}})$ (which is incurred by the contractor when it offers a bid of $t_{\text{min}}$) is not counted because it is included up front in the contractor’s bid and later paid to the agency as delay penalty. We also do not count markup by the contractor. In what follows, we show that when $c_D < c_T$, the A+B mechanism may distort contractor sorting, with the project being awarded to a contractor that does not have the lowest total cost.

**Proposition 4.** When $c_D < c_T$ there is a strictly positive probability that the A+B mechanism incorrectly sorts contractors, with the result that the lowest-cost contractor may not win the contract in all instances.

We outline a proof of the above assertion with the help of an example involving 2 bidders, denoted by subscripts $i = 1, 2$. Based on Proposition 3, both contractors would bid $t_b = t_{\text{min}}$. We assume that to be the case, although this assumption does not drive the result and the result would hold if $t_b = t_e$ for each contractor. When bidding a short completion time both contractors expect to pay penalties and the estimated completion time is based on an incentive rate of $c_D$. The expected completion time depends on $t_{0,i}$ and expediting level $e_i$, with $t_{e,i} = t_{0,i} - e_i$, and the expected completion times of the two contractors may be different because of either a different $t_0$ or a different $h$.

Suppose Contractor 1 has a lower cost. That is, $A_1 + h_1 e_{1}^2 + c_T t_{e,1} < A_2 + h_2 e_{2}^2 + c_T t_{e,2}$. 

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Rewriting the condition for Contractor 1 to have lower total cost we get:

\[
A_1 + (c_D/2)e_1 + c_T(t_{0,1} - e_1) < A_2 + (c_D/2)e_2 + c_T(t_{0,2} - e_2)
\]
\[
\iff A_1 - A_2 < c_T(t_{0,2} - t_{0,1}) - (c_T - c_D)(e_2 - e_1).
\]

(10)

The inequality above is obtained upon using the fact that \(e_i = c_D/2h_i\).

Incorrect bid sorting requires that the project is awarded to Contractor 2, even though Contractor 1 has a lower total cost. Contractor 2 will win the contract, in spite of being the higher cost contractor, if \(s_{0,2} < s_{0,1}\). Using (7), with a bid time of \(t_{\min}\) and anticipating penalties we get

\[
s_{0,i} = A_i - c_D^2/4h_i + (c_U - c_D)t_{\min} + c_D t_{0,i}
\]
\[
= A_i - (c_D/2)e_i + (c_U - c_D)t_{\min} + c_D t_{0,i}.
\]

The inequality \(s_{0,2} < s_{0,1}\) occurs when

\[
A_2 - (c_D/2)e_2 + (c_U - c_D)t_{\min} + c_D t_{0,2} < A_1 - (c_D/2)e_1 + (c_U - c_D)t_{\min} + c_D t_{0,1}
\]
\[
\iff A_1 - A_2 > c_D(t_{0,2} - t_{0,1}) - (c_D/2)(e_2 - e_1).
\]

(11)

It can be shown that both (10) and (11) can be true, with a non-empty range for \(A_1 - A_2\).

In particular, the range of \(A_1 - A_2\) is non-empty if \((c_T - c_D)(t_{0,2} - t_{0,1}) > (c_T - c_D)(e_2 - e_1)\), or equivalently, \(t_{e,2} = t_{0,2} - e_2 > t_{e,1} = t_{0,1} - e_1\). It is quite possible and perhaps often the norm in A+B bidding environments that different contractors would have different expected completion times after factoring in their optimal amount of effort. Thus, we have shown that the A+B mechanism is not guaranteed to award the project to the lowest total-cost contractor.

