Fast-Ship Commitment Contracts in Retail Supply Chains

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We analyze three types of supply contracts between a supplier and a retailer when both agree as follows — if a customer experiences a stockout, then the purchased item can be shipped to the customer on an expedited basis at no extra cost. This practice is referred to as the fast-ship option in this paper. In the first contract (Structure A), the supplier specifies a total supply commitment and allows the retailer to choose its split between the initial order and the amount left to satisfy fast-ship orders. In the other two contracts (Structures B and C), the supplier agrees to fully supply the retailer’s initial order but places a restriction on the quantity available for fast-ship commitment. The difference between the second and third contracts is that in contract Structure B, the supplier moves first, whereas in contract Structure C, the supplier determines its commitment after observing the retailer’s order. We characterize the supplier’s and the retailer’s optimal decisions and preferences. We also discuss how the supplier and the retailer may resolve their conflict regarding the preferred contract type. Supplementary materials are available for this article. Go to the publisher’s online edition of IIE Transaction for proofs.

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1. Introduction

Stockout occurs when supply falls short of demand. Stockouts are not uncommon in retail supply chains (Gruen et al. 2002 report a worldwide average out-of-stock rate of 8.3 percent) and often result in customer dissatisfaction (Grant and Fernie 2008). In industries such as toys and apparel, matching supply with demand is especially challenging due to fast-changing customer preferences and market trends, which lead to high inventory costs, markdowns, and lost sales (Johnson 2001). The options available to customers when they experience stockouts affect profits of supply chain partners.

When customers learn that the retail store they are in has stocked out of a desired item, they may respond in a number of different ways. Some customers may switch brands and buy a substitute product, others may buy from a different retail store, and some others may postpone purchase decision or choose an entirely different product (Emmelhainz et al. 1991). Such stockout-triggered purchasing behaviors may hurt retailers even when some customers purchase substitutes because stockout events can negatively affect the overall sales of products in the same category due to lack of selections available to customers (Kalyanam et al. 2007). Retailers’ profit margins in many industry segments are low (for example, see Datamonitor 2008 for electronics industry profile and profit margins of key players), which suggests that cost-effective means of capturing sales that are lost during stockout events would be of interest to retailers.

To reduce loss of sales during stockout periods, retailers may negotiate a flexible supply contract that allows them to either adjust the order size before the start of the selling season or to place multiple orders during the selling season. The latter includes offering a fast-ship option to customers, which is the focus of this study. A retailer that offers the fast-ship option arranges to have out-of-stock items shipped directly from the supplier to customers at no additional cost to the customers, thereby creating a hybrid between traditional and drop-ship channels. For example, Best Buy, an electronic products’ retailer, offers an Instant Ship option to in-store customers when mobile devices they intent to purchase are not available in store. Many apparel retailers, such as J.Crew and Gap Inc., offer to home-ship items when customers cannot find desired items in the right sizes on the store shelf, absorbing the shipping cost. Office Depot ships ink and toner cartridges free to customers if it stocks out of such items.
The fast-ship option allows the channel to use the retailer-held inventory as the primary source of supply (traditional approach) and supplier-held backup inventory as the secondary source of supply (drop-ship approach). The latter is used only when the primary source is exhausted. This contrasts with the two extremes of traditional and drop-ship channels in which all inventory is kept either at the retailer location or at the supplier location (Wilson 2000). Drop-ship channels are commonly encountered in the context of Internet-based retailers (e.g., Zappos, an Internet footwear store). Similarly, the supplier may also procure additional inventory at a higher cost to replenish its stock as needed.

In the traditional approach, the retailer bears all of the inventory cost and its stocking decision determines channel performance. Use of drop-ship approach reduces retailers’ inventory cost. However, this option does not meet the needs of those customers who prefer to touch and feel the items before buying and those who do not want to wait. In fact, depending on the product, between 47 to 92 percent of retail sales happen in “brick-and-mortar” stores (Schonfeld 2010). The fast-ship option, which is the focus of this paper, combines the advantages of both traditional and drop-ship channels and provides a mechanism by which supply-demand mismatch cost may be apportioned between the retailer and the supplier. Models presented in this paper show that the fast-ship option has the potential to improve profits of both the retailers and the suppliers.

In a channel that supports the fast-ship option, both the retailer and the supplier have two opportunities to replenish. The retailer places an initial order before the start of the selling season and multiple fast-ship orders that occur later in the selling season. The fast-ship orders are placed, as needed, after inventory at the retail store runs out. Similarly, the supplier procures (or produces) a certain quantity of items before the selling season, which can be greater than the retailer’s initial order size, and may procure additional items after demand realization, if needed.

From the retailer’s perspective, the fast-ship option may be particularly attractive for high-value items for which the obsolescence cost is high and the additional cost of direct shipping to customers is relatively small. This is because the fast-ship option can reduce the retailer’s cost of meeting its demand. A supplier who cooperates with the retailer to support the fast-ship option may also benefit from this practice because the total sales may be higher. However, because the retailer may decrease its initial order size and the procurement cost for the fast-ship orders may be higher, suppliers may wish to limit the fast-ship commitment via the terms of a supply contract.
In this paper, we compare three possible supply commitment contract structures between a single supplier (S) and a single retailer (R) that support the fast-ship option for a product with a short selling season. Within each structure, a particular set of values of the retailer’s and the supplier’s parameters is referred to as a contract. These three structures are variants of quantity flexibility provisions that are common in supply contracts. We describe related literature at a later point in this section.

In the first structure (referred to as Structure A contract), the supplier commits to a maximum total quantity $p \geq 0$. The retailer can then choose any initial order quantity $q$ and place any number of fast-ship requests so long as the total amount ordered does not exceed $p$. The second structure, referred to as Structure B, limits only the supplier’s fast-ship commitment. That is, the supplier commits to supply no more than $z \geq 0$ via the fast-ship option. It also supplies any amount $q$ ordered by the retailer before the start of the selling season. Both A and B are supplier-driven structures because the supplier makes its choice first and the retailer orders $q$ after learning either $p$ or $z$. The third structure, referred to as Structure C, may be viewed as a retailer-led analog of Structure B because the supplier chooses its fast-ship commitment $\gamma$ after receiving R’s initial order $q$.

The three contract structures belong to a family of affine supply commitment contracts in which the supplier’s total commitment is an affine function of the form $aq + b$, and $a$ and $b$ are contract parameters. Different values of $a$ and $b$ give rise to different relationships between the initial order size and the fast-ship supply commitment. Specifically, Structure A contract arises when $a = 1$ and $b = p - q$, whereas Structures B and C structures arise when $a = 1$ and $b$ is either $z$ or $\gamma$. The actual number of items that are fast shipped depends on the parameter values chosen by the two players in each supply structure.

The setting in our paper is motivated by contractual restrictions, also referred to as vertical restraints (Rey and Verge 2005). While different types of vertical restraints have been studied in the literature (Rey and Tirole 1986), we consider variants of quantity fixing contracts, that include minimum quantity purchases, quantity forcing, as well as quantity rationing contracts. Our focus is on specific forms of quantity rationing contracts where the supplier imposes restrictions on the quantity available to the buyer. In particular, the three structures are variants of several quantity fixing contracts used in practice. For example, the Indefinite Delivery, Indefinite Quantity
(ID/IQ) contracts offered by suppliers to government agencies are similar to Structure A (see http://www.gsa.gov/portal/content/103926). The ID/IQ contract specifies a maximum total quantity that would be purchased within a fixed time period. The buyer can place multiple orders and each order can be of an arbitrary size. However, orders must be placed during the contract period and the negotiated price applies to the orders whose cumulative quantity is less than the maximum specified in the contract. Similarly, in a practice similar to Structures B and C, state government agencies require suppliers of road salt (used for de-icing roads during winter) to agree to supply more than the amount initially ordered at the negotiated price (e.g., BART 2008). The supplier caps extra supplies at values that are proportional to the initial order quantity. The ID/IQ and road salt supply contracts are similar in spirit, though not exactly the same, as the three structures studied in this paper.

We develop mathematical models that help explain how the supplier and the retailer would choose values of their parameters within each contract structure when they maximize their individual profits. Specifically, we establish certain properties of the retailer’s and the supplier’s parameter optimization problems, which allow us to solve these problems using nonlinear optimization techniques. We then compare the three contracts under optimal parameter choices to determine which structure is preferred by the supplier and which structure is preferred by the retailer. Within reasonable ranges of problem parameters, we show that from the supplier’s viewpoint, B is the most preferred structure and A is the least preferred when both the supplier and the retailer make individually optimal decisions. This is because the retailer orders less up front under Structure A contract and fast-ship sales are less profitable for the supplier. As a result, among the two supplier-led structures, the supplier will not offer Structure A, even though A may provide greater flexibility to the retailer. Structure B is more profitable than C for the supplier because the supplier receives a larger initial order in Structure B as compared to C.

