IE-4521 Homework Assignment 3 (Due Thursday September 29th)

Probability Review

1. Do the following problems from the book: 3.1.4, 3.1.8, 3.2.4, 3.2.11, 3.3.3, 3.3.6, 3.4.7, 3.5.4, 4.1.3, 4.2.6, 4.3.5, 4.4.4, 4.5.4.

2. • Suppose $X$ is a continuous random variable with cdf $F$. What is the distribution of $F(X)$? Show your work.
   • Let $U$ be a uniform random variable on $[0, 1]$. For a continuous cdf $F$, find the cdf of the random variable $F^{-1}(U)$.

3. For $X$ and $Y$ independent exponential random variables with respective parameters $\lambda$ and $\mu$ find
   • $P(X < Y)$
     Big hint: evaluate the following integral
     \[
     \int_0^\infty \mu e^{-\mu y} \int_0^y \lambda e^{-\lambda x} dx dy.
     \]
     This is an integral of the joint pdf over the region where $X < Y$.
   • (optional) $P(\min\{X, Y\} < t)$

4. One way to describe the distribution of a discrete random variable $X$ is to find the function $g(s) = E[s^X]$ for all $s \in [0, 1]$.
   • Suppose $X$ has a Poisson($\lambda$) distribution. Find $g(s) = E[s^X]$.
     Hint: This is just evaluating $E[h(X)]$ where $h(x) = s^x$, therefore if $X$ is a Poisson random variable
     \[
     g(s) = \sum_{n=0}^{\infty} s^n P(X = n),
     \]
     now fill in the value of $P(X = n)$ for a Poisson random variable with parameter $\lambda$ and figure out the value of the series from your calculus book.
   • Suppose $X$ and $Y$ are independent Poisson random variables with respective parameters $\lambda$ and $\mu$. Find $g(s) = E[s^X+Y]$, does this give you any ideas about the distribution of $X + Y$.
     Hint: Recall that if $X$ and $Y$ are independent that $E[h(X)g(Y)] = E[h(X)]E[g(Y)]$ for any functions $h$ and $g$, and notice that $s^{x+y} = s^x s^y$.
   • (Optional) If $X$ and $Y$ are independent Poisson random variables with respective parameters $\lambda$ and $\mu$, show that the conditional distribution of $X$ given that $X + Y = n$ is binomial, and find the parameters.

5. The geometric distribution is a discrete distribution with the ‘memoryless’ property. In particular, if $X \sim Geom(p)$ show that
   \[
   P(X > m + n | X > m) = P(X > n).
   \]