An immediate corollary of Proposition 4 is that STAs can avoid incorrect bid sorting by setting \(c_D = c_U = c_T\) because in that case, the above range is empty. This choice of STA parameters has the additional advantage of strengthening contractor incentives for expediting.
Example 5: Suppose there are two contractors bidding on a project and the minimum bid time 
\( t_b = t_{\text{min}} = 175 \) days. Contractor 1 is the same contractor that we introduced in Example 1. 
Recall that for this contractor, \( A_1 = \$3 \) million, \( t_{0,1} = 250 \) days, and \( h_1 = \$50 \). Contractor 
2, in contrast, has mobilization and line-item components cost of \( A_2 = \$3.1 \) million, shorter 
nominal completion time of \( t_{0,2} = 205 \) days, and a significantly higher expediting cost of 
\( h_2 = \$5,000 \).

If the STA were to choose \( c_I = c_D = c_U = c_T = \$7,000 \), both contractors would bid 
\( t_b = t_{\text{min}} = 175 \) days. Contractor 1 would expedite by 70 days, completing the project in 180 
days and incurring a penalty for being late by 5 days. Using (8), its minimum bid score would 
be \( s_{0,1}(180) = \$4.505 \) million. Similarly, Contractor 2 would expedite by 0.7 days, incurring 
a penalty for being late by 29.3 days. Using (8) once again, this contractor’s minimum 
bid score would be \( s_{0,2}(204.3) = \$4.53255 \) million. The total project cost to the agency of 
choosing either contractor, without considering the markup, is equal to each contractor’s 
minimum bid score. Therefore, in this case, the bids will be sorted according to the agency 
costs and neither contractor makes an unreasonable profit.

Next, consider the situation in which the agency sets parameters as before, i.e. \( c_I = 
\$3,000 \), \( c_D = \$4,000 \), and \( c_U = \$5,000 \). Contractor 1 would complete the project in 210 
days, with a break-even bid score of \$4.095 million. Contractor 2 would complete the project 
in 204.6 days and its bid will be more attractive with a score of \$4.0942 million. Therefore, 
contractor 2 may win the bid. That is, the bids may be sorted differently depending on what 
value the agency chooses for \( c_I, c_D \) and \( c_U \).

Agency Expenditures

Next, we analyze the expenditures incurred when A+B bidding mechanism is used. Three 
questions arise in this context.

1. What are the total expenditures under A+B?
2. How do expenditures vary with changing bid parameters?

3. Does the A+B mechanism increase contractor profit?

The total expenditure for the STA when using A+B depends on the winning contractor’s break-even score, $s_0$. Contractor profit above break-even, $s_b - s_0$, is due to the competitive environment, not A+B. With a bid of $t_b = t_{\text{min}}$, the compensation from the STA is related to the winning bidder’s $s_0(t_{\text{min}})$. Rearranging (8), we obtain

$$s_0(t_{\text{min}}) = A - (c_D^2/4h) + c_U t_{\text{min}} + c_D(t_0 - t_{\text{min}})$$

$$= A + c_D^2/4h + c_U t_{\text{min}} + c_D(t_0 - c_D/2h - t_{\text{min}})$$

(12)

The second equality above is obtained by adding and subtracting $c_D^2/2h$ from the right-hand side of the first equality. Evaluating each component clarifies the impact on expenditure. The first two components are the material, labor and mobilization costs and the costs of expediting, respectively. As this mechanism provides incentives for early completion, the STA needs to compensate the contractor for investments in shortening project completion. Stronger incentives (higher values of $c_D$) hasten completion, but this cost is transferred to the STA. The third component is the value of the B component, which would be added to the bid score irrespective of the actual completion time. The fourth component is the penalty that the contractor anticipates when bidding.

We now turn to the impact of parameter choice. The above equation helps us answer this. STAs frequently set the incentive rate lower than the penalty rate $c_I < c_D$, hoping to reduce project expenditure. When contractors bid optimally, the incentive rate has no impact on expenditure. The incentive rate affects completion time when contractors bid sub-optimally, i.e. an amount greater than their expected completion time. In that case, a lower value of $c_I$ induces contractors to expend less effort on expediting project completion, which further supports setting equal rates, $c_I = c_D$. Setting the penalty rate lower than the
road user cost, i.e. $c_D < c_U$, has two counteracting effects. Expenditure decreases because
the contractor spends less on expediting, i.e. the second term in (12) is smaller. However,
the fourth term may be higher or lower, depending on parameter values because the per-day
penalty is lower, but the number of days late relative to bid is higher. Note that $t_b$ remains
equal to $t_{\text{min}}$ so long as $c_I \leq c_D \leq c_U$. Therefore, the overall impact can be determined only
after knowing parameter values.