From the retailer’s perspective, Structure A is usually preferred, except in cases where the total promised supply ($p$) is smaller than the promised supply under other contract structures (i.e. $p$ is less than either $q + z$ or $q + \gamma$). However, because Structure A will not be chosen by the supplier, we compare only Structures B and C from the retailer’s perspective. We show that when the retailer faces a choice between Structures B and C, it prefers Structure C because it can secure a greater supply commitment under Structure C as compared to B. We also test whether
contract structure preference changes if the supplier (resp. retailer) chooses the wholesale price in supplier-led (resp. retailer-led) contracts.

Clearly, the two players have different contract structure preferences, which gives rise to the problem of contract type selection. We present two approaches for resolving such differences. In the first case, one of the two players is assumed to be the dominant player (hold-out). The behavior of a hold-out player can be explained as follows. When the retailer is the hold-out, it would only accept a contract in which its profit is at least as much as its best profit under contract Structure C. Similarly, when the supplier is the hold-out, it would only accept a contract in which its profit is at least as much as its best profit under contract Structure B. The profits of the two players in this case are referred to as hold-out profits.

The second case arises when there is no dominant player and neither player can insist on a minimum profit threshold equal to its optimal profit under its preferred contract structure. Instead, the two players are willing to negotiate and each player has a disagreement profit level. For the retailer, the disagreement profit level is the best profit it would make under contract Structure B and for the supplier, the disagreement profit level is the profit it would make under contract Structure C. This is because the retailer (respectively supplier) is guaranteed to strike a contract if it agrees to Structure B (respectively C).

For the first case, we show that the player who is not the hold-out player can propose a contingent contract that improves its profit over its hold-out profit while maintaining or improving the hold-out player’s profit. In the second case with no hold-out player, we prove the existence of negotiated contracts, which guarantee that each player will make more than its disagreement profit. We do not dwell on the division of profits because many solutions are possible depending on the negotiating power of each party. However, we show that under a negotiated contract, it is in each player’s best interest to maximize total supply chain profit, i.e. the negotiation approach is equivalent to vertical integration of the two players and contract structure preferences become irrelevant. These two conflict resolving approaches remain valid when the wholesale price is chosen by the leader in the contract. Specifically, this means the wholesale price is determined by the supplier in Structure B and the retailer in Structure C.

We also briefly discuss the effect of partial backorder rate. Specifically, we observe that a higher partial backorder rate is beneficial for the supplier in Structure B and for the retailer in Structure
C. For other cases, the results depend on parameters. However, it is more likely that both players can benefit from higher partial backorder rate in Structure C.

**Related Literature**

Supply contracts are widely used in industry (e.g., White et al. 2005) and commitment flexibility, similar to that implied by the three contract structures we study, is a common theme in supply contracts’ literature; see, for example, Van Mieghem (2003), Wu et al. (2005), and Stevenson and Spring (2007). Quantity flexibility (QF) allows the buyer to adjust the purchase quantity in a certain range without penalty, reducing the channel’s cost of matching supply and demand (Wu 2005).

In a QF contract, the buyer announces an early tentative order $q^T$ before the production period begins. Knowing $q^T$, the supplier commits to supply $q^S$. After receiving a more accurate demand forecast, which occurs before the selling season starts, the buyer then adjusts its order size and comes up with a final (firm) order $q^F$. The buyer (respectively supplier) is not penalized if $q^F \geq q^T - a$ (respectively $q^S \geq \min\{q^F, (q^T + b)\}$), where $a$ and $b$ are called flexibility parameters (see, e.g., Tsay 1999). That is, the buyer in a QF contract commits to purchasing no less than a certain amount/percent below the forecast and the seller commits to supply up to a certain amount/percent above the forecast.

The supply commitment contracts we study serve a different purpose than QF contracts. The fast-ship option is triggered only after stockout occurs and its purpose is to provide a mechanism for serving unmet demand. In contrast, the reason for allowing quantity adjustments in QF contracts is to reduce the expected cost of overage and shortage. QF contracts do not help retailers meet demand that occurs after stockouts. Another difference is that fast-ship sales induce additional costs for both the supplier and the retailer in our supply commitment contracts, whereas, in QF contracts, quantity adjustments within permissible ranges do not induce additional costs. Finally, supply flexibility parameters are often exogenously determined in QF contracts; for example, $a \geq 0$ and $b \geq -a$ are exogenous in Tsay (1999). In our setting, within each contract structure, the supply commitment is determined by parameters $p$, $z$, or $\gamma$, which are chosen by the supplier, and both players pick individually optimal contract parameters.

Structure A contract is also related to Eppen and Iyer’s (1997a) two-period stochastic dynamic
programming model of a backup agreement contract. In the first period, the buyer commits to buy up to some amount $q^T$ for the selling season and claims immediate ownership of $(1 - \sigma)q^T$ units where $\sigma$ is exogenous. After period-1 demand is realized, the buyer can adjust its inventory by placing a second order of up to $\sigma q^T$ units at the original price in period 2. In each period, a small portion of sales is returned and a constant fraction of returned units can be reused to satisfy demand. In addition, the buyer pays a penalty $\ell$ for any reserved units that are not purchased.

Our approach is similar because we also allow the retailer to place a second order up to some pre-determined total quantity commitment by the supplier. However, our problem is different because (1) the total supply commitment is a decision made by the supplier in our models and consequently the buyer does not pay a penalty for not purchasing all of the promised supply, (2) we model both the supplier’s and the retailer’s problems and obtain their optimal parameters, whereas Eppen and Iyer do not address the supplier’s problem, and (3) Eppen and Iyer focus on the impact of backup fraction $\sigma$ and penalty $\ell$ on the buyer’s expected profit and commitment $q^T$, whereas we study the interactions between the supplier and the retailer for three different contract structures when both players make individually-optimal decisions within each structure.

Our model is also related to previous works involving more than one replenishment opportunity; see, for example, Eppen and Iyer (1997b), Gurnani and Tang (1999), and Donohue (2000). These authors have studied the use of two ordering opportunities for fashion products when both opportunities arise prior to the start of the selling season. The retailer, after placing an initial order, observes a signal that is correlated with the demand during the selling period. With this new information, the demand forecast is updated and the second replenishment is used to lower supply-demand mismatch costs. The focus of the papers cited above is to model the effect of the retailer getting additional (but incomplete) demand information after placing its first order. In contrast, in our setting, the purpose of the second replenishment (which takes place after demand realization) is to serve customers that agree to wait for out-of-stock items (see Gupta et al. 2010 for similar settings).

The dual strategy model in Netessine and Rudi (2006) is also related to our model. When dual strategy is adopted, each retailer uses its stockpile as the primary source of items needed to satisfy demand and drop shipping as a backup source when its stock runs out. However, there are important differences between our work and Netessine and Rudi (2006). First, Netessine and Rudi
(2006) assumes that when in-store inventory runs out, all remaining customers agree to receive their items from the drop-ship channel, which corresponds to setting $\alpha = 1$ in our model. Our model is more general and can be applied to situations in which some customers do not take advantage of the fast-ship option. Second, the supplier in our model has two replenishment opportunities whereas the supplier in Netessine and Rudi (2006) has a single replenishment opportunity. Put differently, the scenario discussed in Netessine and Rudi is a special case in our model. Netessine and Rudi (2006) compares the dual strategy with both pure traditional (i.e. where $z$ or $\gamma = 0$) and pure drop-ship (i.e. where $q = 0$) environments. In contrast, we analyze different contract structures within which the fast-ship commitment level is chosen by the supplier. Netessine and Rudi (2006) paper identifies the best channel strategy as a function of supply chain parameters whereas we provide insights into supply chain partners’ contract structure preferences and parameter choices.

The rest of this paper is organized as follows. Notation and model formulations for the three contract structures are introduced in Section 2. We analyze the two player’s optimal decisions for Structures A and B in Section 3, and Structure C in Section 4. In Section 5, we contrast the three structures from the retailer’s and the supplier’s perspectives and address the problem of contract structure selection. Section 6 discusses the effect of leader-selected wholesale price and the effect of different values of partial backorder rate. Summary of main results, and directions for future research are provided in Section 7. All proofs are presented in an Online Supplement within the publisher’s online edition of IIE Transaction.

2. Notation and Model Formulation

The sequence of events for the three contract structures are illustrated in Figure 1. The retailer offers the fast-ship option until the available fast-ship commitment (as guaranteed via its contract with the supplier) is exhausted. Structures A and B are supplier-led contracts and commitments $p$ and $z$ are decided before the retailer decides the order quantity. Structure C is a retailer-led contract in which commitment $\gamma$ is decided after the retailer’s order quantity decision.

