Managing Disruptions in Decentralized Supply Chains with Endogenous Supply Process Reliability

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Abstract

Supply disruptions are all too common in supply chains. To mitigate delivery risk, buyers may either source from multiple suppliers or offer incentives to their preferred supplier to improve its process reliability. These incentives can be either direct (investment subsidy) or indirect (inflated order quantity). In this paper, we present a series of models to highlight buyers’ and suppliers’ optimal parameter choices. Our base-case model has deterministic demand and the supplier has two possible yield outcomes: partial disruption or the special case of all-or-nothing supply. For the all-or-nothing model, we show that the buyer prefers to only use the subsidy option which obviates the need to inflate order quantity. However, in the partial disruption model, both incentives - subsidy and order inflation - may be used at the same time. Although single sourcing provides greater indirect incentive to the selected supplier because that avoids order splitting, we show that the buyer may prefer the diversification strategy under certain circumstances. We also quantify the discount needed in the wholesale price to ensure that dual sourcing strategy dominates sole sourcing. Finally, we extend the model to the case of stochastic demand. Structural properties of ordering/subsidy decisions are derived for the all-or-nothing model, and in contrast to the deterministic demand case, we establish that the buyer may increase use of subsidy and order quantity at the same time.

Key words: supply disruption; process improvement; incentive mechanism; dual sourcing.

1. Introduction

The success of supply chain partners in decentralized settings depends on their ability to coordinate production and purchasing decisions to match supply with demand. This paper focuses on supply disruption risk, which contributes to the complexity of this task. Supply disruption is increasingly
an important concern for procurement managers because supply chains have become longer and price competition is driving firms to purchase from cheaper but unproven suppliers (see survey results reported in McKinsey 2006, and Deloitte 2008). In a recent example, disruptions in flight schedules due to the spread of volcanic ash from Iceland had a significant impact on operational performance of firms (Banker 2010). Similarly, a survey completed in 2011 found that 85% of firms had suffered at least one supply disruption (Veysey 2011). Hendricks and Singhal (2003; 2005a,b) and Powell (2011) have highlighted the negative impact of supply glitches on operational performances of firms leading to increased equity risk and loss of shareholder wealth.

How might a buyer firm cope with supply disruption risk? One set of risk mitigating strategies includes multi-sourcing, backup sourcing, and emergency purchases. Such strategies have been extensively studied in the Operations Management (OM) literature (see Anupindi and Akella 1993; Gurnani et al. 1996; 2000; Tomlin 2006; Dada et al. 2007). An alternate, but increasingly popular approach is one in which buyer firms invest in improving suppliers’ processes to lower costs, increase quality, and improve reliability (Handfield et al. 2000; Liker and Choi 2004; Liu et al. 2010; Wang et al. 2010). For instance, in the automotive industry, it is common for companies like Honda, Toyota, BMW, and Hyundai to work with their suppliers to improve performance (Handfield et al. 2000, Krause et al. 2007).

Wang et al. (2010) provides guidance concerning when the dual sourcing approach is favored relative to the process improvement approach. This comparison assumes that the buyer firm has developed a close relationship with the supplier firm so that it can affect the adoption of a particular production process in the supplier’s production facility. Such close partnership between firms is often hard to achieve. In most cases, each firm has autonomy over key operational decisions such as process and technology choices, and production and order quantities. This motivates us to study the problem of managing disruptions in a decentralized supply chain in which the buyer may provide incentives to influence supply reliability, but it is the supplier firm that makes process/technology choices and production decisions. We study the following research questions: Which levels of different types of incentives should the buyer use to enable higher investment in reliability by the supplier? When would the buyer prefer a cooperative, incentive-based, approach with a single supplier as compared to an arms length approach of diversifying orders from two suppliers? As such, the contribution of this paper is to determine the optimal levels of different types of incentives,
and compare the process improvement strategy with the supplier diversification strategy.

We consider a series of models starting with a base model comprising of a single buyer and a single supplier. The buyer faces a deterministic demand and the supplier’s production process is subject to catastrophic disruption (i.e., an all-or-nothing yield) with the probability of disruption a function of the supplier’s investment in process improvement. The buyer may influence the supplier’s process choice by providing incentives. We focus on two types of incentives in the paper, which are commonly observed in industry and studied in the OM literature (see details in Krause 1997). In the first instance, the order quantity itself serves as an incentive because larger orders create the promise of higher payments if delivery is successful. Contracts like these are called quantity-only contracts in this paper and the corresponding incentive mechanism is called indirect incentive because there is no direct investment by the buyer in the supplier’s process improvement. In the second approach, which we call subsidy contract or direct incentive mechanism, the buyer subsidizes the supplier’s investment in process improvement.

In each case, we use a Stackelberg game formulation to study the interactions between the two firms. The buyer, as the leader, first places an order with the supplier. In addition, it may specify a subsidy rate if it decides to offer subsidy. Then, the supplier, as the follower, makes the technology choice, which determines the process reliability level. Finally, the supplier decides how much to produce. We obtain an equilibrium solution of this game, which allows us to study the two firms’ preferences between the two incentive mechanisms and the effectiveness of each mechanism in improving supplier reliability. The analysis shows that subsidy not only improves process reliability, but also obviates the buyer’s need to inflate its order.

Our base-case model has deterministic demand and all-or-nothing disruption. This model is a special case of a more general model with partial disruption, that is, the case in which the supplier has a positive minimum yield equal to $\alpha$ fraction of the quantity produced. The latter is referred to as the $(1, \alpha)$ supply risk model, as opposed to the $(1, 0)$ base-case model. We also consider the case in which the buyer’s demand is random. In order to keep the models tractable, the stochastic demand model considers $(1, 0)$ risk, and the $(1, \alpha)$ model considers deterministic demand. These two variants of the base-case model confirm some of the insights from the base case, but also reveal new insights.

Because buyers often improve supply reliability by procuring from multiple suppliers, we
also analyze a dual-sourcing model in which the buyer does not invest in suppliers’ processes. It is important to note that our dual sourcing model is different from those commonly seen in the literature because in our model the supplier reliability is endogenous, whereas in previous studies, reliability is modeled as an exogenous parameter. We use this model to compare the effectiveness of process improvement and dual sourcing approaches in dealing with supply risk. The buyer’s tradeoff is as follows. If it places its entire order with a single supplier and possibly offers subsidy to reduce supply risk, it ensures greater supplier effort, higher reliability and a better chance of meeting demand. In contrast, if it diversifies, it lowers supply risk because both suppliers are not likely to experience disruption at the same time. However, with endogenous reliability choice, there is a potential downside of supply diversification — a lower order allocation to each supplier may reduce suppliers’ incentive to invest in reliability-improving effort. When comparing single and dual sourcing models, we assume that the wholesale price may be lower in the dual sourcing model because of supplier competition. In particular, we investigate the level of discount, if any, that will be needed to make dual sourcing more attractive to the buyer.

Our analysis shows that despite the benefit of larger order in single sourcing mode, dual sourcing may lead to higher expected profit for the buyer under the same wholesale price. This happens because the benefit of risk diversification together with the savings from lower overage cost can outweigh the loss resulting from lower supplier reliability in some cases. Conversely, there exist cases in which dual sourcing is attractive only if wholesale price is lower when sourcing from two suppliers. One may think that when dual sourcing is dominated for some value of the wholesale price, it should become the preferred strategy if price were to drop below a certain level. We also show that this intuition is not always true because the buyer’s profit is increasing in wholesale price in some cases. This happens because the supplier’s reliability increases in the wholesale price, and this benefit outweighs the decrease in the buyer’s margin from higher wholesale price.

The key contribution of this paper lies in evaluating certain supply risk management strategies in a decentralized supply chain setting. We develop rigorous analysis of the suppliers’ and the buyers’ parameter optimization problems under different settings. We use these results to provide guidance to operations managers regarding which risk mitigation strategies are appropriate under which situations. Before presenting the models, we briefly discuss the related literature in §2.
2. Related Literature

There has been a growing stream of literature on supply disruptions. Most papers deal with the impact of disruptions on the performance of firms and various risk management strategies adopted by purchasing firms; see Vakharia and Yenipazarli (2008) and references therein for a summary of common approaches to managing disruptions. Even though this topic has attracted interest in the operations management community, with a few exceptions, a common assumption in the literature is that the supplier is unable to improve upon its reliability performance. This is in contrast to the evidence in practice. As mentioned earlier, there are examples in automotive and other industries where buyers and suppliers have invested in improving performance - lowering costs, and improving quality and reliability. Our paper considers supply disruption mitigation strategies when suppliers can make autonomous process choice.

Our paper is also related to the literature on supply process improvement. Krause (1997; 1999) developed a classification of different types of supplier development activities including cooperative strategies such as incentives and direct involvement to improve performance, and non-cooperative strategies such as multiple sourcing to promote competition. Recent empirical evidence in the US points to use of direct involvement by firms as an effective mechanism to improve supplier reliability (Krause et al. 2007). Our paper provides an analytical framework for two most commonly used incentive mechanisms mentioned in these papers—direct investment and larger business commitment. The literature also contains papers that have modeled different impacts of process improvement activities. Using the example of semiconductor manufacturing industry, Bohn and Terwiesch (1999) discussed the importance of improving process yields in that it not only lowers product costs but also provides maximum benefit during periods of constrained capacity, such as during new product ramp-up. In another paper, Terwiesch and Bohn (2001) developed an analytical model to explain the effect of process learning on interactions between capacity utilization and yield improvement. A recent paper by Liu et al. (2010) modeled reliability improvement in the form of stochastic ordering between two scenarios in a centralized setting. Different from these papers, we assume that process reliability is defined as the probability of successful delivery, and a buyer firm can affect suppliers’ reliability-enhancing investment effort by providing suitable incentives. Thus, reliability itself is endogenous in our paper, and we study its selection in a decentralized setting.
Our paper is also closely related to extensive literature on dual sourcing, as mentioned in §1. Among these papers, the most relevant one that also compares the effectiveness of dual sourcing and process improvement strategies like we do in our paper, is Wang et al. (2010). In a newsvendor setup, Wang et al. consider a problem where a supplier may experience random capacity or random yield, and a firm may undertake costly effort to improve its supplier’s reliability (resulting in a stochastically larger output), or source from two suppliers. Their motivation is similar to the problem addressed in our paper but there are significant differences in the model setting and the ensuing analysis between the two papers. First, in our model, the process improvement/investment decision is made by the supplier and the buyer influences it by determining the level of subsidy/order quantity. This is consistent with a decentralized decision-making structure encountered in practice and in contrast to the centralized problem setting in Wang et al. (2010). Moreover, in our paper, both the buyer and the supplier make two decisions – the buyer chooses the order size and the level of investment subsidy (if used), whereas the supplier chooses the production quantity and the investment in process improvement. As such, we focus on the effect of endogenizing supplier reliability by extending the centralized newsvendor setting in Wang et al. (2010) to a decentralized setting.

The rest of the paper is organized as follows. In §3, we formulate the base-case model with a single buyer, a single supplier, deterministic demand, and endogenous reliability. §4 considers stochastic demand, and §5 presents the dual-sourcing model in which the buyer may split its order over two suppliers, but does not invest in process improvement of either supplier. We conclude the paper and discuss future research directions in §6.