The STA by setting $c_I = c_D = c_U$ eliminates the benefits to contractors of misrepresenting
completion time, and $t_{\text{min}}$ is no longer a factor in determining the agency’s total expenditure.
This can be seen by substituting $c_I = c_D = c_U$ into (9). We also recommend that all time-
based parameters should be set equal to the true road user cost, i.e. $c_I = c_D = c_U = c_T$.
This has advantages, e.g. a socially optimal project expediting effort. However, doing so also
increases STA expenditure. When STAs do not include the full cost of public inconvenience
in their calculations, or when there are doubts about the accuracy of calculations leading up
to an estimate of $c_T$, there may be justification for setting $c_I = c_D = c_U < c_T$.

Finally, we turn to the third question. As discussed at the end of the section titled
Contractor Bids, contractor profit depends on the competitive landscape of the project.
Incorrect bid sorting, which may provide unreasonable advantage to some contractors in
certain situations, may occur, but it can be eliminated by choosing parameters correctly.

INSIGHTS FROM DATA

In this section, we present empirical evidence that highlights the importance of under-
standing strategic bidding in procurement auctions. For this purpose, we obtained data on
all A+B projects let by the Minnesota DOT (Mn/DOT) between April 2000 and August
2008. There were 38 such projects during this timeframe. Due to differences in data col-
lection practices across districts, the data includes actual completion times for 27 projects,
bid details for 22 projects, and payment information for only 15 projects. Among these
projects, the smallest winning bid was $601,000 with a completion time of 31 days. The
largest bid was $103 million at 987 days. The mean winning bid was $10.72 million. The median number of bidders on these projects was five, indicating that there is substantial competition in this space. Mn/DOT engineers developed estimates of both project cost and completion time. Engineers’ cost estimates were not revealed to the contractors until after the winning bid was determined and bid abstracts were posted. The estimated completion time was frequently chosen as the maximum allowed bid time.

The analysis presented in preceding sections of this paper offers both policy implications for STAs and bidding strategies for contractors. Using the Mn/DOT dataset we tested whether current practices are consistent with our recommendations. The data allows evaluation of the following issues: Do STAs have consistent policies for setting time-based parameters and do they choose \( c_I = c_D = c_U \)? Do contractors offer short bid times, frequently incurring penalties? Does increased competition lower contractors’ profits? Next, we present results from data analysis.

The data exhibits large variations in the choice of time-based parameters. For example, the daily road user costs in these projects ranged from a minimum of $3,000 to a maximum of $28,000. More importantly, data pointed toward a lack of coherent policy for setting time-based parameters. In nearly half the projects, incentives for early completion were zero, while the daily RUC and penalties were specified. Also, \( c_D = c_U \) was chosen in only about half the projects, with \( c_D \) being lower in nearly all of the remaining projects. In the latter subset of projects, Mn/DOT did not set all three time-based incentive parameters equal, i.e. \( c_I, c_D, c_U \) were not equal.