The notation used in model formation is listed in Table 1. R’s demand $X \in \mathcal{R}_+$ is continuous with probability density and distribution functions $f(\cdot)$ and $F(\cdot)$, respectively. We also assume that $f(\cdot) > 0$ over the support of $X$. Note that only a fraction of customers utilize the fast-ship option (referred as the partial backorder rate in this paper), and the rest do not make a purchase
at the retailer’s store. This is similar to partial backorders where the retailer backorders only when a customer is willing to wait (see, for example, Abad 1996). The supplier procures $q + y$ items before the selling season, and may procure additional items during the selling season as needed. The supplier’s replenishment costs are $c_1$ and $c_2$ for the two replenishment options. The supplier pays a holding cost $h$ for the additional $y$ units it procures because these items are sold only at the end of the selling period. The supplier replenishes during the selling season only when $y$ cannot satisfy all promised fast-ship orders.

The shipping costs for regular and fast-ship orders are $\tau_1$ and $\tau_2$, respectively. Both $\tau_1$ and $\tau_2$ are paid by the supplier to a third party logistics service provider. Note that $\tau_1$ and $\tau_2$ are independent of origin and destination because shipping charges depend on the item’s size and/or weight but not origin and destination. Such pricing schemes are common in the US; see for example U.S. Postal Service’s (www.usps.com) shipping rates for standard-sized boxes of certain maximum weight regardless of origin and destination. The retailer sells items to customers at a unit retail price $r$ regardless of whether the item is sold from on-hand inventory or by using the fast-ship option. The supplier sells items to the retailer at a unit wholesale price $w$ for initial orders, and $w + \delta$ for fast-ship orders; that is, the wholesale price for fast-ship items is obtained by adding a mark-up $\delta \geq 0$ to the base price $w$ and markups of 15-20% are common (Scheel 1990). For notational simplification, we also use $w_2 = w + \delta$ to denote the wholesale price of fast-ship items.

Parameters $r$, $w$, and $\delta$ are are assumed exogenous in this paper. Scenarios with endogenous $w$ are discussed in Section 6. In addition, we make the following assumptions about key parameters.

**Assumption 1.** $\tau_1 \leq \tau_2$.

**Assumption 2.** $c_1 \leq c_2$.

**Assumption 3.** $c_1 + h \leq c_2$. 
Table 1: Summary of Notation

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decision Variables:</td>
<td></td>
</tr>
<tr>
<td>$p$</td>
<td>S’s total commitment under Structure A, $p \geq 0$</td>
</tr>
<tr>
<td>$z$</td>
<td>S’s fast-ship commitment under Structure B, $z \geq 0$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>S’s fast-ship commitment under Structure C, $\gamma \geq 0$</td>
</tr>
<tr>
<td>$y$</td>
<td>S’s extra production quantity</td>
</tr>
<tr>
<td>$q$</td>
<td>R’s order quantity, $q \geq 0$</td>
</tr>
<tr>
<td>Parameters:</td>
<td></td>
</tr>
<tr>
<td>$X$</td>
<td>Demand with density and distribution functions $f(\cdot) &gt; 0$ and $F(\cdot)$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Partial Backorder Rate, $\alpha \in [0, 1]$</td>
</tr>
<tr>
<td>$y$</td>
<td>Additional units procured during the first replenishment, $y \geq 0$</td>
</tr>
<tr>
<td>$w$</td>
<td>Wholesale price per unit for regular orders, $w \geq c_1 + \tau_1$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Markup per unit for fast-ship orders, $\delta \geq 0$</td>
</tr>
<tr>
<td>$w_2$</td>
<td>Wholesale price per unit for the fast-ship orders, $w_2 = w + \delta$</td>
</tr>
<tr>
<td>$\tau_1, \tau_2$</td>
<td>Regular/fast-ship order shipping cost per unit, $\tau_1, \tau_2 \geq 0$</td>
</tr>
<tr>
<td>$c_1, c_2$</td>
<td>S’s first/second replenishment costs per unit, $c_1, c_2 \geq 0$</td>
</tr>
<tr>
<td>$h$</td>
<td>Holding cost incurred by S for stocking each extra unit, $h \geq 0$</td>
</tr>
<tr>
<td>$r$</td>
<td>Retail price per unit, $r \geq 0$</td>
</tr>
</tbody>
</table>

Assumption 1 reflects the fact that fast-ship orders utilize premium shipping (more expensive) with expedited delivery, whereas regular orders are sent to retailers utilizing an economical transportation system. Assumption 2 implies that the second batch procurement cost is higher than the first because suppliers have shorter time windows within which to obtain the second replenishment. Conditional upon needing an item to satisfy excess demand, Assumption 3 makes it cheaper for the supplier to procure this item in the first batch and hold it till the end of the selling season. Although our model remains valid without Assumption 3, it serves to make $h$ relevant because otherwise the supplier would not stock extra units and no holding cost would be incurred.

We also assume that parameters are chosen such that the fast-ship option is attractive to both the supplier and the retailer, which is assured by the following two assumptions:

**Assumption 4.** $w < r - \delta$.

**Assumption 5.** $w - \tau_1 - c_1 \geq \alpha(w_2 - \tau_2 - c_1 - h) \geq 0$.

Assumption 4 ensures that both initial and fast-ship orders are profitable for the retailer. In absence of this requirement, the retailer may choose not to offer the fast-ship option to customers when a stockout occurs. Because $w_2 \geq w$ (i.e., $\delta \geq 0$) and the retail price does not change when items are supplied via the fast-ship option, the retailer’s unit profit is greater if an item is supplied
from stock. Assumption 5 is a sufficient condition under which the fast-ship orders are also less profitable for the supplier. If this condition were violated, the supplier might not supply orders before the start of the selling season and offer an infinite fast-ship commitment under all three contract structures, which would obviate the need to study different contract structures. That is, Assumption 5 identifies a range of problem parameters within which the questions posed in this paper are non-trivial. We explain the logic behind Assumption 5 in the ensuing paragraph.

The supplier’s contribution margin from a sale from the first replenishment is \((w - \tau_1 - c_1)\) and at most \(\alpha (w_2 - \tau_2 - c_1 - h)\) from the second replenishment. This is because only \(\alpha\) fraction of customers would purchase the item when experiencing stockout. Therefore, if \(w - \tau_1 - c_1 \geq \alpha (w_2 - \tau_2 - c_1 - h) \geq 0\), then the supplier definitely earns more from each unit ordered by the retailer in its initial order.

Two scenarios arise when Assumption 5 is violated. If \(w - \tau_1 - c_1 < \alpha (w_2 - \tau_2 - c_2)\), then we can show that the three structures are identical and the supplier would choose \(p, z, \) and \(\gamma = \infty\) because fast-ship order produces higher profits. However, if \(\alpha (w_2 - \tau_2 - c_2) < w - \tau_1 - c_1 \leq \alpha (w_2 - \tau_2 - c_1 - h)\), then we are not able to obtain results analytically because the marginal benefits for both types of orders depend on other parameters. Numerically, we observe that there exists a threshold between \(\alpha (w_2 - \tau_2 - c_2)\) and \(\alpha (w_2 - \tau_2 - c_1 - h)\) such that our results hold if \((w - \tau_1 - c_1)\) is greater than the threshold. Later in Sections 3 and 4, we show that \(w_2 \geq \tau_2 + c_2\) is sufficient for \(p^* = \gamma^* = \infty\) within Structures A and C, but that \(z^*\) is not necessarily unbounded under the same condition.

Because the expedited transportation cost is linear in the number of fast-ship items, modeling several fast-ship orders as a single second replenishment does not affect the profit functions of the two players. In other words, the total fast-ship demand can be represented as \(\alpha (X - q)^+\), where \(b^+ = \max(0, b)\). Let index \(i \in \{A, B, C\}\) denote contract structure. In expressions that apply to all contract structures, we use parameter \(j \in \{p, z, \gamma\}\) to denote supply commitment. The retailer’s expected profit given that contract structure \(i\) has been selected, supplier has committed \(j\), and retailer has ordered \(q\), can be written as follows.

\[
\pi_i(q, j) = \pi^* = rE[X \land q] - wq + (r - w_2)E[\alpha (X - q)^+ \land \zeta_j^*(q)],
\]

(1)

where \((X \land q)\) denotes \(\min(X, q)\), and \(\zeta_j^*(q)\) is the maximum fast-ship supply committed by S. That is, \(\zeta_j^*(q) = p - q\), or \(z\), or \(\gamma\) when \((i, j) = (A, p), (B, z), \) and \((C, \gamma)\), respectively. Moreover,
\( r E[X \wedge q] - wq \) is the expected profit from the initial order, \( (\alpha(X - q)^+ \wedge \zeta_j^i(q)) \) is the magnitude of fast-ship demand and \( (r - w_2)E[\alpha(X - q)^+ \wedge \zeta_j^i(q)] \) is the expected profit from the fast-ship orders. Equation (1) assumes that both players have made their decisions. The optimal values of decision variables depend on the sequence in which the two players make their decisions, but the resulting profit for the retailer after decisions are made can be expressed as (1) for all contract types.