3. Endogenous Reliability Model

A buyer, who faces a known (deterministic) demand \( d \) for a customized product, can source this product from a supplier at a wholesale price \( w \). The buyer sells the product to its customers at an exogenous market price \( r \geq w \). The supply is subject to a random yield loss: for a production quantity \( x \), the output is \( Yx \), where the yield rate \( Y \) has the following distribution:

\[
Y = \begin{cases} 
1, & \text{with probability } z, \\
\alpha, & \text{with probability } 1 - z,
\end{cases}
\] (1)


where $0 \leq \alpha < 1$ denotes the imperfect yield rate, and $0 \leq z \leq 1$ denotes the perfect-yield probability and can be viewed as the supplier’s reliability. The supplier gets paid only for quantities delivered and cannot deliver more than the amount ordered by the buyer. We assume that any excess production has no salvage value. In this paper, we mostly focus on the case in which $\alpha = 0$, as studied in Baiman et al. (2000), Babich et al. (2007), Dada et al. (2007), Chaturvedi and Martínez-de-Albéniz (2008), Gurnani and Shi (2006), Yang et al. (2009), and Gumus et al. (2012). This special case corresponds to a situation where the supplier’s output is completely destroyed or unusable when some disruptive events happen. We denote this model as $(1, 0)$ model. Note that the $(1, 0)$ environment is frequently encountered and it is a tractable special case of the $(1, \alpha)$ model. Later in this section, we present results for the more general $(1, \alpha)$ model.

The supplier incurs a unit production cost $c \leq w$ and a fixed cost $\beta z^2 / 2$ of acquiring the technology to enable the production of custom items with reliability $z$, where $\beta / 2$ corresponds to the investment cost of choosing a technology with perfect yield. Similar cost functions have been widely used in the OM and marketing literature to model diminishing impact of investment efforts that are directed at improving demand and quality (e.g., Li 2011, Heese and Swaminathan 2006, and Moothy 1988). Neither the buyer nor the supplier incurs a shortage penalty from having insufficient supply. It can be shown that if penalty were strictly positive and linear, then the contract terms for that setting can be obtained by appropriately increasing either $r$ or $w$ by the amount of the penalty. In all our models, the two players are risk neutral and their interaction is modeled as a Stackelberg game with the buyer as the leader and the supplier as the follower. We use $\pi$ to denote expected profit. Subscripts B and S identify the buyer and the supplier, respectively.

Since the supplier can influence its reliability by exerting costly effort which benefits both the buyer and the supplier, the buyer may want to use direct financial subsidy to provide incentive for the supplier’s investment. Specifically, we consider the following type of subsidy — the buyer places an order of size $q$ with the supplier and also offers to pay for $\gamma$ fraction of the supplier’s cost of technology investment. We assume that the buyer is able to verify the investment cost incurred by the supplier, for instance, when the technology costs are well known in the industry. As mentioned earlier, we refer to the use of such subsidy as the direct incentive mechanism to differentiate it from order inflation, which serves as an indirect incentive mechanism.$^1$

$^1$The profit potential associated with a higher order size serves as an incentive to the supplier to improve reliability
Upon participating in a contract \((q, \gamma)\) with the buyer, the supplier chooses the optimal reliability level \(z\) and production quantity \(x\) to maximize the expected profit. The supplier’s problem can be written as follows:

\[
\max_{x \geq 0, 0 \leq z \leq 1} \pi_S(x, z; q, \gamma, \alpha) = zw \min\{q, x\} + (1 - z)w \min\{q, \alpha x\} - cx - (1 - \gamma) \frac{\beta z^2}{2}.
\]  (2)

The buyer, taking into account the supplier’s best response \((x^*(q, \gamma), z^*(q, \gamma))\) to the contract terms, solves the following profit maximization problem:

\[
\max_{q \geq 0, 0 \leq \gamma \leq 1} \pi_B(q, \gamma) = z^* \left[ r \min\{d, q, x^*\} - w \min\{q, x^*\} \right] + (1 - z^*) \left[ r \min\{d, q, \alpha x^*\} - w \min\{q, \alpha x^*\} \right] - \gamma \frac{\beta (z^*)^2}{2}.
\]  (3)

In what follows we analyze the special case of the formulation in (2) and (3) with all-or-nothing reliability, i.e., the \((1, 0)\) model.

### 3.1 The all-or-nothing \((1, 0)\) model

To focus the study on situations in which incentives provided by the buyer are for the purpose of improving supply reliability rather than guaranteeing supplier participation, we normalize the unit production cost \(c\) to be zero. Setting \(c = 0\) also improves exposition. We first analyze the supplier’s problem. Since the supplier only gets paid when there is no disruption and for deliveries up to the received order \(q\), the supplier’s optimal production quantity in the \((1, 0)\) model is simply \(q\). The supplier’s problem then can be written as a univariate maximization problem as follows:

\[
\max_{0 \leq z \leq 1} \pi_S(z; q, \gamma) = zwq - (1 - \gamma) \frac{\beta z^2}{2}.
\]  (4)

Lemma 1 characterizes the supplier’s optimal technology choice.

**Lemma 1** Upon receiving a contract \((q, \gamma)\), the supplier’s optimal technology choice is \(z^*(q, \gamma) = \min\{wq/(1 - \gamma)\beta, 1\}\).

The proofs of all Propositions and Lemmas are included in the appendix A.1.

The optimal investment for the supplier is based on the unconstrained solution to problem (4) and its upper limit of one. Clearly, the supplier chooses a more reliable technology when a larger order is received and/or a larger subsidy is provided by the buyer (i.e., higher \(\gamma\)). So from and deliver the items.
the supplier’s perspective, the two mechanisms provide substitutable incentives. Note that when the subsidy option is not available, \( \gamma = 0 \) in Lemma 1.

Taking into account the supplier’s best response \( z^*(q, \gamma) \) specified in Lemma 1, the buyer’s problem (3) can be written as follows:

\[
\max_{q \geq 0, 0 \leq \gamma \leq 1} \pi_B(q, \gamma) = z^*(q, \gamma) \left[ r \min(q, d) - wq \right] - \frac{\gamma \beta [z^*(q, \gamma)]^2}{2}. \tag{5}
\]

Proposition 1 characterizes the buyer’s optimal subsidy contract.

**Proposition 1** In the \((1, 0)\) model, the buyer’s optimal order quantity is \( q^* = d \), and the optimal subsidy level is given as follows:

1. For a low wholesale price \((w \leq 2r/3)\),

\[
\gamma^* = \begin{cases} 
\frac{2r-3w}{2r-w}, & \text{if } d \leq \frac{2\beta}{2r-w}, \\
1 - \frac{w}{\beta}d, & \text{if } \frac{2\beta}{2r-w} < d \leq \frac{\beta}{w}, \\
0, & \text{if } d > \frac{\beta}{w}.
\end{cases} \tag{6}
\]

2. For a high wholesale price \((w > 2r/3)\), \( \gamma^* = 0 \).

At a low wholesale price level, when demand exceeds a certain threshold \((d > \beta/w)\), because the large size of the order needed to satisfy demand provides a natural incentive for the supplier, the buyer does not need to provide any extra incentive beyond the regular order (i.e., the known demand). However, when demand is below the threshold, \( \beta/w \), the buyer has to provide incentive by either inflating its order beyond \( d \) (indirect incentive) or by providing a subsidy (direct incentive). An immediate observation from Proposition 1 is that the buyer always prefers the latter. Using the optimal subsidy level from (6), we can compute the optimal induced reliability level from Lemma 1: \( z^* = (2r-\gamma)\beta/(2r-w) \) when \( d \leq 2\beta/(2r-w) \), and \( z^* = 1 \) otherwise. This implies that as demand increases, the buyer would like to induce a higher supply reliability. Similar observations can be made for high wholesale price. The induced reliability level is \( z^* = wd/\beta \) when \( d \leq \beta/w \), and \( z^* = 1 \) otherwise. The buyer never needs to provide extra incentives, since the high wholesale price makes the natural incentive large enough to reach the desired reliability level.

In order to further investigate the difference between the direct and indirect mechanisms, we next study a quantity-only contract where the investment-cost-sharing option is not available.
The supplier’s optimal technology choice for a given contract $q$ follows immediately from Lemma 1 with $\gamma = 0$, i.e., $z^*(q) = \min\{wq/\beta, 1\}$. The buyer’s problem is:

$$\max_{q \geq 0} \pi_B(q) = z^*(q) \left[ r \min(q, d) - wq \right].$$

(7)

The optimal order quantity in this instance is summarized below.

**Proposition 2** In the $(1, 0)$ model, the buyer’s optimal quantity-only contract can be characterized as follows:

1. For a low wholesale price ($w \leq r/2$),

   $$q^* = \begin{cases} \frac{rd}{2w}, & \text{if } d \leq \frac{2\beta}{r}, \\ \frac{\beta}{w}, & \text{if } \frac{2\beta}{r} \leq d \leq \frac{\beta}{w}, \\ d, & \text{if } d > \frac{\beta}{w}. \end{cases}$$

   (8)

2. For a high wholesale price ($w \geq r/2$), $q^* = d$.

Several observations can be made by comparing Propositions 1 and 2. First, the boundary between low and high wholesale price is different in the subsidy ($2r/3$) and quantity-only ($r/2$) contracts. Second, a buyer who uses a quantity-only contract needs to order more if it wants greater reliability. This happens because the buyer has no other means of affecting supplier’s decisions within the quantity-only mechanism. In fact, when the wholesale price and demand are below a certain threshold ($w \leq r/2, d \leq \beta/w$), the buyer finds it economical to inflate its order above demand $d$. Third, subsidy not only is more cost-effective than order inflation in providing incentives, but it also leads to a better (in the weak sense) technology choice, i.e., $z^*_{\text{subsidy}} \geq z^*_{\text{quantity-only}}$, that is, by using subsidy, the buyer benefits from higher (in the weak sense) reliability as compared to using quantity incentive only.

In summary, in the case of all-or-nothing yield, although the direct and indirect incentive mechanisms are substitutable from the supplier’s perspective, the buyer has a clear preference – the former is weakly better than the latter. The induced reliability level is also weakly higher when the subsidy option is used. Finally, it is interesting to note that the buyer never increases the use of both incentives at the same time. As demand increases, the order quantity set at $q^* = d$ also increases, but the optimal subsidy is non-increasing.
3.2 The guaranteed-minimum-yield \((1, \alpha)\) model

In the \((1, 0)\) model, the supplier always sets the production quantity equal to the received order \(q\). The only strategy it can use to increase the expected quantity delivered is to invest in a more reliable technology so that the probability \(z\) of perfect yield is higher. In the \((1, \alpha)\) model (with \(\alpha > 0\)), since the supplier’s minimum output is \(\alpha x\), the supplier may use an alternative strategy, namely, to set a higher production quantity \(x\), so that the amount it can deliver (i.e., \(\min\{\alpha x, q\}\)) is higher. The supplier may also choose a combination of these strategies. We note that the production inflation strategy is viable only when the unit production cost is not too high, i.e., when \(c < \alpha w\); otherwise, all analysis and insights derived for the \((1, 0)\) model can be directly applied to the general \((1, \alpha)\) model.