Turning to the second question regarding contractor bids, the Mn/DOT data suggests that contractors are not bidding strategically, although contractors that win projects do bid shorter completion times. Figure 1 shows three normalized metrics: (1) bid times for contractors who won the project, (2) bid times for all other contractors, and (3) actual completion times. We normalize these times such that 1.0 is the maximum allowed bid time.
This figure shows that among winning contractors more than one-third bid at the maximum allowed time. For those contractors that do not win the project (147 bids), this fraction is much higher, above 50%. Together, this suggests that contractors did not bid strategically on their completion times. In comparing the distribution of bid times we see that contractors that are awarded the contract place significantly shorter bid times than other contractors ($p < 0.05$), consistent with the benefits of offering short bid times shown in Section 3. The third column compares the actual completion time to the maximal allowed. We can see from here that the actual completion time is frequently shorter than the bid completion time, even for contractors who win the project. This suggests that while winners place shorter bids, they are not bidding completion times as short as suggested by the models, and frequently earn bonuses at project completion. We believe that as contractors become more experienced at bidding in the A+B environment, future bids are likely to be shorter relative to actual completion times.
An example of contractors learning to bid strategically can be seen in state project 8611-18. In this project four contractors placed a time-bid of either zero days or one day, while three other contractors placed bids of 30 days or more. One of the contractors that offered the short completion time was awarded the contract. However, the contractors that offered very short completion times also had lower A-component bids. This suggests that they may not have incorporated anticipated penalties into their bids.

Analysis of data in an attempt to shine light on the third question raised earlier suggests that competition reduces contractor profit. We cannot evaluate this directly, since contractors’ costs are not disclosed. We can instead evaluate the ratio of the lowest bid to the second lowest bid, which should be less than 1. As this ratio nears 1, contractors have less opportunity to raise their bids and increase profit. In our dataset, the mean for this ratio is 0.941, but it varies substantially with the number of bidders. In instances where there are four or fewer bidders, the mean for the ratio is 0.91, indicating that the winning contractor could have increased the bid by about 10% and would still have been awarded the project. For projects with more than four competitors, the mean is 0.96, indicating that even small mistakes, or attempts to increase profit, result in losing a contract. Figure 2 shows how the ratio of the lowest to the second bid changes as a function of the number of bidders, confirming the predicted effect of increased competition.

A specific illustration of the importance of incorporating competitive factors when choosing bid parameters can be developed from the data provided for state project 8285-88. In this project the daily time-based parameters were $c_U = $15,000, $c_D = $10,000, and $c_I = $10,000 with $t_I = 10$ days. A minimum bid time was not set. Six contractors bid on this project. The winning contractor offered a price of approximately $9.9 million, with a completion time of 100 days, generating a score of $11.4 million. The second-lowest bidder offered a price of $10.1 million, and a target completion of 90 days, yielding a score of $11.5 million. In this project, bidding a shorter completion time by Contractor 2 would have enabled that contractor to win the project without affecting its profit. The break-even bid completion time
would be 70 days. Bidding 20 days shorter would lower Contractor 2’s score by $300,000, but would increase penalties (or lower incentives) by $200,000. With the $100,000 windfall, Contractor 2 could lower its score to just below $11.4 million without changing its profit, offering the lowest bid.

CONCLUDING REMARKS

Construction project letting mechanisms that include time in bid score require state transportation agencies to specify time-based incentive parameters at the time of announcing a request for bids. Contractors must take into account possible bonuses or penalties resulting from early or late completion, as well as the competitive environment when placing their bids. Similarly, agencies must choose their time-based incentive parameters to achieve best value for the tax-paying public. We investigate questions relating to how strategic contractors would bid and how agencies would set their parameters knowing that contractors bid strategically. Although there is significant literature dealing with A+B bidding, this literature does not address such questions. In fact, there is little if any guidance to contractors.
on how to bid in this environment, and to agencies on how to set incentive parameters. The paper fills this gap in the literature by presenting a normative analysis of the contractors’ and the agencies’ decisions.

Our key recommendation is that all time-based incentive parameters should be set equal, with $c_I = c_D = c_U$. This results in winners’ profits being determined solely by competitive factors — costs and number of bidders. When agencies set these unequal, short bid times are preferred resulting in contractors paying penalties for late completion. We also show that when agencies do not choose $c_I = c_D = c_U$, this may cause a contractor who is more costly to win the contract with the winner earning unreasonable profits. The A+B mechanism offers state agencies a method to expedite projects, benefiting the commuting public. This paper provides guidance for managing these innovative contracts.

ACKNOWLEDGMENTS

This material is based upon work supported, in part, by the National Science Foundation under Grant No. CMMI-0653451 to Diwakar Gupta.

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