Similarly, given that contract structure \( i \) has been selected, the retailer has ordered \( q \), and the supplier has chosen \( y \) and \( j \), the supplier’s expected profit is given by

\[
\pi_S^i(y, j, q) = (w - \tau_1 - c_1)q - (c_1 + h)y + (w_2 - \tau_2)E[\alpha(X - q)^+ \wedge \zeta_j^i(q)] - c_2E[(\alpha(X - q)^+ - y)^+ \wedge (\zeta_j^i(q) - y)^+] .
\] (2)

In (2), \( (w - \tau_1 - c_1)q \) is the profit from R’s initial order, \( (c_1 + h)y \) is the cost of procuring and stocking extra \( y \) items, and \( (w_2 - \tau_2)E[\alpha(X - q)^+ \wedge \zeta_j^i(q)] \) is the revenue from fast-ship demand. The last term comes from the fact that S has an uncovered commitment of \( (\zeta_j^i(q) - y)^+ \) and the leftover fast-ship demand after stockpile \( y \) is exhausted equals \( (\alpha(X - q)^+ - y)^+ \). Therefore, \( c_2E[(\alpha(X - q)^+ - y)^+ \wedge (\zeta_j^i(q) - y)^+] \) is the extra procurement cost for the fast-ship orders that are not served from the amount stocked by the supplier in response to the retailer’s initial order.

Similar to Equation (1), Equation (2) also assumes that both players have made their decisions. Although the optimal values of either player’s decision variables depend on the sequence of events, the resulting profit of the supplier can be expressed as (2) for all contract types.

In Structures A and B, the supplier is the first mover. Therefore, when \( (i, j) \in \{(A, p), (B, z)\} \), the retailer’s problem is to find \( q^i(j) = \arg \max \pi_R^i(q, j) \) for each supplier-selected \( j \) whereas the supplier’s problem is to find \( y^i(j) = \arg \max \pi_S^i(y, j, q^i(j)) \) and \( j^* = \arg \max_j \pi_S^i(y^i(j), j, q^i(j)) \). In contrast, the retailer is the first mover in Structure C. Therefore, the supplier’s problem in Structure C is to find \( y^C(q) = \arg \max_y \pi_S^C(y, \gamma, q) \) and \( \gamma(q) = \arg \max_\gamma \pi_S^C(y^C(q), \gamma, q) \) for each retailer-selected \( q \) whereas the retailer’s problem is to find \( q^C = \arg \max_q \pi_R^C(q, \gamma(q)) \).

With expressions (1) and (2) in hand, we are ready to find optimal parameter values for each player under each contract structure. In the ensuing analysis, we use \( e_j^i(q) = q + \zeta_j^i(q)/\alpha \) for notational convenience and assume, without loss of generality, that both the retailer and the supplier pick the smallest among possible optimal parameter values when multiple such values exist. Because
in Structures A and B, the supplier moves first and in Structure C the retailer moves first, we present the analysis of Structures A and B in the same section (i.e., Section 3). The analysis of Structure C is presented separately in Section 4.

3. Parameter Optimization: Structures A and B

In the main body of the paper, we describe our key results and explain why the results hold. An optimal order quantity for the retailer \( q^i(j) = \arg \max \pi^i_R(q, j), (i, j) \in \{(A, p), (B, z)\} \) can be obtained as follows.

**Proposition 1.** For \((i, j) \in \{(A, p), (B, z)\}, \pi^i_R(q, j) \) is concave in \( q \) and \( R \)'s optimal order quantities under contract Structures A and B are as follows.

\[
q^A(p) = \begin{cases} 
p & \text{if } p < \bar{F}^{-1}\left(\frac{w}{w_2}\right), \\
\bar{F}^{-1}\left(\frac{w+(1-\alpha)(r-w_2)\bar{F}(e^A_p(q^A(p)))}{r-\alpha(r-w_2)}\right) & \text{otherwise.} \end{cases}
\tag{3}
\]

\[
q^B(z) = \bar{F}^{-1}\left(\frac{w-\alpha(r-w_2)\bar{F}(e^B_z(q^B(z)))}{r-\alpha(r-w_2)}\right). \tag{4}
\]

The second part of Equation (3) and Equation (4) are obtained from the first order optimality conditions. The first part of Equation (3) can be explained as follows. When \( p \) is small, the retailer prefers to have all item sold from the initial stockpile, i.e., \( q^A(p) = p \). This is because the retailer expects to sell most of the available supply and the marginal benefit of satisfying a demand from the initial stockpile is higher than or equal to that of satisfying demand by taking advantage of the fast-ship option (because \( w \leq w_2 \)). Define \( q^A(p)' = \partial q^A(p)/\partial p \) and \( q^B(z)' = \partial q^B(z)/\partial z \). Using (3)-(4), we obtain two inequalities in Lemma 1.

**Lemma 1.** The following inequalities hold:

1. \( 0 \leq q^A(p)' \leq (1 - \alpha)^{-1}. \)
2. \( -\alpha^{-1} \leq q^B(z)' \leq 0. \)

Lemma 1 shows that the retailer orders more (reps. less) under Structure A (reps. B) when the supplier’s commitment increases. The inequalities \( q^A(p)' \leq (1 - \alpha)^{-1} \) and \( -\alpha^{-1} \leq q^B(z)' \)
themselves do not have meaningful explanations but are required to establish parameter choices and structural preferences in this paper.

Let $\pi_R^i(j) = \max_q \pi_R^i(q, j)$ denote R’s optimal expected profit as a function of $j$ when $i \in \{A, B\}$. From (1), we observe that $\pi_R^i(q, j)$ is increasing in $j$ if we were to keep $q$ fixed because $r - w_2 \geq 0$. Hence, $\pi_R^i(j) = \pi_R^i(q^i(j), j)$ must be increasing in $j$ as well. This makes sense on an intuitive level. A higher value of $j$ implies greater supply flexibility for the retailer. As a result, it incurs a smaller cost of coping with demand uncertainty because the retailer is able to satisfy more demand after inventory runs out.

The supplier’s expected profit $\pi_S^i(y, j, q^i(j))$ shown in (2) is concave in $y$. Therefore, we obtain an optimal $y^i(j) = \arg \max \pi_S^i(y, j, q^i(j))$ as follows.

$$y^i(j) = [\alpha(\eta_S - q^i(j))^+ \wedge \zeta^i_j(q^i(j))],$$  

(5)

where $\eta_S = \bar{F}^{-1}((c_1 + h)/c_2)$ and $\bar{F}^{-1}(x) = 0$ if $x \geq 1$. The quantity $\eta_S$ has a straightforward explanation. If the supplier stocks out (relative to its commitment), then it incurs a unit shortage cost of $(c_2 - c_1 - h)$. If, in contrast, it stocks too much, then its overage cost is $(c_1 + h)$. Thus, $\bar{F}(\eta_S) = (c_1 + h)/(c_1 + h + c_2 - c_1 - h)$ represents the fractile of demand that the supplier should stock in absence of constraints. However, its commitment is limited to $\zeta^i_j(q^i(j))$, only $\alpha$ fraction of customers use fast-ship option, and $y$ is needed only after the retailer’s initial stockpile $q^i(j)$ runs out. This explains expression (5).

Let $p_L = \bar{F}^{-1}(w/w_2)$. Also, define $p_1 = \min\{p : p - q^A(p) = \alpha(\eta_S - q^A(p))\}$, $z_1 = \min\{z : z = \alpha(\eta_S - q^B(z))\}$, $p_2 = \min\{p : q^A(p) = \eta_S\}$, and $z_2 = \min\{z : q^B(z) = \eta_S\}$. Note that $p_2$ (respectively $z_2$) exists only if $\lim_{p \to \infty} q^A(p) \geq \eta_S$ (respectively $\lim_{z \to \infty} q^B(z) \leq \eta_S \leq \lim_{z \to 0} q^B(z)$). Lemma 2 below explains how the value of $y^i(j)$ changes in $p$ or $z$.

**Lemma 2.** The following statements are true:

1. (a) Inequality $p - q^A(p) < \alpha(\eta_S - q^A(p))$ holds if and only if $p < p_1$. (b) Inequality $\eta_S - q^A(p) > 0$ holds if and only if $p_2$ exists and $p < p_2$.