When \(c < \alpha w\), the supplier’s optimal decision can be summarized as follows. Define:

\[
q_L := \left(1 - \frac{c}{\alpha w}\right) \frac{2\beta}{w(1 - \alpha)}, \text{ and } q_U := \frac{\beta}{w(1 - \alpha)}.
\]

(9) (10)

**Proposition 3** In the \((1, \alpha)\) model, given the contract \((q, \gamma)\), the supplier’s optimal decision is:

1. when the wholesale price is high \((w \geq \frac{2c}{\alpha})\):

\[
\begin{align*}
x^* &= \frac{q}{\alpha}, \quad z^* = 0, \quad \text{if } q \leq \frac{(1-\gamma)\beta}{2c(1-\alpha)}, \\
x^* &= q, \quad z^* = 1, \quad \text{if } q \geq \frac{(1-\gamma)\beta}{2c(1-\alpha)}.
\end{align*}
\]

2. when the wholesale price is low \((\frac{c}{\alpha} \leq w \leq \frac{2c}{\alpha})\):

\[
\begin{align*}
x^* &= \frac{q}{\alpha}, \quad z^* = 0, \quad \text{if } q \leq (1 - \gamma)q_L, \\
x^* &= q, \quad z^* = \frac{w(1-\alpha)q}{(1-\gamma)\beta}, \quad \text{if } (1 - \gamma)q_L \leq q \leq (1 - \gamma)q_U, \\
x^* &= q, \quad z^* = 1, \quad \text{if } q \geq (1 - \gamma)q_U.
\end{align*}
\]

(11)

Proposition 3 shows that when the wholesale price is high, the supplier has enough incentives to fulfill the buyer’s entire order. This is achieved by either inflating production quantity when the order size is small, or using a perfect yield process when the order size is large and inflating production becomes more costly. When the wholesale price is low, the supplier may become reluctant in taking these efforts to meet the buyer’s entire order, especially when the order size is not so large.
The above result also implies that for a given contract \((q, \gamma)\), a supplier with a higher value of \(\alpha\) is less likely to invest in reliability improving effort and may even choose the production inflation strategy, forgoing any investment in reliability improvement.

Next, we discuss the buyer’s problem. When the wholesale price is high, i.e., \(w \geq 2c/\alpha\), or when the order size is either large (i.e., \(q \geq q_U\)) or small (i.e., \(q \leq q_L\)), the optimal contract is \(q^* = d, \gamma^* = 0\). In the former two cases, the supplier chooses the strategy of using a perfect yield process; and in the last case, the supplier chooses the first strategy of inflating production in (11). However, when demand is within the range \([q_L, q_U]\), the supplier has three different responses specified in (11). The buyer then needs to choose the contract terms that induce a response in the buyer’s best interest and achieve the highest expected profit. A detailed analysis of the case in which \(d \in [q_L, q_U]\) is deferred to the appendix A.2 and we focus on some important insights next.

The fact that the supplier has an additional production inflation option in the \((1, \alpha)\) model has implications for both the buyer’s incentive offer and the profits for both players. First, in contrast to the \((1, 0)\) model, we observe that the buyer’s optimal order quantity can be either smaller than, or equal to, or greater than the demand \(d\) (see the solid line for the optimal order quantity and the dashed line for demand in Figure 1(a)). The reason for intentionally creating a shortage by ordering less than demand is to induce the supplier to choose the production inflation strategy. Without lowering the order size below demand, the supplier would have chosen the investment strategy which is not in the buyer’s best interest. Second, we see that the indirect incentive mechanism may be more attractive than the direct one. For example, in Figure 1 when \(d = 2.7\) the optimal subsidy contract has \(q^* > d\) and \(\gamma^* = 0\). In this case, order inflation is more effective than subsidy in improving supplier reliability. This is also in contrast to the \((1, 0)\) model where order inflation is never used. Third, there are situations (examples included in the appendix A.3) where the buyer may make lower expected profit in the \((1, \alpha)\) model as compared to the \((1, 0)\) model, that is, a supplier with an all-or-nothing disruption may be preferred to one with some guaranteed minimum yield. This could happen when the lower-yield supplier (i.e., with all-or-nothing yield) is induced to use a perfect yield process, or more interestingly, when the supplier with guaranteed minimum yield is induced to use the production inflation strategy. Similarly, within the \((1, \alpha)\) model, the effect of \(\alpha\) is not monotone, that is, the buyer’s expected profit may decline for higher values of \(\alpha\). These results are direct consequences of the endogenous investment
in reliability which is the focus of our paper and differentiates our work from the extant literature.

4. Stochastic Demand

In this section, we consider stochastic demand for the (1, 0) yield model. We use $f(\cdot)$, $F(\cdot)$ and $\bar{F}(\cdot)$ to denote the pdf, cdf, and ccdf of the buyer’s demand distribution. Note that the supplier’s best response remains the same as that in the deterministic demand case, which is given in Lemma 1.

For ease of exposition, we introduce the notation $d[p] = \bar{F}^{-1}(p)$, with the understanding that $d[p]$ is the smallest value that satisfies the defining equality. In other words, $d[p]$ is a quantity such that the probability that demand exceeds $d[p]$ is $p$. It is easy to confirm that $d[p]$ is decreasing in $p$ and that for a fixed $p$, $d[p]$ is larger when demand is stochastically larger in the usual sense\(^2\).

According to Lemma 1, the supplier has two different technology choices based on the contract it receives. We discuss the resulting cases one by one. In the first case, the buyer’s contract offer satisfies $wq/(1 - \gamma)\beta \geq 1$, and the supplier chooses a perfect yield process, i.e., $z^* = 1$. The buyer’s problem in that case is:

$$\max_{q, \gamma} \pi_B = rE_D\left[\min(q, D)\right] - wq - \gamma\frac{\beta}{2}$$

$$\text{s.t. } q \geq \frac{(1 - \gamma)\beta}{w}, \quad 0 \leq \gamma \leq 1.$$  \hspace{1cm} (12)

\(^2\)For random variables $A$ and $B$, $A$ is stochastically larger than $B$ in the usual sense, denoted $A \geq_{st} B$, if $E\phi(A) \geq E\phi(B)$ for all increasing functions $\phi$ for which the expectations exist. An equivalent characterization of $A \geq_{st} B$ is that $F_A(x) \geq F_B(x)$ for every $x$. 

\[\]
The optimal contract is characterized below.

**Proposition 4** Under stochastic demand, the buyer’s optimal subsidy contract when the contract terms \((q, \gamma)\) satisfy \(wq/(1-\gamma)\beta \geq 1\) (and hence the supplier optimally sets \(z^* = 1\)) is:

\[
q^* = \begin{cases} 
  d\left(\frac{w}{r}\right), & \gamma^* = 0, \\
  \frac{\beta}{w}, & \gamma^* = 0, \\
  d\left(\frac{w}{r}\right), & \gamma^* = 1 - \left(\frac{w}{r}\right)(d\left(\frac{w}{r}\right)),
\end{cases}
\]

\[
q^* = \begin{cases} 
  \frac{\beta}{w}, & \text{if } d\left(\frac{w}{r}\right) \leq \frac{\beta}{w}, \\
  \frac{\beta}{w}, & \text{if } d\left(\frac{w}{r}\right) \leq \frac{\beta}{w} \leq d\left(\frac{w}{r}\right), \\
  d\left(\frac{w}{r}\right), & \text{if } d\left(\frac{w}{r}\right) \leq \frac{\beta}{w}.
\end{cases}
\]

Proposition 4 indicates that the buyer provides a subsidy only when demand is relatively low, i.e., when \(d\left(\frac{w}{r}\right) \leq \beta/w\). Put differently, if demand were to be stochastically larger and all other parameters were to remain invariant, then \(d\left(\frac{w}{r}\right)\) would be larger and the buyer would be less likely to offer subsidy. This finding is consistent with Proposition 1 for deterministic demand.

In the second case, the buyer’s contract offer is such that \(wq/(1-\gamma)\beta < 1\), and the supplier sets the reliability level to be \(z^* = wq/(1-\gamma)\beta\) (see Lemma 1). The buyer’s problem in that case is:

\[
\max_{q,\gamma} \pi_B = \frac{wq}{(1-\gamma)\beta} \left[ rE_D[\min(q, D)] - wq \right] - wq \left[ \frac{wq}{(1-\gamma)\beta} \right]^2 \\
s.t. 0 \leq q \leq \frac{(1-\gamma)\beta}{w}, \quad 0 \leq \gamma \leq 1.
\]

(13)

It is convenient to rewrite the buyer’s problem in terms of optimization over parameters \(q\) and \(z\), rather than \(q\) and \(\gamma\). We first obtain the optimal \(q\) for a given \(z\): \(q^*(z) = \min\{d\left(\frac{w}{r}\right), \beta z/w\}\). Let \(z_0\) be the solution to the following first order condition:

\[
rE[D \mid D \leq \frac{\beta z_0}{w}] + \frac{2r\beta z_0}{w} \bar{F}(\frac{\beta z_0}{w}) - 2\beta z_0 = 0.
\]

(14)

When the demand distribution satisfies the IGFR (increasing generalized failure rate) property (see Lariviere 2006), there exists a unique positive value of \(z_0\) satisfying (14). Many common distributions such as uniform, normal, and exponential satisfy the IGFR property. The following result characterizes the buyer’s optimal contract:

**Proposition 5** Under stochastic demand, when the contract terms \((q, \gamma)\) satisfy \(wq/(1-\gamma)\beta < 1\) (and hence the supplier optimally sets \(z^* = wq/(1-\gamma)\beta\)), the buyer’s optimal order quantity \(q^*\)
and induced reliability level $z^*$ are given as follows:

$$
\begin{cases}
  z^* = \min(z_0, 1), q^* = \frac{\beta z^*}{w}, & \text{if } \frac{\beta}{w} \leq d_{\left[\frac{w}{r}\right]}, \\
  z^* = z_0, q^* = \frac{\beta z^*}{w}, & \text{if } \frac{\beta}{w} E[D | D \leq d_{\left[\frac{w}{r}\right]}] \leq d_{\left[\frac{w}{r}\right]} \leq \frac{\beta}{w}, \\
  z^* = \frac{\beta}{w} E[D | D \leq d_{\left[\frac{w}{r}\right]}], q^* = d_{\left[\frac{w}{r}\right]}, & \text{if } d_{\left[\frac{w}{r}\right]} \leq \frac{\beta}{w} E[D | D \leq d_{\left[\frac{w}{r}\right]}] \leq \frac{\beta}{w}, \\
  z^* = \frac{\beta}{w} d_{\left[\frac{w}{r}\right]}, q^* = d_{\left[\frac{w}{r}\right]}, & \text{if } d_{\left[\frac{w}{r}\right]} \leq \frac{\beta}{w} \leq \frac{\beta}{w} E[D | D \leq d_{\left[\frac{w}{r}\right]}].
\end{cases}
$$

The optimal subsidy level is $\gamma^* = 1 - \frac{wq^*}{\beta z^*}$.

Note that in the first two cases in Proposition 5, the optimal subsidy level is zero. Because $d_{\left[\frac{w}{r}\right]}$ increases in stochastically increasing demand, it is straightforward to see that the first case becomes more likely and the last case becomes less likely as demand increases. However, the effect of stochastically larger demand on the two cases in the middle is mixed because both $E[D | D \leq d_{\left[\frac{w}{r}\right]}]$ and $d_{\left[\frac{w}{r}\right]}$ increase in stochastically increasing demand and their relative rates depend on the manner in which the demand distribution changes. Therefore, while it is reasonable to expect the buyer to provide (respectively, not provide) subsidy only when demand is very small (respectively, large), the precise impact of stochastically larger demand is not entirely predictable without specifying the manner in which demand distribution changes.

To determine the optimal subsidy contract, the buyer needs to compare its profits under the two situations analyzed in Propositions 4 and 5. We find some differences in the properties of the optimal contract as compared to the deterministic demand case. For instance, in the deterministic demand case, order quantity and subsidy are substitutes in the sense that the buyer never increases the use of both at the same time. This may not be the case when demand is stochastic. An example is given in Figure 2 which shows the optimal order quantity $q^*$ and the optimal subsidy level $\gamma^*$ as functions of $r$ ($q^*$ is shown by a solid line, and $\gamma^*$ by a dotted line). In this example, it is noteworthy that when $r$ changes from 23 to 28, the buyer increases order quantity and subsidy level at the same time.

5. Sole Sourcing or Dual Sourcing?

We have studied the use of subsidy to affect a single supplier’s technology choice in the presence of supply disruption. Another commonly seen strategy used to hedge against the underlying delivery
risk is dual sourcing. The motivation for using dual sourcing is to avoid/reduce underage costs if a single supplier is unable to deliver. Moreover, if demand for the product is high, dependence on a single supplier becomes more risky and supplier diversification would be the optimal strategy (Anupindi and Akella 1993).

In the sole sourcing model, the buyer develops a cooperative approach with its supplier by subsidizing the supplier’s cost of reliability improving effort. In contrast, dual sourcing is the conventional diversification approach where the buyer splits orders to hedge against the disruption risk but does not provide any extra incentive to the suppliers. This, however, has implications when supply reliability is endogenous, because as shown in §3, suppliers’ technology choice is affected by the order size, and so the use of dual sourcing may become less attractive to the buyer because smaller orders placed with individual suppliers may diminish their incentive to choose a more reliable process. In this section, we first derive the buyer’s optimal dual sourcing strategy for the (1, 0) yield model. Then, we discuss the buyer’s optimal sourcing strategy. We start with the deterministic demand case first.

5.1 Dual sourcing with deterministic demand

In the dual sourcing model, the buyer has the option of placing two orders, one with each supplier having the yield process described in (1) with $\alpha = 0$. We continue to assume the unit production cost to be zero. Upon receiving the order, each supplier decides on its own reliability level $z$. Throughout this section, we assume that the suppliers are symmetric, in the sense that they have identical production and investment costs; and they face the same wholesale price. However, up
front we do not exclude the possibility that the suppliers may end up choosing different investment levels, because the buyer may order different quantities from each supplier. Also, the buyer may choose to inflate the order to provide incentive to the supplier to improve reliability.

By design, each supplier in the dual sourcing setting receives a quantity-only contract, hence the optimal technology choice for a given order quantity \( q \) is given by Lemma 1 with \( \gamma = 0 \), i.e., \( z^*(q) = \min\{wq/\beta, 1\} \). Since our purpose is to compare the use of dual sourcing to the subsidy approach, when analyzing the dual sourcing model, we restrict ourselves to the case where \( z^*_i(q_i) < 1 \) (\( i = 1, 2 \)), because when \( z^*_i(q_i) = 1 \), it is obvious that the buyer should only order from supplier \( i \) because of its perfect yield process. So the dual sourcing model reduces to a sole sourcing model with quantity-only contract, which is weakly dominated by using a subsidy contract. In other words, dual sourcing cannot do better than the subsidy approach in this case. From Proposition 2, we know that when \( d \geq \beta/w \) a single supplier will optimally set \( z^* = 1 \). Therefore, in analyzing the dual sourcing model, we will assume that \( d < \beta/w \). Note that since the buyer always equally split the order between the suppliers when dual sourcing (as shown later in this section), the two sourcing strategies are equivalent for the buyer when \( d \geq 2\beta/w \).

Based on the above discussion, we can write the buyer’s problem as follows:

\[
\max_{q_1, q_2 < \beta/w} \frac{w^2 q_1 q_2}{\beta^2} \left[ r \min(d, q_1 + q_2) - w(q_1 + q_2) \right] + \sum_{i \neq j} \frac{w q_i}{\beta} \left( 1 - \frac{w q_j}{\beta} \right) \left[ r \min(d, q_i) - w q_i \right], \ (i, j = 1, 2).
\]

Notice that since the reliability levels at both suppliers are strictly less than 1, there is always a yield loss. Therefore, the optimal dual sourcing solution must have \( q_1 + q_2 \geq d \), i.e., \( \min(d, q_1 + q_2) = d \). In solving the buyer’s problem, we can separate it into three sub-problems: (1) where \( q_1 \leq d \) and \( q_2 \leq d \); (2) \( q_1 \geq d \) and \( q_2 \leq d \) (or vice versa by symmetry); and (3) \( q_1 \geq d \) and \( q_2 \geq d \).

For the purpose of comparing the use of dual sourcing to the subsidy approach, we find that it suffices to focus on symmetric solutions in the dual sourcing model for the following reasons (formal proof of this argument appears as part of the proof for Proposition 6): First, in case (1) above, if we express any asymmetric solution as: \( q_1 = Q/2 - \epsilon \), and \( q_2 = Q/2 + \epsilon \), where \( Q \) is the total order quantity; then it can be shown that the buyer’s profit function is increasing in \( \epsilon \). So the optimal asymmetric solution must be sole sourcing from a single supplier. By the argument above, we can ignore asymmetric solutions in our analysis. Second, in case (3), when \( d > 2\beta/r \) the same argument as case (1) applies and sole sourcing is the only asymmetric solution. When \( d \leq 2\beta/r \), we can show that the buyer’s profit function is decreasing in \( \epsilon \), the optimal dual sourcing
solution must be symmetric. Finally, in case (2), we can show that the optimal solution is either single sourcing from supplier 2, or ordering \(d\) from supplier 1 or 2. In the latter case, the solution is weakly dominated by either case (1) or (3).

We summarize the dual sourcing solution which is of our interest as follows:

**Proposition 6** If a buyer orders a positive quantity \(q_i\) (\(i = 1, 2\)) from supplier \(i\), then \(q_1^* = q_2^* = q^*\) and \(q^*\) is given by:

\[
q^* = \begin{cases} 
\frac{\beta rd}{(2\beta + rd)w_r}(> d), & \text{if } d \leq \frac{\beta(r-2w)}{wr}, \\
\frac{\beta(r-2w)}{wr} \leq d \leq \frac{\beta(r-w)}{wr}, & \text{if } \frac{2\beta(r-w)+wrd}{3wr}(< d),
\frac{d}{2}, & \text{if } d \geq \frac{4\beta(r-w)}{wr},
\end{cases}
\]

The above result indicates that if the buyer uses two suppliers and both suppliers have endogenous reliability, then the order quantity is largely driven by the demand. When demand is below a certain threshold, i.e., \(d \leq \frac{\beta(r-2w)}{wr}\) (notice that this is only relevant when \(r > 2w\)), the buyer orders more than the demand from each supplier. In this way, the buyer can satisfy the demand with a high probability. The only exception would be when both suppliers suffer from disruption at the same time, which has a much lower probability of occurrence. Furthermore, the extra units ordered from each supplier, although not needed when disruption does not happen at either supplier, induce the suppliers to choose a more reliable technology so that the probability of disruption is even smaller. We note that such order inflation never occurs in an exogenous reliability dual sourcing model because with a deterministic demand \(d\), a buyer would never order more than \(d\) from each supplier if the supplier cannot affect the reliability of their production process.

As demand increases further, the natural incentive provided by ordering the desired quantity \(d\) from each supplier becomes larger; in addition, the cost of paying for unwanted products increases. So order inflation is no longer used. When demand exceeds \(\frac{\beta(r-w)}{wr}\) the suppliers receive even larger orders and the buyer benefits from a lower disruption risk (due to the investment made by the suppliers in improving reliability). Therefore, the probability of getting \(2q^*\) delivered is higher and the buyer orders less than \(d\) from each supplier. Eventually, each supplier’s order size reduces to \(d/2\), which is the minimum that the supplier needs to order to meet its demand \(d\).
5.2 Comparison of dual and sole sourcing strategies

In this section, we study the buyer’s preference between sole and dual sourcing strategy. Since it is likely that when suppliers compete, the wholesale price will be lower than that under single sourcing, we assume in the sequel that the wholesale prices are different under different sourcing strategies. In particular, it is interesting to ascertain how much of a discount in the wholesale price, if any, is needed to make dual sourcing strategy as good or better than sole sourcing. Of course, dual sourcing strategy may dominate sole sourcing at the same wholesale price level. In Figure 3 (a), we show the buyer’s optimal sourcing strategy when it pays the same wholesale price regardless of the sourcing strategy, and we can see that when $d$ and $w$ are in the unshaded region dual sourcing does better than sole sourcing. In such cases, dual sourcing will surely be adopted if competition between suppliers further drives down the wholesale price.

We notice that in Figure 3(a) the effect of wholesale price is not monotone. To understand this result, recall that when supply reliability is endogenous, dual sourcing has a two-fold effect. On the up side, it diversifies the risk of disruption leading to zero supplies. On the downside, it lowers the incentive given to individual suppliers, and hence leads to less reliable supply processes. When wholesale price is high, the ability to deliver each item is more profitable for the supplier. This reduces the need for the buyer to provide incentives. That is, a buyer upon sole sourcing can expect the supplier to choose the desired reliability level with relatively small incentive. Alternatively, if the buyer were to use dual sourcing, it will order a smaller amount from each supplier ($q^*$ decreases in $w$, see Proposition 6), which further lowers suppliers’ incentive to invest in reliability improvement. This explains why as wholesale price increases, dual sourcing becomes less attractive compared to sole sourcing. However, when wholesale price is lower than some threshold, larger incentive must be given to the supplier(s) to induce reliability improvement. This translates into a larger order and greater cost of order inflation. Both Proposition 2 and 6 suggest that when subsidy is not used, the buyer’s order quantity from each supplier increases to a level above $d$ if $w$ is low. In this case, the use of subsidy is preferred because it avoids the problem of over-ordering. Therefore, compared to sole sourcing with the option to provide subsidy, dual sourcing is only optimal when $w$ is neither too high nor too low. Not surprisingly, dual sourcing becomes more attractive if a quantity-only contract is used under sole sourcing.

One may expect that when dual sourcing strategy is dominated at the same wholesale price
level, it may become the preferred strategy if the wholesale price were to drop below certain level. Surprisingly, this is not always the case. It can be shown that if dual sourcing is used, the buyer’s expected profit is actually increasing in $w$ when $w < r/2$ and $d > \beta(r + 2w)/wr$. This might be because suppliers’ reliability level is increasing in $w$, and this benefit may outweigh the decrease in the margin $(r - w)$.

When the buyer’s expected profit with dual sourcing is decreasing in $w$, the dual sourcing strategy may indeed dominate the sole sourcing strategy provided that the competition between suppliers drives down the wholesale price below a threshold. This conjecture is confirmed by Figure 3(b). In this figure, we fix $w$ in the sole sourcing model, and calculate the amount of discount in $w$ needed for dual sourcing to be better than sole sourcing. The horizontal axis is demand, and the vertical axis is the wholesale price below which dual sourcing dominates (referred to as $w_d$). For low demand, dual sourcing is already dominant (as shown in Figure 3(a)), so $w_d = w$ as shown in the left region. For high demand, the supplier(s) always uses a perfect yield process and the two sourcing strategies become equivalent as shown in the right region. When demand is in between, $w_d$ is first decreasing in demand, and then increasing as $d > \beta/w$. The latter result is because when $d > \beta/w$, sole sourcing leads to perfect yield, a constant independent of $d$. With dual sourcing, at a higher level of $d$, buyer provides larger incentive to suppliers through bigger orders. So less discount in $w$ is needed for dual sourcing to dominate. Finally, $w_d$ may be decreasing in $d$ because using dual sourcing the buyer splits the order in half, so each supplier gets less incentive than a sole-sourcing supplier through buyer’s order. The diversification benefit of dual sourcing cannot
offset the above effect. Therefore, as demand increases, a deeper discount in \( w \) is needed for dual sourcing to dominate.