2. (a) If $\lim_{p \to \infty} q^A(p) \geq \eta_S$ and $p_L \leq p_2$, then $p_L \leq p_1 \leq p_2$. (b) If $\lim_{p \to \infty} q^A(p) \geq \eta_S$ and $p_L > p_2$, then $y^A(p) = 0$ for all $p$. (c) If $\lim_{p \to \infty} q^A(p) < \eta_S$, then $p_L \leq p_1$. 

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3. (a) Inequality $z < \alpha(\eta_S - q^B(z))$ holds if and only if $z < z_1$. (b) Inequality $\eta_S - q^B(z) < 0$ holds if and only if $z_2$ exists and $z < z_2$.

4. (a) If $\lim_{z \to \infty} q^B(z) > \eta_S$, then $y^B(z) = 0$ for all $z$. (b) If $\lim_{z \to 0} q^B(z) < \eta_S$, then $y^B(z) = z$ for $z < z_1$ and $y^B(z) = \alpha(\eta_S - q^B(z))$ for $z \geq z_1$. (c) If $\lim_{z \to \infty} q^B(z) \leq \eta_S \leq \lim_{z \to 0} q^B(z)$, then $y^B(z) = 0$ for $z < z_2$ and $y^B(z) = \alpha(\eta_S - q^B(z))$ for $z \geq z_2$.

The results in Lemma 2 are important because they help obtain Proposition 3 shown later in this section. We summarize the values of $y^i_j$ for different values of $p$ and $z$ in Table 2. For ease of presentation, A1, A2, · · · B3 are used to denote different ranges of parameter values in Table 2. Note that when $p < p_L$, $y^A(p) = p - q^A(p) = 0$ because the fast-ship option is not offered by the supplier.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Conditions</th>
<th>Range of $p$</th>
<th>Value of $y^A(p)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>$\lim_{p \to \infty} q^A(p) \geq \eta_S$ and $p_2 \leq p_L$</td>
<td>$p \in [0, p_L)$ $p \in (p_L, \infty)$</td>
<td>$y^A(p) = p - q^A(p) = 0$ $y^A(p) = 0$</td>
</tr>
<tr>
<td>A2</td>
<td>$\lim_{p \to \infty} q^A(p) \geq \eta_S$ and $p_2 &gt; p_L$</td>
<td>$p \in [0, p_L)$ $p \in (p_L, p_1)$ $p \in (p_1, p_2)$ $p \in (p_2, \infty)$</td>
<td>$y^A(p) = p - q^A(p) = 0$ $y^A(p) = p - q^A(p)$ $y^A(p) = \alpha(\eta_S - q^A(p))$ $y^A(p) = 0$</td>
</tr>
<tr>
<td>A3</td>
<td>$\lim_{p \to \infty} q^A(p) &lt; \eta_S$</td>
<td>$p \in [0, p_L)$ $p \in (p_L, p_1)$ $p \in (p_1, \infty)$</td>
<td>$y^A(p) = p - q^A(p) = 0$ $y^A(p) = p - q^A(p)$ $y^A(p) = \alpha(\eta_S - q^A(p))$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Conditions</th>
<th>Range of $z$</th>
<th>Value of $y^B(z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>$\lim_{z \to \infty} q^B(z) &gt; \eta_S$</td>
<td>$z \in [0, \infty)$</td>
<td>$y^B(z) = 0$</td>
</tr>
<tr>
<td>B2</td>
<td>$\lim_{z \to 0} q^B(z) &lt; \eta_S$</td>
<td>$z \in (0, z_2)$ $z \in [z_1, \infty)$</td>
<td>$y^B(z) = z$ $y^B(z) = \alpha(\eta_S - q^B(z))$</td>
</tr>
<tr>
<td>B3</td>
<td>$\lim_{z \to \infty} q^B(z) \leq \eta_S \leq \lim_{z \to 0} q^B(z)$</td>
<td>$z \in [0, z_2)$ $z \in (z_2, \infty)$</td>
<td>$y^B(z) = 0$ $y^B(z) = \alpha(\eta_S - q^B(z))$</td>
</tr>
</tbody>
</table>

Let $\pi^i_S(j) = \pi^i_S(y^i(j), j, q^i(j))$. This implies that $j^* = \arg\max_j \pi^i_S(j)$ for each $(i, j) \in \{(A, p), (B, z)\}$. We are now ready to solve for $j^*$. Let $q^i(j)'$ denote the rate of change in $q$ as a function of $j$. We first point out a sufficient condition in Proposition 2 in which the supplier does not restrict its total commitment under contract Structure A. This happens when $w_2 \geq c_2 + \tau_2$. 

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Proposition 2. If $w_2 \geq c_2 + \tau_2$, then $p^*$ is unbounded.

When $w_2 \geq c_2 + \tau_2$, the supplier can earn a positive profit from fast-ship orders even if it does not produce any extra quantity up front (i.e. $y = 0$). As a result, there is no economic reason for the supplier to limit the size of its commitment. One may be tempted to extend this intuition to contract Structure B. That is, to expect that when $w_2 \geq c_2 + \tau_2$, the supplier always chooses $z^* = \infty$. As we show below, the above result may not always hold for Structure B.

Next, we prove that the supplier’s profit under Structures A and B is unimodal in $p$ and $z$ for exponential and uniform demand distributions. A profit maximizing value of $p$ can be unbounded as seen in Proposition 2, but the optimal value of $z$ is always finite. In addition to these distributions, we studied Gaussian and Gamma distributions numerically (the latter with shape parameter $\geq 1$) and found that the result in Proposition 3 held in all our numerical experiments. In the sequel, we shall assume that the supplier’s profit under Structures A and B is unimodal. Note that the results presented in the rest of this paper do not depend on a particular demand distribution so long as unimodality of supplier’s profit function holds.

Proposition 3. If the demand is either exponentially or uniformly distributed, then the following two statements are true:

1. the supplier’s profit under a Structure A contract has at most one local maximum. The global maximizer $p^*$ is either equal to the local maximizer, or $p^*$ is unbounded.

2. the supplier’s profit under a Structure B contract has at most one local maximum. The global maximizer $z^*$ is either equal to the local maximizer, or $z^* = 0$, or $z^*$ is unbounded.

Proposition 3 implies that the optimal $p$ (respectively $z$) can be obtained efficiently via simple line searches and by comparing the supplier’s profit at the local maximizer with that at $p = \infty$ (respectively $z = 0$ or $\infty$). Note that when $w_2 \geq c_2 + \tau_2$, the supplier can earn additional profit by satisfying fast-ship demand beyond its commitment specified in contract terms. However, the retailer would then anticipate greater availability and lower its initial order, which would lower the supplier’s profit. Hence, the supplier has an incentive to uphold contract terms and supply no more than its commitment.

Before closing this section, we present a comparison of Structures A and B in terms of their impact on retailer’s stocking decision. Recall that $0 \leq q^A(p)' \leq (1 - \alpha)^{-1}$ and $-\alpha^{-1} \leq q^B(z)' \leq 0$,.
which shows that R responds differently within the two structures if the supplier were to increase available supply — \( q \) is non-decreasing in \( p \) and non-increasing in \( z \). The different responses come from different ways in which the retailer can react to changes in supply commitments under Structures A and B. These observations also provide greater insights into the relative size of initial orders, see Proposition 4 below.

**Proposition 4.** For any \( p \) and \( z \), \( q^A(p) \leq q^B(z) \).

Proposition 4 shows that independent of contract parameter values within each structure, the supplier receives a larger initial order under Structure B as compared to A. This result is used to establish the preference of the supplier for Structure B over A in Proposition 8.

4. **Parameter Optimization: Structure C**

In Structure C, the supplier chooses \( \gamma \) after knowing \( q \). Recall that the extra supply \( y \) is chosen by the supplier after observing \( q \) in all three structures. Because \( \pi^C_S(y, \gamma, q) \) is concave in \( y \) (proof is omitted for brevity), it can be shown that the supplier chooses \( y^C \) according to

\[
y^C(q) = [\alpha(\eta_S - q)^+ \wedge \gamma].
\]

Let \( \eta_R = \bar{F}^{-1}((c_1 + h)/(w_2 - \tau_2)) \). We obtain \( \gamma(q) = \arg \max_{\gamma} \pi^C_S(y^C(q), \gamma, q) \) in Proposition 5 below.

**Proposition 5.** The supplier’s profit \( \pi^C_S(y^C(q), \gamma, q) \) is either unbounded or unimodal in \( \gamma \). In addition, the optimal \( \gamma(q) \) can be obtained as follows.

\[
\gamma(q) = \begin{cases} 
\infty & \text{if } w_2 \geq c_2 + \tau_2, \text{ and} \\
\alpha(\eta_R - q)^+ & \text{otherwise},
\end{cases}
\]

The quantity \( \eta_R \) can be explained in a manner similar to \( \eta_S \). The reason that \( c_2 \) does not appear in the expression for \( \eta_R \) is that when \( w_2 < c_2 + \tau_2 \), the supplier always selects \( y = \gamma \) and there is no need to obtain more items at unit cost \( c_2 \).