### 5.3 Dual sourcing strategy under stochastic demand

Next, we discuss the dual sourcing strategy under stochastic demand. The buyer’s problem in this case is:

\[
\max_{q_1, q_2} \frac{w^2 q_1 q_2}{\beta^2} \left[ r E[D|\min(D, q_1 + q_2)] - w(q_1 + q_2) \right] + \sum_{i,j=1,2,i\neq j} \frac{w q_i}{\beta} \left( 1 - \frac{w q_j}{\beta} \right) \left[ r E[D|\min(D, q_i)] - w q_i \right].
\]

Unlike the result in Proposition 6 that concerned deterministic demand, we numerically observe that an optimal dual sourcing strategy may split orders unequally under stochastic demand. That is, it may be optimal for the buyer to order a different quantity from each supplier even though the two suppliers have identical costs and yield distributions. One example is given in Figure 4 which shows the optimal order quantities when demand is normally distributed on \([0, 5]\). We observe that asymmetric solutions tend to occur when \( r \) is high relative to \( w \). From the newsvendor model, we know that as \( r \) increases the buyer’s order target (i.e., order quantity under perfect delivery) also increases. If the buyer orders the same amount from the two suppliers, it has to increase its order from each at the same time. This becomes more costly as the order target increases. The buyer may find it more attractive to allocate a larger portion of the order to one supplier, but order a small amount from the other supplier to take advantage of risk diversification. In this way, the supplier who gets the larger order has greater incentive to choose a more reliable process, and it can deliver a quantity close to the buyer’s order target with a higher probability. The second smaller order serves as a hedge — it offers the possibility (albeit small) of having enough supply on hand to meet a high demand realization scenario (which also occurs with a small probability). This has the potential to increase buyer’s revenue without causing its costs to go up a great deal.

When comparing the dual sourcing strategy and the subsidy contract under sole sourcing, we find that some but not all effects observed earlier for deterministic demand case can be generalized to the stochastic demand case. In particular, consistent with the deterministic demand case, Figure 5(a) shows that dual sourcing can dominate subsidy contract at the same wholesale price level, and (b) shows how much of a discount in the wholesale price, if any, is needed for dual sourcing to become dominant. However, the effect of \( w \) is a little different. When demand is deterministic,
dual sourcing is preferred to the subsidy contract only when \( w \) is neither too high nor too low. When demand is stochastic, we observe that dual sourcing is also preferred when \( w \) is relatively high. One possible reason why dual sourcing is preferred over a greater range of parameters when demand is stochastic is that dual sourcing limits supply risk differently – by offering several possible realizations of quantity delivered – as compared to single sourcing in which risk mitigation occurs by reducing the chance of receiving no delivery.

Figure 5: Comparison of buyer’s optimal sourcing strategies when demand is stochastic \( (r = 10, \beta = 20, D \sim \text{Unif}[0, d_{\text{max}}]) \)
6. Conclusion

As supply disruption becomes more common, caused in part by longer and more complex global supply chains, many mitigation strategies have been proposed and observed in practice. Among them, the most popular ones are supplier diversification and supply process improvement. Previous works on this topic have placed emphasis on how purchasing firms should effectively employ these strategies and when one strategy mitigates supply disruption better than the other. These studies typically assume that either the reliability of the supply process is exogenous (i.e., it cannot be affected by buyers and suppliers) or that buyers can dictate supplier’s process improvement effort. In this paper, we revisited these common strategies but in a decentralized setting where the supplier determines the supply reliability through its choice of production process.

Our base-case model assumed a two-echelon supply chain with a buyer facing deterministic demand and the supplier having all-or-nothing yield. When modeling the process improvement strategy, we considered two different ways a buyer could influence its supplier’s process selection: (1) order quantity, which provided an indirect incentive to the supplier, and (2) subsidy, which allowed the buyer to share the supplier’s direct investment in process improvement. We studied firms’ preferences between the two incentive mechanisms and the effectiveness of each in improving supplier reliability. When modeling the supplier diversification strategy, we considered a dual sourcing model where the buyer does not provide direct incentive to its suppliers, but the supply reliability is still affected by the buyer through its order size. The endogenous supply reliability differentiates our dual sourcing model from those commonly seen in the literature. We derived the buyer’s optimal order quantity under the dual sourcing strategy and compared its choice between the single and dual sourcing strategies. The comparisons assumed that the wholesale price would be different (lower) under dual sourcing because of supplier competition. We also generalized the base model to a partial disruption setting and a stochastic demand setting.

In the base-case model, we first showed that although the direct and indirect incentives were substitutable from the supplier’s perspective, the buyer always preferred the use of direct subsidy to inflating its order size above \( d \). Furthermore, we found that the direct incentive indeed resulted in a more reliable supply process. Although dual sourcing typically benefits the buyer when reliability is exogenous, by splitting its orders in an endogenous reliability environment, the buyer reduces indirect incentive offered to each supplier to invest in reliability improvement. Therefore, it is not
straightforward to know which option will be preferred by the buyer on an intuitive level. We showed that despite the benefit of larger order in single sourcing mode, dual sourcing might lead to higher expected profit for the buyer under the same wholesale price. This happened in our examples because the benefit of risk diversification together with the savings from lower overage cost outweighed the loss from lower supplier reliability. Conversely, we found examples in which dual sourcing was attractive only when wholesale price was lower under dual sourcing. We also found examples in which a lower wholesale price was not attractive to the buyer because of lower supplier reliability. Thus, in some examples, dual sourcing never became the preferred strategy even upon lowering the wholesale price.

When the base model was extended to the general partial disruption model, more interesting results were revealed. Since the minimum yield was positive, the supplier had one more degree of freedom to increase delivery, i.e., it could inflate production quantity in addition to investing in its production process. Because of this, the buyer might order less than the known demand, although this would surely lead to shortage. In this way, the buyer could induce the supplier to inflate production rather than improve reliability, where the latter might require the buyer to provide some incentives. Furthermore, we observed that the buyer might prefer the indirect to direct incentive, inflating its order above \( d \) rather than offering a subsidy, which was again in contrast to the base model.

For the case of stochastic demand, some of our findings in the base model continue to hold, but some others do not. For example, the comparison between single and dual sourcing strategy is more or less consistent with the base model. However, we showed that use of both direct subsidy and order quantity could increase at the same time in contrast to a consistent preference for the former in the base model. Another interesting result is that when sourcing from two identical suppliers (with all-or-nothing yield and same cost structure), the buyer may order a different quantity from each supplier. We conjectured that the asymmetry is partly caused by the fact that there are more possible delivery outcomes which may lead to less supply-demand mismatch when demand itself is stochastic. Also, when the buyer’s order target is high, it makes sense to give a lion’s share of the order to one supplier who would in turn choose a more reliable process. In this way, the buyer increases its chance of receiving enough delivery, meanwhile still benefiting from supply diversification.
This paper contributes to the supply risk management literature by modeling some common supply risk mitigation strategies in a decentralized supply chain setting. More importantly, our research obtains managerially relevant insights for buyer firms. Our models confirm the intuition that direct investment by buyers in suppliers’ process improvement efforts are superior to indirect incentives, but at the same time point out that this is not uniformly true, especially when the minimum yield is strictly positive rather than zero. This research also provides guidance to supply chain managers for choosing the appropriate risk mitigation strategy. There are a number of ways in which our modeling efforts could be generalized to consider settings different from those we model. For example, future efforts could consider different investment cost functions and competition among multiple buyers and/or suppliers. A potential choice for the former is a function of the type $1/(1 - z)$, which would prevent the supplier from choosing a perfectly reliable process. Yet another possibility would be to consider a long-term contract in which investments in reliability improvement affect multiple purchase periods and the supplier’s yield improves over time on account of learning even without investment.

References


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A.1. Proofs for the main results in the paper

Proof of Lemma 1  The supplier’s expected profit function given in (4) is concave in $z$. The optimal solution to the unconstrained problem is $wq/(1 - \gamma)\beta$. Since $z$ is defined on $[0, 1]$, the optimal solution is $z^* = \min\{wq/(1 - \gamma)\beta, 1\}$. ■

Proof of Proposition 1  The buyer can solve problem (5) by solving the following problem: max $\{P1, P2\}$, where the two sub-problems correspond to different supplier’s best response given in Lemma 1. Problem P1 corresponds to the best response $x^* = q, z^* = 1$, i.e.,

$$P1: \max_{q \geq 0, 0 \leq \gamma \leq 1} \left[r \min\{q, d\} - wq - \frac{\gamma \beta}{2}, \ s.t. \ q \geq \frac{(1 - \gamma)\beta}{w}\right].$$

Solve P1 in two steps: first solve the optimal $q$ for fixed $\gamma$, we have $q^*(\gamma) = \max\{d, (1 - \gamma)\beta/w\}$. Then reduce P1 to a one-dimensional problem in $\gamma$ and solve for the optimal $\gamma$, we have $\gamma^* = \max\{0, 1 - wd/\beta\}$. Notice that $q^*(\gamma^*) = d$, and the buyer’s expected profit resulted from P1 is $(r - w)d - \max\{0, 1 - wd/\beta\}\beta/2$.

Problem P2 corresponds to the best response $x^* = q, z^* = wq/(1 - \gamma)\beta$. Since there is one-to-one relationship between $z^*$ and $\gamma$, we can write P2 as a problem optimizing on $q$ and $z$ instead, i.e.,

$$P2: \max_{q \geq 0, 0 \leq z \leq 1} z \left[r \min\{q, d\} - wq\right] - \frac{\beta z^2}{2} \left(1 - \frac{wq}{\beta z}\right), \ s.t. \ z \geq \frac{wq}{\beta}.$$

The constraint is due to the fact that $\gamma \in [0, 1]$ in $z^*$.

Again, we solve P2 in two steps: First, notice that for any fixed $z$, the objective function in P2 is decreasing in $q$ when $q \geq d$ and is increasing in $q$ when $q \leq d$. So the optimal $q$ for fixed $z$ is $q^*(z) = \min\{d, \beta z/w\}$. Then we solve for the optimal $z$. For this, the buyer solves two sub-problems, P21 and P22, corresponding to the two different $q$ responses derived above. P21 corresponds to the case of $q^*(z) = d$, i.e.,

$$P21: \max_z z(r - w)d - \frac{\beta}{2} z^2 + \frac{wdz}{2}, \ s.t. \ 0 \leq z \leq 1, \ z \geq \frac{wd}{\beta} \quad (1.1)$$

It can be shown that the objective function is concave in $z$ and the interior solution is $(2r - w)d/2\beta$. The optimal choice of $z$ can be characterized as follows, (the resulted optimal objective function in
P21 is attached)

\[ z^* = \begin{cases} \frac{(2r-w)d}{2 \beta}, & \text{if } d \leq \frac{2 \beta}{2r-w} \text{ and } 2r > 3w, \\ \frac{wd}{\beta}, & \text{if } d < \frac{2 \beta}{2r-w} \text{ and } 3w > 2r, \\ 1, & \text{if } d > \frac{2 \beta}{2r-w}. \end{cases} \]

\( \pi_B = \frac{(2r-w)^2 d^2}{8 \beta} \)

P22 corresponds to the case of \( q(z) = \beta z/w \):

P22: \[ \max z (r-w) \frac{\beta z}{w} \]

s.t. \( 0 \leq z \leq 1, \ z \leq \frac{wd}{\beta} \)

The objective function is convex and increasing in \( z \). So the optimal choice of \( z \) is \( wd/\beta \) when \( d \leq \beta/w \) and is 1 otherwise. The resulted optimal objective function in P22 is \((r-w)wd^2/\beta \) when \( d \leq \beta/w \) and \((r-w)\beta/w \) otherwise.

By comparing the optimal objective function of P21 and P22, we get the optimal \( z \) for problem P2. When \( 3w \leq 2r \),

\[ z^* = \begin{cases} \frac{(2r-w)d}{2 \beta}, & \text{if } d \leq \frac{2 \beta}{2r-w}, \\ 1, & \text{if } d > \frac{2 \beta}{2r-w} \end{cases} \]

When \( 3w \geq 2r \), \( z^* = \min\{1, wd/\beta\} \).

Finally, by comparing to the solution of P1, we obtain the optimal solution. If the optimal solution emerges from P2, the optimal subsidy level is recovered by the equation \( z^* = wq^*/(1 - \gamma)^* \beta \).

\[ \blacksquare \]

Proof of Proposition 2 The solution procedure is similar to the proof of Proposition 1. The buyer can solve problem (7) by solving the following problem: \( \max \{P3, P4\} \), where P3 corresponds to the best response \( x^* = q, z^* = 1 \), and P4 for \( x^* = q, z^* = wq/\beta \). For P3:

P3: \[ \max_{q \geq 0} r \min\{q, d\} - wq, \ s.t. \ q \geq \frac{\beta}{w}. \]

The solution can be easily derived to be: \( q^* = \min\{d, \beta/w\} \). The resulted buyer’s profit is \( rd - \beta \) if \( d \leq \beta/w \) and \((r-w)d \) otherwise.

For P4:

P4: \[ \max_{q \geq 0} \frac{wq}{\beta} \left[ r \min\{q, d\} - wq \right], \ s.t. \ q \leq \frac{wq}{\beta}. \]
When \( d \geq \beta/w \), the constraint implies \( q < d \): it can be shown that the objective function is convex and increasing in \( q \). So the optimal solution is \( q^* = \beta/w \). When \( d < \beta/w \): in the domain \( q \leq d \), the objective function is convex and increasing in \( q \). So the optimal solution must satisfy \( q \geq d \). In this region, the objective function is concave in \( q \), so the optimal solution is:

\[
q^* = \begin{cases} 
\frac{rd}{2w}, & \text{if } d \leq \frac{rd}{2w} \leq \frac{\beta}{w} \text{ i.e., } r \geq 2w \& d \leq \frac{2\beta}{r}, \\
d, & \text{if } \frac{rd}{2w} < d \text{ i.e., } r < 2w, \\
\frac{\beta}{w}, & \text{if } \frac{rd}{2w} > \frac{\beta}{w} \text{ i.e., } d > \frac{2\beta}{r}.
\end{cases}
\]  

\[(1.2)\]

Next, we compare the optimal expected profit in P3 and P4. It is immediate that when \( d \geq \beta/w \), the optimal \( q \) is \( d \) since the resulting profit is the maximum possible profit the buyer can earn. When \( d \leq \beta/w \) and \( w > r/2 \), the optimal \( q \) is \( d \) due to (1.2). Part 2 follows.

When \( d \leq \beta/w \) and \( w \leq r/2 \), the optimal \( q \) is \( \beta/w \) when \( d \geq 2\beta/r \), and \( rd/2w \) when \( d \leq 2\beta/r \), due to (1.2).

**Proof of Proposition 3** We first solve the optimal production quantity for any given \( z \). The objective function \( \pi_S(x, z, q, \alpha) \) in (2) is increasing in \( x \) when \( x \leq q \) and decreasing in \( x \) when \( x \geq \frac{2}{\alpha} \).

When \((1 - z)w\alpha \leq c\), the objective function is decreasing in \( x \) when \( x \in [q, \frac{2}{\alpha}] \), hence \( x^* = q \). When \((1 - z)w\alpha \geq c\), the objective function is increasing in \( x \) when \( x \in [\frac{2}{\alpha}, q] \), hence \( x^* = \frac{2}{\alpha} \).

In solving for the optimal \( z \), the supplier first solves two subproblems, P5 and P6, where P5 corresponds to \( x^* = \frac{2}{\alpha} \) and P6 for \( x^* = q \). In P5, the objective function is decreasing in \( z \), hence \( z^* = 0 \) and the resulting supplier profit is \( \pi^0_S = wq - c \frac{2}{\alpha} \). In P6, \( x^* = q \), the supplier’s problem becomes:

\[
\max_z \quad zwq + (1 - z)w\alpha q - cq - \frac{\beta z^2}{2}, \quad \text{s.t. } 1 - \frac{c}{w\alpha} \leq z \leq 1
\]

The objective function is concave in \( z \) and the interior solution is denoted as \( q_0 = \frac{wq(1-\alpha)}{\beta} \). The optimal choice of \( z \) is:

\[
\begin{align*}
\pi^1_S &= (w\alpha - c)q + \frac{w^2(1-\alpha)^2q^2}{2\beta}, & \text{if } 1 - \frac{c}{w\alpha} \leq q_0 \leq 1, \\
\pi^2_S &= (w - c)q - \frac{2}{\beta}, & \text{if } q_0 \geq 1, \\
\pi^3_S &= 1 - \frac{c}{w\alpha}, & \text{if } q_0 \leq 1 - \frac{c}{w\alpha}.
\end{align*}
\]

(1.3)

Notice that \( \pi^1_S > \pi^0_S \) if and only if \( q \geq \frac{2\beta(1-c/w\alpha)}{\alpha(1-\alpha)} \), and \( \pi^2_S > \pi^0_S \) if and only if \( q \geq \frac{\beta\alpha}{2\alpha(1-\alpha)} \).
When \( \alpha w \geq 2c \), we can show that \( \frac{\beta}{w(1-\alpha)} < \frac{\beta\alpha}{2c(1-\alpha)} \). So when \( q \leq \frac{\beta\alpha}{2c(1-\alpha)} \), the optimal solution emerges from P5, i.e. \( x^* = \frac{q}{\alpha} \) and \( z^* = 0 \). Otherwise, the optimal solution emerges from P6, and \( x^* = q \) and \( z^* = 1 \). Part 1 follows.

When \( \alpha w \leq 2c \), we can show that
\[
(1 - \frac{c}{w^2}) \frac{2\beta}{w(1-\alpha)} < \frac{\beta}{w(1-\alpha)}, \quad \text{and} \quad \frac{\beta\alpha}{2c(1-\alpha)} < \frac{\beta}{w(1-\alpha)}.
\]
So when \( q \leq \frac{2\beta(1-c/w^2)}{w(1-\alpha)} \), the optimal solution emerges from P5, i.e. \( x^* = \frac{q}{\alpha} \) and \( z^* = 0 \). Otherwise, the optimal solution emerges from P6. Part 2 follows from (1.3).

**Proof of Proposition 4**  When the contract terms \((q, \gamma)\) satisfy \( \frac{wq}{(1-\gamma)^2} \geq 1 \), the buyer’s problem is given in (12). We solve the buyer’s problem in two steps: First, we find the optimal \( q \) for fixed \( \gamma \). It can be shown that the objective function is concave in \( q \), so \( q^*(\gamma) = \max\{F^{-1}(w/r) \cdot (1-\gamma)^2 \cdot \frac{1}{w^2} \} \).

Then we optimize on \( \gamma \).

When \( \frac{(1-\gamma)^2}{w^2} \leq \tilde{F}^{-1}(\frac{w}{r}) \), \( q^* = \tilde{F}^{-1}(\frac{w}{r}) \) and it can be shown that the objective function is decreasing in \( \gamma \). So the optimal \( \gamma \) is achieved at the left extreme point \( \max\{0, 1 - \frac{w}{r} \tilde{F}^{-1}(\frac{w}{r})\} \).

When \( \frac{(1-\gamma)^2}{w^2} \geq \tilde{F}^{-1}(\frac{w}{r}) \) (only relevant when \( \tilde{F}^{-1}(\frac{w}{r}) < \frac{q}{\alpha} \)), \( q^* = \frac{(1-\gamma)^2}{w^2} \) and it can be shown that the objective function is concave in \( \gamma \) and the interior solution is given as \( 1 - \frac{w}{r} \tilde{F}^{-1}(\frac{w}{r}) \). Notice that \( \gamma \) is constrained as \( 0 \leq \gamma \leq 1 - \frac{w}{r} \tilde{F}^{-1}(\frac{w}{r}) \), and it can be shown that \( 1 - \frac{w}{r} \tilde{F}^{-1}(\frac{w}{r}) < 1 - \frac{w}{r} \tilde{F}^{-1}(\frac{w}{r}) \).

So the optimal \( \gamma \) in this case is given by \( \max\{0, 1 - \frac{w}{r} \tilde{F}^{-1}(\frac{w}{r})\} \).

To summarize, when \( 1 - \frac{w}{r} \tilde{F}^{-1}(\frac{w}{r}) < 0 \), i.e., \( \tilde{F}(\frac{q}{\alpha}) \geq \frac{w}{r} \), the optimal solution is given by the former case and \( \gamma^* = 0 \), \( q^* = \tilde{F}^{-1}(\frac{w}{r}) \). When \( 1 - \frac{w}{r} \tilde{F}^{-1}(\frac{w}{r}) < 0 \) and \( 1 - \frac{w}{r} \tilde{F}^{-1}(\frac{w}{r}) > 0 \), i.e., \( \frac{w}{r} \leq \tilde{F}(\frac{q}{\alpha}) \leq \frac{w}{r} \), the optimal solution is given by the latter case and \( \gamma^* = 0 \), \( q^* = \frac{q}{\alpha} \). Finally, when \( 1 - \frac{w}{r} \tilde{F}^{-1}(\frac{w}{r}) > 0 \), i.e., \( \tilde{F}(\frac{q}{\alpha}) \leq \frac{w}{r} \), the optimal solution is given by the latter case and \( \gamma^* = 1 - \frac{w}{r} \tilde{F}^{-1}(\frac{w}{r}) \), \( q^* = \tilde{F}^{-1}(\frac{w}{r}) \).

**Proof of Proposition 5**  When the contract terms \((q, \gamma)\) satisfy \( \frac{wq}{(1-\gamma)^2} \leq 1 \), the buyer’s problem is given in (13). Rewrite it as one optimizing on \( q \) and \( z \), as follows:

\[
\max_{q,z} \pi_B = z \left[ rE_D \min\{q, D\} - wz \right] - \frac{\beta z^2}{2} (1 - \frac{wq}{\beta z})^2, \quad \text{s.t.} \quad 0 \leq z \leq 1, z \geq \frac{wq}{\beta},
\]

The last constraint is due to the fact that \( \gamma \geq 0 \) in \( z^* \). We solve the above problem in two steps: First, we find the optimal \( q \) for fixed \( \gamma \). It can be shown that the objective function is concave in \( q \), so \( q^*(\gamma) = \min\{\frac{q}{\alpha}, \tilde{F}^{-1}(\frac{w}{r})\} \). Then we optimize on \( z \).

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When \( F^{-1}(\frac{w}{w}) \leq \beta z \) (only relevant when \( F^{-1}(\frac{w}{w}) \leq \beta \)), \( q^* = F^{-1}(\frac{w}{w}) \) and it can be shown that the objective function is concave in \( z \). The interior solution is given as \( \frac{r}{w} \int_0^{F^{-1}(\frac{w}{w})} \xi f(\xi) d\xi \). The optimal \( z \) in this case is given by \( \max\{\frac{w}{w} F^{-1}(\frac{w}{w}), \frac{r}{w} \int_0^{F^{-1}(\frac{w}{w})} \xi f(\xi) d\xi\} \).