Next, we obtain an optimal order quantity, \( q^C = \arg \max_q \pi^C_R(q, \gamma(q)) \), as shown in Proposition 6 below. Hereafter, we use \( \pi^C_R(q) = \pi^C_R(q, \gamma(q)) \) and \( \pi^C_S(q) = \pi^C_S(y^C(q), \gamma(q), q) \) for convenience.
Proposition 6. The retailer’s profit is bimodal in \( q \) and there exists a \( \hat{c}_1 \in [w(w_2 - \tau_2)/(r-h), w(w_2 - \tau_2)/(r-\alpha(r-w_2)) - h] \) such that the optimal order quantity \( q^C \) can be obtained as follows.

\[
q^C = \begin{cases} 
F^{-1} \left( \frac{w}{r-\alpha(r-w_2)} \right) & \text{if either } w_2 \geq c_2 + \tau_2, \text{ or } w_2 < c_2 + \tau_2 \text{ and } c_1 \leq \hat{c}_1, \text{ and} \\
F^{-1} \left( \frac{w}{\tau} \right) & \text{if } w_2 < c_2 + \tau_2 \text{ and } c_1 > \hat{c}_1.
\end{cases}
\] (8)

The intuition behind Proposition 6 is as follows. When either the wholesale price is sufficiently large \( (w_2 \geq c_2 + \tau_2) \), or the unit cost of supplier’s initial purchase is sufficiently small \( (w_2 < c_2 + \tau_2 \text{ and } c_1 \leq \hat{c}_1) \), the supplier makes an ample fast-ship commitment to the retailer. In such cases, the retailer’s decision is based upon an assumption of ample availability of fast-ship supply. That is, the vast majority of customers who exercise the fast-ship option are served in this case. However, when \( w_2 < c_2 + \tau_2 \text{ and } c_1 > \hat{c}_1 \), the supplier chooses a conservative value of \( \gamma(q) \) because its second replenishment cost is higher. Anticipating this response, the retailer orders more up front.

Note that when \( q^C = F^{-1}(w/r) \) and \( \gamma(q^C) = 0 \), a Structure C contract is identical to a Structure B contract with \( z^* = 0 \). Similarly, when \( w_2 \geq c_2 + \tau_2 \), Structures A and C are identical because \( q^A = q^C \) and \( p - q^A = \gamma(q^C) = \infty \). That is, in some cases, the ability to be the first to choose contract parameters (also called market leadership) does not affect either party’s expected profit. Equation (7) and Proposition 6 also help obtain the following inequalities.

Proposition 7. For a fixed pair of \((w, \delta)\) values, the following inequalities hold: (1) \( q^A(p) \leq q^C \) for any \( p \), (2) \( \gamma(\hat{q}) \geq z^* \) where \( \hat{q} = q^B(z^*) \), and (3) \( q^B(\hat{z}) \geq q^C \) where \( \hat{z} = \gamma(q^C) \).

The arguments that lead to Part 1 of Proposition 7 are similar to those presented immediately after Proposition 4. Because the retailer enjoys greater freedom to adjust the supply between initial order and fast-ship orders under contract Structure A, it is not required to commit to an order quantity as large as that in contract Structure C. The intuition behind Part 2 of Proposition 7 is that because the supplier chooses \( \gamma \) after knowing \( q^C \), it can commit to a higher supply than that under Structure B without worrying about the possibility that a higher supply commitment may induce the retailer to order less up front. For similar reasons, the retailer chooses a smaller order quantity under Structure C when the fast-ship supply commitment under Structure C is the same as that under Structure B (Part 3 of Proposition 7). Proposition 7 is important because it leads to key results related to contract structure preferences (Proposition 8) and the possibility of resolving
5. **Contract Structure Preference and Selection**

We first investigate which contract structures are preferred by each player. In Proposition 8, we show that the supplier weakly prefers Structure B contracts and the retailer weakly prefers Structure A contracts unless the total promised supply under Structure A is lower than that under the other two contract types. In Proposition 8, the relationship “≽” denotes a weak preference.

**Proposition 8.** *For a fixed pair of* \((w, \delta)\) *values, the following statements are true.*

1. The supplier’s preference ordering of contract structures is \(B ≽ C ≽ A\).
2. If the total promised supply under Structure A is at least as much as that under Structures B and C, then the retailer prefers A.
3. When Structure A is unavailable, the retailer prefers \(C ≽ B\).

Proposition 8 can be explained by first observing that for the same total supply commitment, the supplier’s profit is higher within a contract structure that induces the retailer to order more up front. This is because higher initial purchase quantity simultaneously increases initial sales revenue and reduces the need for fast-ship supply, which can be costly to the supplier. Conversely, the retailer’s profit is higher when a contract structure allows it to order slightly less up front without sacrificing supply commitment, or else when a structure allows it to obtain a greater fast-ship supply commitment for the same initial purchase quantity. From Proposition 3 and Part 1 of Proposition 7, we observe that the retailer orders less when Structure A is utilized, regardless of supplier’s total commitment. Therefore, it is clear that Structure A is the least preferred structure for the supplier. Moreover, from Part 3 of Proposition 7, we observe that when supply commitment is held the same, the retailer orders more under Structure B than Structure C. This explains the preference ordering of contract structures from the supplier’s viewpoint. Within supplier-led structures, \(B ≽ A\) even when \(w\) is chosen by the supplier within the constraint that \(w ≤ r − \delta\) for each fixed \(\delta\).

We consider the retailer’s viewpoint next. If the total supply under Structure A is no less than that under the other two structures, it is clear that this would be the preferred structure for the retailer because the retailer can choose to order less up front. In other words, although the retailer is the first mover in Structure C, Structure A provides greater flexibility as long as the supply is sufficient. We also proved in Proposition 8 that the retailer prefers Structure C over B because the retailer can secure greater supply commitment for the same initial purchase quantity. Therefore R
prefers Structure C to B even when it has the right to choose wholesale price (see details in Section 6).

5.1 Contract Structure Selection

We observed above that between the two supplier-led structures, the supplier prefers B over A. Therefore, a supplier will not select A so long as the option to select B is available. We also observed that among Structures B and C, the supplier prefers B, whereas the retailer prefers C. This creates a potential conflict. In this section, we discuss how such conflict may be resolved.

We present two approaches for resolving conflict in contract selection preferences. The contract resolution approach depends on whether one player is dominant over the other. First, we show that when there is a dominant (hold-out) player, the other player can offer a modified contract to increase its profit without hurting the hold-out player’s profit. Second, we show that when there is no hold-out player, that is, both players have equal power, they can use a bargaining framework to decide the split of profit between themselves such that both players’ profits are higher than their disagreement profits.

Scenarios with a Hold-Out Player

If the channel has a hold-out player, then this means that the hold-out player will cooperate only if its profit equals its maximum profit under its preferred contract structure. However, this may lead to a lower profit for the other player. We argue next that as a non-hold-out player, either the supplier or the retailer may offer a modified contract that improves its profit and simultaneously make the other player weakly prefer the modified contract. We show below that such recourse is always available. In this Proposition, $z^*$ (respectively $q^C$) is the optimal choice of supplier’s (respectively retailer’s) decision variable under contract Structure B (respectively C).

Proposition 9.

1. There exists a $z \geq z^*$ such that $\pi^B_R(z) \geq \pi^C_R(q^C)$ and $\pi^B_S(z) \geq \pi^C_S(q^C)$.
2. There exists a $q \geq q^C$ such that $\pi^B_R(z^*) \leq \pi^C_R(q)$ and $\pi^B_S(z^*) \leq \pi^C_S(q)$.

The results in Proposition 9 can be explained as follows. If the retailer is the hold-out player and it strictly prefers Structure C contract, then the supplier may offer a higher supply commitment only if the retailer agrees to the choice of Structure B and ensure that the retailer earns a slightly higher profit under modified B than that under C. In addition, the order quantity under the modified Structure B contract (accepted by the retailer) can be proven to be higher than that under Structure C contract (arguments are similar to those underlying Part 3 of Proposition 7).
Therefore, the modified Structure B contract is still a better choice for the supplier. Similarly, if the supplier is the hold-out player, the retailer can find a \( q \geq q^C \) such that the modified Structure C contract is preferred by both the supplier and the retailer. This happens because the supply commitment under modified Structure C contract remains higher than \( z^* \) (arguments are similar to Part 2 of Proposition 7).

Next, we use an example to illustrate the results shown in Proposition 9. Consider a case in which \( X \) is Gamma distributed with \( E[X] = 400 \) and \( Var(X) = 8000 \). Other problem parameters are \( r = 12, w = 8, w_2 = 10.5, \tau_1 = 0.1, \tau_2 = 2, c_1 = 1, c_2 = 9, h = 0, \) and \( \alpha = 0.5 \). The results are shown in Table 3.

Table 3: Examples of Conflict Resolution by Providing Modified Contracts
(a) The Supplier is the Hold-out Player and The Retailer Offers Modified Contract C

<table>
<thead>
<tr>
<th>Individual Optimum</th>
<th>( z )</th>
<th>( q^B(z) )</th>
<th>( \gamma(q) )</th>
<th>( q )</th>
<th>( \pi^S_S(z) )</th>
<th>( \pi^B_R(z) )</th>
<th>( \pi^C_S(q^C) )</th>
<th>( \pi^C_R(q^C) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>R’s Counter Offer</td>
<td>–</td>
<td>–</td>
<td>79.52</td>
<td>349.0</td>
<td>–</td>
<td>–</td>
<td>2614.4</td>
<td>1267.6</td>
</tr>
</tbody>
</table>

(b) The Retailer is the Hold-out Player and The Supplier Offers Modified Contract B

<table>
<thead>
<tr>
<th>Individual Optimum</th>
<th>( z )</th>
<th>( q^B(z) )</th>
<th>( \gamma(q) )</th>
<th>( q^C )</th>
<th>( \pi^S_S(z) )</th>
<th>( \pi^B_R(z) )</th>
<th>( \pi^C_S(q^C) )</th>
<th>( \pi^C_R(q^C) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>S’s Counter Offer</td>
<td>85</td>
<td>347.6</td>
<td>–</td>
<td>–</td>
<td>2608.0</td>
<td>1268.6</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

The first rows of Tables 3(a) and 3(b) show both parties’ profits and preferences when each makes individually optimal decision. That is, \( z^* = 70.5 \) and \( q^C = 346 \). As shown in Proposition 8, the supplier prefers B and the retailer prefers C. In the second row of Table 3(a), we present a modified Structure C contract when the retailer commits to a higher-than-optimal order quantity. We see that when the retailer increases \( q^C \) from 346 to 349, both parties prefer the modified C contract over B. In this example, the retailer’s profit increases from 1265.8 to 1267.6 and the supplier’s profit increases from 2610.7 to 2614.4. The retailer’s profit is only slightly less than its best profit of 1267.8, which would be realized under Structure C.

Similarly, if the supplier offers a modified B contract by increasing \( z \) from 70.5 to 85, results are shown in Table 3(b). In this example, both players prefer the modified B contract as compared to the original C contract. The retailer’s profit increases from 1267.8 to 1268.6 and the supplier’s profit increases from 2601.0 to 2608.0. Note that the modified contracts B and C also generate greater channel profits compared to original contracts.
Scenarios Without a Hold-Out Player

Suppose there is no hold-out player. In this case, each player has a minimum profit expectation, referred to as the disagreement profit. We define disagreement profits next. Because the supplier’s profit is greater under contract Structure B and the retailer’s profit is greater under Structure C, the disagreement profits (minimum profit each party expects to earn) are $\pi_R^B(z^*)$ and $\pi_S^C(q^C)$ for the retailer and the supplier, respectively. We call these disagreement profits because the retailer (respectively the supplier) can always earn a minimum of $\pi_R^B(z^*)$ (respectively $\pi_S^C(q^C)$) by agreeing to the selection of contract Structure B (respectively C).

Let $\pi_T^i(q,y,j)$ denote the total supply chain profit in a negotiated profit allocation scheme when both players agree to use contract structure $i \in \{B,C\}$ and $0 < \sigma_S < 1$ (respectively $\sigma_R = 1 - \sigma_S$) denote the supplier’s (respectively retailer’s) fraction of total profits, i.e., $\pi_{S,T}^i(q,y,j) = \sigma_S \pi_T^i(q,y,j)$ and $\pi_{R,T}^i(q,y,j) = \sigma_R \pi_T^i(q,y,j)$. Because $\sigma_S$ and $\sigma_R$ do not depend on $(q,y,j)$, maximizing individual profit in a negotiated contract is equivalent to maximizing $\pi_T^i(q,y,j)$. That is, let

$$\hat{q}_i = \arg \max_q \pi_{R,T}^i(q,\hat{y},\hat{j}) = \sigma_R \arg \max_q \pi_T^i(q,\hat{y},\hat{j}), \quad (9)$$

and

$$(\hat{y}_i,\hat{j}_i) = \arg \max_{(y,j)} \pi_{S,T}^i(\hat{q},\hat{y},\hat{j}) = \sigma_S \arg \max_{(y,j)} \pi_T^i(\hat{q}_i,y,j). \quad (10)$$

By the definition of $(\hat{q},\hat{y},\hat{j})$, we obtain $\pi_T^i(\hat{q},\hat{y},\hat{j}) \geq \pi_T^i(q,y,j)$ for any $(q,y,j)$. Furthermore, the only difference between Structures B and C is the sequence of decisions. That is, $\pi_T^i(q,y,z) = \pi_T^C(q,y,\gamma)$ if $z = \gamma$. Because the decision sequence does not matter when the two players agree to maximize total profits in the supply chain, it follows that $\hat{q}_B = \hat{q}_C$ and $\hat{z} = \hat{\gamma}$. In other words, when the two player decide to negotiate a profit sharing contract, Structure B and Structure C are identical and there is no contract preference issue. The only question that remains is whether a negotiated contract yields higher profit as compared to each player’s disagreement profit. This question is answered in the following proposition.

**Proposition 10.** There exists some $(\sigma_R,\sigma_S)$ such that $\pi_{S,T}^i(\hat{q},\hat{y},\hat{j}) \geq \pi_S^C(q^C)$ and $\pi_{R,T}^i(\hat{q},\hat{y},\hat{j}) \geq \pi_R^B(z^*)$.

Proposition 10 shows that the two players can always find a set of $(\sigma_R,\sigma_S)$ such that a negotiated contract generates a higher individual profit than each player’s disagreement profit. That is, a conflict can be resolved as long as $\sigma$ lies between $\sigma_R$ and $\sigma_S$. 

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Contract satisfying Proposition 10 can result either from direct negotiation between the retailer and the supplier or when the two players trust this decision to a third party arbiter. If the arbiter were to choose a profit allocation that lies outside the feasible range we identified (e.g., $\sigma < \sigma_R$ or $\sigma > \sigma_S$), then negotiated solution would not exist because in that case one of the two players would do better by accepting the holdout position of the other player in the first place. That is if one of the parties knows that it will not receive a fair share in a negotiated solution, then negotiation will not occur because this party can do better by accepting the holdout position of the other party.

We demonstrate how such negotiation might work through an example. Using parameters introduced in Table 3(a) and 3(b), we observe that $\pi^T_i(\hat{q}, \hat{y}, \hat{j}) = 4218.7$. Hence, any negotiated contract with $\sigma_S \in [0.62, 0.7]$ should be acceptable to the two players when there is no hold-out player. In particular, when $\sigma_S = 0.65$, the supplier earns 2742.2, which is greater than 2601.0 (profit under C). Similarly, the retailer earns 1476.5, which is greater than 1265.8 (profit under B).

Note that the contract conflict still exists when $w$ is chosen by the contract leader. In such scenarios, the two approaches for resolving conflict also remain valid. We discuss those scenarios in Section 6.

6. Insights

6.1 Contract Structure Preferences and Selection with Endogenous $w$

In this section, we allow $w$ to be chosen by the first mover in a contract. That is, $w$ and $z$ are simultaneously selected by the supplier in Structure B. Similarly, $w$ and $q$ in Structure C are selected by the retailer before the supplier decides $\gamma$. The purpose of these variants is to study if the structural results obtained when $w$ was assumed exogenous remain intact. We do not consider cases in which $\delta$ is chosen by the supplier because it can be argued that the supplier would then set $\delta = r - w$. The retailer would make zero profit from fast-ship orders and all contracts would become heavily skewed in favor of the supplier.

**Contract Structure Preferences**

When $w$ is exogenous, we showed that Structure A will not be chosen by the supplier. We can argue that the same result holds when $w$ is chosen by the supplier. Hence, we only focus on Structures B and C in this section. Recall that the supplier prefers Structure B over C for a fixed $w$ (see Proposition 8). Hence, the same ordering holds with endogenous $w$ because the supplier’s profit for Structure B is even higher when $w$ for Structure B is also selected by the supplier. Similarly, the retailer prefers Structure C over B when it chooses $w$ within Structure C. In other words, the
conflict between the supplier’s and the retailer’s preference identified in Proposition 8 still exists when the wholesale price is endogenous and the leader is endowed with pricing power.