When \( F^{-1}(\frac{w}{w}) \geq \beta z \), \( q^* = \beta z \) and it can be shown that
\[
\frac{\partial^2 \pi_B}{\partial z^2} = \frac{r\beta}{w} \left[ 2F(\frac{\beta z}{w}) - \frac{\beta z}{w} f(\frac{\beta z}{w}) \right] - 2\beta,
\]
and \( \frac{\partial^2 \pi_B}{\partial z^2}|_{z=0} = 2\beta(\frac{r}{w} - 1) > 0 \). Assume that \( \pi_B \) satisfies IGFR property (i.e., \( \frac{xf(x)}{F(x)} \) increasing in \( x \)). Then \( \pi_B \) is convex increasing and then concave. So the first order condition (14) characterizes the optimal solution to the unconstrained problem. Combine with the constraints, we get the optimal solution in this case: when \( z_0 \leq \frac{w}{w} F^{-1}(\frac{w}{w}) (z_0 \) defined in (14)), \( z^* = \min\{z_0, 1\} \); otherwise \( z^* = \frac{w}{w} F^{-1}(\frac{w}{w}) \). Notice that \( z_0 \leq \frac{w}{w} F^{-1}(\frac{w}{w}) \) is equivalent to \( \frac{r}{w} \int_0^{F^{-1}(w/2r)} \xi f(\xi) d\xi \leq F^{-1}(\frac{w}{w}) \).

To summarize, when \( \frac{\beta}{w} \leq F^{-1}(\frac{w}{w}) \), the optimal solution is given by the latter case (with constraint always satisfied) and \( z^* = \min\{z_0, 1\} \), \( q^* = \beta z^* \). When \( \frac{\beta}{w} \geq F^{-1}(\frac{w}{w}) \) and \( \frac{r}{w} \int_0^{F^{-1}(w/2r)} \xi f(\xi) d\xi \leq F^{-1}(\frac{w}{w}) \), the optimal solution is given by the latter case \( z^* = z_0, q^* = \beta z^* \). When \( \frac{\beta}{w} \geq F^{-1}(\frac{w}{w}) \) and \( \frac{r}{w} \int_0^{F^{-1}(w/2r)} \xi f(\xi) d\xi \geq F^{-1}(\frac{w}{w}) \), the optimal solution is given by the former case \( z^* = \max\{\frac{w}{w} F^{-1}(\frac{w}{w}), \frac{r}{w} \int_0^{F^{-1}(\frac{w}{w})} \xi f(\xi) d\xi\} \), \( q^* = F^{-1}(\frac{w}{w}) \). Finally, the optimal subsidy level is \( \gamma^* = 1 - wq^*/\beta z^* \).

**Proof of Proposition 6** First, when \( d \geq \beta/w \) or when \( d \leq \beta/w \) and \( wq_i/\beta \geq 1 \) for \( i = 1 \) or 2, dual sourcing model reduces to quantity-only sole sourcing model which is weakly dominated by the sole sourcing subsidy model. So for comparison purpose, we may only focus on the case where \( d \leq \beta/w \) and \( wq_i/\beta < 1 \) for \( i = 1, 2 \), i.e., the following problem:

\[
\max_{q_1, q_2} \frac{w^2 q_1 q_2}{\beta^2} \left[ rd - w(q_1 + q_2) \right] + \sum_{i,j=1,2, i \neq j} \frac{wq_i}{\beta} \left( 1 - \frac{wq_j}{\beta} \right) \left[ r \min(d, q_i) - wq_i \right],
\]
\[
s.t. \quad q_i \leq \frac{\beta}{w}, i = 1, 2.
\]

Notice that in the first term, we use the fact that \( z_i < 1 \) and hence \( q_1 + q_2 \geq d \).

We solve the above problem by solving three sub-problems:

1. **When \( q_1 \leq d \) and \( q_2 \leq d \)**: The objective function become:

   \[
   \pi_B = \frac{w^2 q_1 q_2}{\beta^2} \left[ rd - w(q_1 + q_2) \right] + \sum_{i,j=1,2, i \neq j} \frac{wq_i}{\beta} \left( 1 - \frac{wq_j}{\beta} \right) \left( r - wq_i \right)
   \]
Suppose \( q_1 \neq q_2 \) and \( q_1 + q_2 = Q \) where \( Q \) is some constant. Write \( q_1 = Q/2 - \epsilon \) and \( q_2 = Q/2 + \epsilon \) \((\epsilon > 0)\) if \( q_1 < q_2 \) and the other way around if \( q_1 > q_2 \). We can show that:

\[
\frac{\partial \pi_B}{\partial \epsilon} = \frac{2w\epsilon[2\beta(r - w) + w\beta(Q - d)]}{\beta^2} > 0
\]

So \( \pi_B \) is increasing in \( \epsilon \). This implies that if the solution is asymmetric, one of the order quantities is 0. In this case, the dual sourcing model reduces to a quantity-only sole sourcing model. So for comparison purpose, we may only focus on symmetric solutions. Let \( q_1 = q_2 = q \).

The objective function becomes:

\[
\pi_B = \frac{w^2q^2}{\beta^2}(rd - 2wq) + \frac{2wq}{\beta}(1 - \frac{wq}{\beta})(r - w)q
\]

It can be shown that \( \pi_B \) has a unique maximum achieved at \( q = \frac{2\beta(r-w)+wrd}{3wr} \). Combine the constraint \( q \leq d \) and \( 2q \geq d \), we get the optimal solution:

\[
q^* = \begin{cases} 
\frac{d}{2}, & \text{if } d \geq \frac{4\beta(r-w)}{wr}, \\
\frac{2\beta(r-w)+wrd}{3wr}, & \text{if } \frac{\beta(r-w)}{wr} \leq d \leq \frac{4\beta(r-w)}{wr}, \\
d, & \text{if } d \leq \frac{\beta(r-w)}{wr}.
\end{cases}
\]

2. When \( q_1 \geq d \) and \( q_2 \geq d \): The objective function becomes:

\[
\pi_B = \frac{w^2q_1q_2}{\beta^2}[rd - w(q_1 + q_2)] + \sum_{i,j=1,2,i\neq j} \frac{wj}{\beta}(1 - \frac{wj}{\beta})(rd - wq_i)
\]

Using the same technique as the first case, we can show that \( \frac{\partial \pi_B}{\partial \epsilon} = \frac{2w^2\epsilon(rd-2\beta)}{\beta^2} \). When \( d \geq 2\beta/r \), \( \pi_B \) is increasing in \( \epsilon \). Therefore an asymmetric solution is equivalent to a quantity-only sole sourcing model, and we may only focus on symmetric solutions. When \( d \leq 2\beta/r \), \( \pi_B \) is decreasing in \( \epsilon \), therefore the optimal solution must be symmetric. Let \( q_1 = q_2 = q \), and the objective function becomes:

\[
\pi_B = \frac{w^2q^2}{\beta^2}(rd - 2wq) + \frac{2wq}{\beta}(1 - \frac{wq}{\beta})(rd - wq)
\]

It can be shown that \( \pi_B \) is concave in \( q \) and the unconstrained maximum is achieved at \( \frac{\beta rd}{(2\beta + rd)w} \). Combine the constraint \( q_i \geq d \), we get the optimal solution:

\[
q^* = \begin{cases} 
\frac{\beta rd}{(2\beta + rd)w}, & \text{if } d \leq \frac{\beta(r-2w)}{wr}, \\
d, & \text{if } d \geq \frac{\beta(r-2w)}{wr}.
\end{cases}
\]
3. When \( q_1 \geq d \) and \( q_2 \leq d \) (the analysis of \( q_2 \geq d \) and \( q_1 \leq d \) is similar): The objective function becomes:

\[
\pi_B = \frac{w^2 q_1 q_2}{\beta^2} [rd - w(q_1 + q_2)] + \frac{wq_1}{\beta} \left(1 - \frac{wq_2}{\beta}\right)(rd - wq_1) + \frac{wq_2}{\beta} \left(1 - \frac{wq_1}{\beta}\right)(r - w)q_2
\]

It can be shown that for any fixed \( q_2 \), \( \pi_B \) is concave in \( q_1 \), so the optimal \( q_1 \) as a function of \( q_2 \) is

\[
q_1^*(q_2) = \max\{d, \frac{r(\beta d - wq_2^2)}{2w^2}\}
\]

The above two-dimensional maximization problem can be reduced to a one-dimensional problem w.r.t. \( q_2 \). When \( \frac{r(\beta d - wq_2^2)}{2w^2} \geq d \), we plug \( q_1 = \frac{r(\beta d - wq_2^2)}{2w^2} \) in the objective function. It can be shown that if \( d \geq \frac{2\beta(r - w)}{r^2} \), \( \pi_B(q_2) \) is increasing in \( q_2 \) and the optimal \( q_2 \) is at the right extreme point, i.e., \( \frac{r(\beta d - wq_2^2)}{2w^2} = d \). In this case the optimal solution has \( q_1^* = d \) and \( q_2^* < d \), which is dominated by the optimal solution in case 1. If \( d \geq \frac{2\beta(r - w)}{r^2} \), \( \pi_B(q_2) \) first decreases then increases in \( q_2 \), so the maximum is achieved at either the left extreme point 0 or the right extreme point \( q_{2\text{max}} \), where \( q_{2\text{max}} \) is determined by the constraints \( \frac{r(\beta d - wq_2^2)}{2w^2} \geq d \) and \( d \geq \frac{2\beta(r - w)}{r^2} \), and is given as follows: \( q_{2\text{max}} = d \) if \( d \leq \frac{\beta(r - 2w)}{wr} \), and satisfies \( \frac{r(\beta d - wq_2^2)}{2w^2} = d \) otherwise. In the former case, the optimal solution is \( q_2^* = d \) and \( q_1^* > d \), which is dominated by the optimal solution in case 2. In the latter case, the optimal solution has \( q_1^* = d \) and \( q_2^* < d \), which is dominated by the optimal solution in case 1.

When \( \frac{r(\beta d - wq_2^2)}{2w^2} \leq d \), we plug \( q_1 = d \) in the objective function. It can be shown that when \( d \geq \frac{\beta(r - w)}{wr} \), \( \pi_B(q_2) \) is decreasing in \( q_2 \), so the optimal solution is \( q_2^* = 0 \) and \( q_1^* = d \), which is dominated by the optimal solution in case 1. When \( d \leq \frac{\beta(r - w)}{wr} \), the objective function is increasing in \( q_2 \), so the optimal solution is \( q_1^* = q_2^* = d \), which is weakly dominated by the optimal solution in either case 1 or 2.

In summary, the optimal solution in case 3 cannot be strictly dominant in the dual sourcing model.