**Contract Selection With a Hold-Out Player**

Using the same parameters that we used for the for example presented in Table 3, we demonstrate how the two conflict resolving approaches shown in Section 5.1 work when $w$ is endogenous. In Table 4, we show that when there is a hold-out player, a counter offer from the non-hold-out player can make both players weakly prefer the modified contract to the hold-out contract. This means that the approach we obtained in Proposition 9 remains intact for endogenous $w$ as well. However, if the retailer is the hold-out and the wholesale price is endogenous, the supplier’s profit may be 0 in some cases. This is because the retailer may choose $w = c_1 + \tau_1$ in Structure C. In such cases, a counter offer from the supplier does not improve either player’s profit. This is illustrated in Table 4(b). When the retailer is the hold-out, we observe that the supplier’s profit remains 0 and the retailer earns the entire supply chain profit of 4166.6 with or without a counter offer.

**Table 4: Examples of Conflict Resolution by Providing Modified Contracts**

(a) The Supplier is the Hold-Out Player and The Retailer Offers Modified Contract C

<table>
<thead>
<tr>
<th></th>
<th>$w$</th>
<th>$z$</th>
<th>$q^B(z)$</th>
<th>$w$</th>
<th>$\gamma(q)$</th>
<th>$q$</th>
<th>$\pi^B_S(z)$</th>
<th>$\pi^R_B(z)$</th>
<th>$\pi^C_S(q^C)$</th>
<th>$\pi^C_R(q^C)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td>9.5</td>
<td>786.4</td>
<td>325.7</td>
<td>1.1</td>
<td>0</td>
<td>523.3</td>
<td>3062.5</td>
<td>712.5</td>
<td>0</td>
<td>4166.6</td>
</tr>
<tr>
<td>R’s Offer</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>9.5</td>
<td>754.2</td>
<td>333.5</td>
<td>–</td>
<td>–</td>
<td>3065.1</td>
<td>714.4</td>
</tr>
</tbody>
</table>

(b) The Retailer is the Hold-Out Player and The Supplier Offers Modified Contract B

<table>
<thead>
<tr>
<th></th>
<th>$w$</th>
<th>$z$</th>
<th>$q^B(z)$</th>
<th>$w$</th>
<th>$\gamma(q)$</th>
<th>$q$</th>
<th>$\pi^B_S(z)$</th>
<th>$\pi^R_B(z)$</th>
<th>$\pi^C_S(q^C)$</th>
<th>$\pi^C_R(q^C)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td>9.5</td>
<td>786.4</td>
<td>325.7</td>
<td>1.1</td>
<td>0</td>
<td>523.3</td>
<td>3062.5</td>
<td>712.5</td>
<td>0</td>
<td>4166.6</td>
</tr>
<tr>
<td>S’s Offer</td>
<td>1.1</td>
<td>0</td>
<td>523.3</td>
<td></td>
<td></td>
<td></td>
<td>0</td>
<td>4166.6</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

**Contract Selection Without a Hold-Out Player**

When there is no hold-out player and the wholesale price is endogenous, we can always find a negotiated contract (with a profit allocation decided either by both players or by the third-party arbiter) that is acceptable to the two players, which is similar to results shown in Proposition 10. Using the same parameters used for the example with exogenous wholesale price, $\pi_T(\hat{q}, \hat{y}, \hat{j}) = 4218.7$ and both players earn more than their disagreement profits (e.g., 0 for the supplier and 712.5 for the retailer) when $\sigma_S$ is in between [0, 0.83].

In summary, we observe that having endogenous $w$ does not change the preferences of the supplier and the retailer. It also does not affect the two conflict resolving mechanisms. However, because the profit difference in the two structures is greater when $w$ is endogenous, the benefit from the two conflict resolving approaches might be small.
6.2 The Effect of Partial Backorder Rate

In this section, we briefly discuss the effect of partial backorder rate which is largely based on a series of numerical examples involving Structures B and C. At the outset, we expect the retailer to benefit from a higher value of $\alpha$ (because both over- and under-stocking costs can be reduced when a greater fraction of customers are willing to exercise the fast-ship option), but we do not necessarily expect the supplier to realize a greater profit (because its cost increases with greater reliance on fast-ship supplies). Our analysis shows that this preliminary intuition is not entirely correct. We observe that the supplier always earns a greater profit in Structure B whereas the retailer earns a greater profit in Structure C when the value of $\alpha$ is high. Other than these two cases, the two players’ profit changes in $\alpha$ depend on problem parameters. We begin with analytical results of this section.

**Proposition 11.** The retailer’s profit for Structure C is non-decreasing in partial backorder rate regardless of whether $w$ is exogenous or endogenous.

The results behind Proposition 11 can be explained as follows. Because the supplier always chooses a greater commitment under a higher $\alpha$ for any fixed $q$ and $w$, the retailer earns more profit from the fast-ship orders. Hence, when the order quantity $q$ (and the endogenous wholesale price $w$) are chosen optimally by the retailer, the retailer’s profit is even higher under higher $\alpha$. Note that the retailer’s profit is not affect by $\alpha$ if $\gamma(q) = 0$.

We can also show that the supplier earns more profit under Structure C for a higher $\alpha$ when $w$ is set equal to $c_1 + \tau_1$ by the retailer. This is because when $w^* = c_1 + \tau_1$, the supplier earns zero profit for the initial order. Therefore, the fast-ship order becomes the only profitable source for the supplier in such cases and its profit increases as $\alpha$ increases provided $\gamma(q) \neq 0$ (that is, when the supplier chooses to support the fast-ship option).

Other than the two scenarios discussed above, the effect of partial backorder rate on the retailer’s and the supplier’s profits can only be obtained numerically. We briefly summarize the results in Table 5, which vary with problem parameters. In general, we observe that the supplier (reps. retailer) earns more profit under higher partial backorder rate within Structure B (reps. C) because it has the advantage of being the leader. However, the follower in a contract usually earns less profit from higher partial backorder rate unless the procurement cost is low.
Table 5: The Effect of Partial Backorder Rate $\alpha$

<table>
<thead>
<tr>
<th></th>
<th>Exogenous $w$</th>
<th>Endogenous $w$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Structure B</td>
<td>Structure C</td>
</tr>
<tr>
<td>$c_1$</td>
<td>Supplier</td>
<td>Retailer</td>
</tr>
<tr>
<td>Low</td>
<td>↑ ↑ ↑ ↑/−</td>
<td>↑ ↑/−</td>
</tr>
<tr>
<td>High</td>
<td>↑/−</td>
<td>↑</td>
</tr>
</tbody>
</table>

↑: increasing in $\alpha$; ↓: increasing in $\alpha$; \(\_\) invariant in $\alpha$

7. Concluding Remarks

In decentralized supply chains, retailers make stocking decisions to meet uncertain demand. However, retailers often experience stockouts leading to costly opportunity loss. To address this issue, some retailers may offer a fast-ship option (i.e. directly ship out-of-stock items) to customers. Offering such an option can be beneficial to both the supplier and the retailer because the total sales may increase. However, the incentives for the two players may be different since the provision of fast ship reduces (respectively increases) inventory cost for the retailer (respectively the supplier).

In this paper, we studied three different contract structures to provide insights into the effect of different contract types and parameters values on each player’s and channel’s performance. The main contribution of this paper lies in presenting a mathematically rigorous framework for comparing different contract structures. We proved that Structure A is dominated by B from the supplier’s perspective. Therefore, we argued that the supplier will not offer Structure A contracts even though they are preferred by the retailer. Among the remaining two structures, we showed that the supplier prefers Structure B whereas the retailer prefers Structure C. We presented two different approaches for resolving differences in contract structure preferences. In the first case, the holdout players contract preference does not prevail because the non-holdout player can make a better counteroffer. In case the two players negotiate in good faith, channel optimality could be reached so long as the profits are shared in a reasonable manner. We establish the existence of range of profit shares that result in such a solution.

This paper presents an initial attempt to address a gap in the literature on models dealing with the fast-ship option. While the fast-ship option is used by some retailers, there is not much academic literature that focuses on different contract structures within this context. In practice, fast-ship option can be implemented in several different ways. Getting direct supply from the original supplier may not be the only option. In fact, supply sources for fast-ship orders include both primary and secondary suppliers, where the latter often specialize in fast delivery of small
orders. Also, fast-ship orders may be filled by retailers who agree to pool excess inventory.

Many avenues of future research remain open. One such direction is to study the supplier-retailer interactions when two or more retailers form an alliance to cover each other’s shortage. Another direction would be to investigate how the availability of fast-ship option affects a retailer’s ordering policy in a multi-period setting. Both these topics are currently under investigation by the authors.

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References