Finally, by comparing the optimal solution in case 1 and 2, we obtain the result in Proposition 6.
A.2. Analysis for the optimal subsidy contract in the \((1, \alpha)\) model

In this appendix, we analyze the buyer’s optimal subsidy contract in the \((1, \alpha)\) model when \(c/\alpha \leq w \leq 2c/\alpha\) and \(q_L \leq d \leq q_U\). Throughout this section, we use “null” to denote that contradiction in constraints. For example, in (2.4) \(q^*\) is a null set because \(q \geq d/\alpha\) and \(d\alpha > \beta z/\left[w(1-\alpha)\right]\) implies \(q \geq \beta z/\left[w(1-\alpha)\right]\) which contradicts with the constraint \(z \geq w(1-\alpha)q/\beta\). According to Proposition 3, Part 2, the supplier has three different responses depending on the buyer’s choice of \(q\) and \(\gamma\):

Case 1. When \(q \leq (1-\gamma)q_L\), the supplier’s best response is \(x^* = q/\alpha\) and \(z^* = 0\). So the buyer’s problem is:

\[
\max_{q, \gamma} \ r \min\{q, d\} - wq, \ \text{s.t.} \ q \leq (1-\gamma)q_L
\]

The optimal solution can be found as:

\[
\begin{align*}
\{q^{(1)} = d, \gamma^{(1)} = 0, \pi_B^{(1)} = (r-w)d\}, & \quad \text{if} \ d \leq q_L, \\
\{q^{(1)} = q_L, \gamma^{(1)} = 0, \pi_B^{(1)} = (r-w)q_L\}, & \quad \text{if} \ d \geq q_L.
\end{align*}
\]

(2.1)

Case 2. When \(q \geq (1-\gamma)q_U\), the supplier’s best response is \(x^* = q\) and \(z^* = 1\). So the buyer’s problem is:

\[
\max_{q, \gamma} \ r \min\{q, d\} - wq - \gamma \frac{\beta}{2}, \ \text{s.t.} \ q \geq (1-\gamma)q_U
\]

The optimal solution can be found as:

\[
\begin{align*}
\{q^{(2)} = d, \gamma^{(2)} = 1 - \frac{d}{q_U}, \pi_B^{(2)} = (r-w)d - \left(1 - \frac{d}{q_U}\right)\beta\}, & \quad \text{if} \ d \leq q_U, \\
\{q^{(2)} = d, \gamma^{(2)} = 0, \pi_B^{(2)} = (r-w)d\}, & \quad \text{if} \ d \geq q_U.
\end{align*}
\]

(2.2)

Case 3. When \((1-\gamma)q_L \leq q \leq (1-\gamma)q_U\), the supplier’s best response is \(x^* = q\) and \(z^* = \frac{w(1-\alpha)q}{(1-\gamma)\beta}\). So the buyer’s problem is:

\[
\max_{q, \gamma} \ \frac{w(1-\alpha)q}{(1-\gamma)\beta} \left\{ r \min\{q, d\} - wq \right\} + \left(1 - \frac{w(1-\alpha)q}{(1-\gamma)\beta}\right) \left\{ r \min\{\alpha q, d\} - w\alpha q \right\} \\
- \frac{\gamma}{2} \frac{w^2(1-\alpha)^2q^2}{\beta^2(1-\gamma)^2}
\]

\[\text{s.t.} \ (1-\gamma)q_L \leq q \leq (1-\gamma)q_U\]
We solve the problem in Case 3 in detail below. We first rewrite the above maximization problem as one optimizing on $q$ and $z$ as follows:

$$
\text{max}_{q, z} \left\{ r \min\{q, d\} - wq \right\} + (1 - z) \left\{ r \min\{\alpha q, d\} - w\alpha q \right\} - \left( 1 - \frac{w(1 - \alpha)q}{\beta z} \right) \frac{\beta z^2}{2}
$$

s.t. $2(1 - \frac{c}{w\alpha}) \leq z \leq 1$

$$
z \geq \frac{w(1 - \alpha)q}{\beta}, \quad 0 \leq z \leq 1
$$

The second constraint is due to the fact that $z = \frac{w(1 - \alpha)q}{(1 - \gamma)\beta}$ and $0 \leq \gamma \leq 1$. We first solve the optimal $q$ for any given $z$ and then optimize on $z$.

**Lemma 2.1.** Given $Y_{(1, \alpha)}$ yield rate distribution, the buyer’s optimal order quantity for a given reliability level $z$ is:

(i). If $\frac{(r - w)\alpha}{2r\alpha + w(1 - \alpha)} \leq 1 - \frac{c}{w\alpha}$: $\hat{q}^*(z) = \min\{d, \frac{\beta z}{w(1 - \alpha)}\}$;

(ii). Otherwise,

$$
\hat{q}^*(z) = \begin{cases} 
\min\{d, \frac{\beta z}{w(1 - \alpha)}\}, & \text{if } 2(1 - \frac{c}{w\alpha}) \leq z \leq \frac{2(r - w)\alpha}{2r\alpha + w(1 - \alpha)} \\
\min\{d, \frac{\beta z}{w(1 - \alpha)}\}, & \text{if } d \leq \frac{\beta z}{w(1 - \alpha)} \\
\text{null}, & \text{if } d > \frac{\beta z}{w(1 - \alpha)}
\end{cases}
$$

**Proof.**

(1). First, when $q \geq d/\alpha$, the objective function is decreasing in $q$. So the optimal solution is:

$$
q^* = \begin{cases} 
\frac{d}{\alpha}, & \text{if } \frac{d}{\alpha} \leq \frac{\beta z}{w(1 - \alpha)} \\
\text{null}, & \text{if } \frac{d}{\alpha} > \frac{\beta z}{w(1 - \alpha)}
\end{cases}
$$

(2). Second, when $q \leq d$, the objective function is increasing in $q$. So the optimal solution is:

$$
q^* = \begin{cases} 
d, & \text{if } d \leq \frac{\beta z}{w(1 - \alpha)}, \\
\frac{\beta z}{w(1 - \alpha)}(\leq d), & \text{if } d \geq \frac{\beta z}{w(1 - \alpha)}
\end{cases}
$$

(3). Finally, when $d \leq q \leq d/\alpha$, the objective function is increasing in $q$ if $z \leq z_0$ and is decreasing in $q$ otherwise, where $z_0$ is defined as follows:

$$
z_0 := \frac{2(r - w)\alpha}{2r\alpha + w(1 - \alpha)}.
$$
If $z_0 < 2[1 - c/(w\alpha)]$, the constraint in (2.3) implies that the optimal solution in (c) is:

$$q^* = \begin{cases} 
  d, & \text{if } d \leq \frac{\beta z}{w(1-\alpha)}, \\
  \text{null}, & \text{if } d > \frac{\beta z}{w(1-\alpha)}. 
\end{cases} \quad (2.7)$$

Combine (2.4), (2.5) and (2.7), the optimal solution when $z_0 < 2[1 - c/(w\alpha)]$ is:

$$q^*(z) = \begin{cases} 
  d, & \text{if } d \leq \frac{\beta z}{w(1-\alpha)}, \\
  \frac{\beta z}{w(1-\alpha)}(\leq d), & \text{if } d > \frac{\beta z}{w(1-\alpha)}. 
\end{cases} \quad (2.8)$$

Lemma 2.1 part (i) follows.

If $z_0 \geq 2[1 - c/(w\alpha)]$, (2.8) characterizes the optimal solution when $z \geq z_0$. When $z \leq z_0$, the optimal solution in (c) is:

$$q^*(z) = \begin{cases} 
  \frac{d}{\alpha}, & \text{if } \frac{d}{\alpha} \leq \frac{\beta z}{w(1-\alpha)}, \\
  \frac{\beta z}{w(1-\alpha)}(\geq d), & \text{if } d \leq \frac{\beta z}{w(1-\alpha)} \leq \frac{d}{\alpha}, \\
  \text{null}, & \text{if } d > \frac{\beta z}{w(1-\alpha)}. 
\end{cases} \quad (2.9)$$

Combine (2.4), (2.5) and (2.9), the optimal solution when $z_0 \geq 2[1 - c/(w\alpha)]$ and $z \leq z_0$ is:

$$q^*(z) = \begin{cases} 
  \frac{d}{\alpha}, & \text{if } \frac{d}{\alpha} \leq \frac{\beta z}{w(1-\alpha)}, \\
  \frac{\beta z}{w(1-\alpha)}(\geq d), & \text{if } d \leq \frac{\beta z}{w(1-\alpha)} \leq \frac{d}{\alpha}, \\
  \frac{\beta z}{w(1-\alpha)}(\leq d), & \text{if } d \geq \frac{\beta z}{w(1-\alpha)}. 
\end{cases} \quad (2.10)$$

Lemma 2.1 part (ii) follows. ■

Next, we solve for the optimal $z$. Lemma 2.1 implies that depending on the choice of $z$, the optimal $q$ value is different. So we solve the optimal $z$ for each optimal $q$ response first.

(a) From Lemma 2.1, when $z_0 \leq z \leq 1$ and $z \geq d/qU$, we have $q^*(z) = d$. Plug this into the objective function in (2.3), we can show that it is concave in $z$. The solution to the first order condition is:

$$z_1 := \frac{(1-\alpha)(2r-w)d}{2\beta}. \quad (2.11)$$
If \(2r \leq 3w\),

\[
z^* = \begin{cases} 
\max\{\frac{d}{q_U}, z_0\}, & \text{if } d \leq q_U, \\
\text{null}, & \text{if } d > q_U.
\end{cases}
\] (2.12)

If \(2r \geq 3w\), we can show that \(z_1 > d/q_U\), so

\[
z^* = \begin{cases} 
\text{null}, & \text{if } d > q_U \\
1, & \text{if } \frac{2\beta}{w(1-\alpha)(2r-w)} \leq d \leq q_U, \\
\max\{z_1, z_0\}, & \text{if } d \leq \frac{2\beta}{w(1-\alpha)(2r-w)}.
\end{cases}
\] (2.13)

(b) From Lemma 2.1, when \(\frac{\beta z}{w(1-\alpha)} \leq d\) and \(2(1 - \frac{c}{w\alpha}) \leq z \leq 1\), the optimal \(q^* = \frac{\beta z}{w(1-\alpha)}\) which is smaller than \(d\). Plug this into the objective function in (2.3), we can show that it is convex increasing in \(z\). So the optimal \(z^*\) is:

\[
z^* = \begin{cases} 
\text{null}, & \text{if } d < 2(1 - \frac{c}{w\alpha})q_U, \\
\min\{\frac{d}{q_U}, 1\}, & \text{if } d \geq 2(1 - \frac{c}{w\alpha})q_U.
\end{cases}
\] (2.14)

(c) From (2.10), when \(2(1 - \frac{c}{w\alpha}) \leq z \leq z_0\) and \(\frac{\beta z}{w(1-\alpha)} \geq \frac{d}{\alpha}\), the optimal \(q^* = \frac{d}{\alpha}\). Plug this into the objective function of (2.3), we can show that it is concave decreasing in \(z\). So the optimal \(z^*\) is:

\[
z^* = \begin{cases} 
\text{null}, & \text{if } d > z_0\alpha q_U, \\
\max\{\frac{d}{q_U}, 2(1 - \frac{c}{w\alpha})\}, & \text{if } d \leq z_0\alpha q_U.
\end{cases}
\] (2.15)

(d) From (2.10), when \(2(1 - \frac{c}{w\alpha}) \leq z \leq z_0\) and \(d \leq \frac{\beta z}{w(1-\alpha)} \leq \frac{d}{\alpha}\), the optimal \(q^* = \frac{\beta z}{w(1-\alpha)}\) which is between \(d\) and \(\frac{d}{\alpha}\). Plug this into the objective function in (2.3), we can show that it is concave in \(z\). The solution to the first order condition is

\[
z_2 := \frac{rw(1-\alpha)d + \alpha \beta (r-w)}{2\beta[(r-w)\alpha + w]}
\] (2.16)

So the optimal \(z^*\) is:

\[
z^* = \begin{cases} 
\text{null}, & \text{if } d < 2(1 - \frac{c}{w\alpha})\alpha q_U \text{ or } d > z_0q_U, \\
\max\{\frac{d}{q_U}, 2(1 - \frac{c}{w\alpha})\}, & \text{if } z_2 \leq \max\{\frac{d}{q_U}, 2(1 - \frac{c}{w\alpha})\}, \\
z_2, & \text{if } \max\{\frac{d}{q_U}, 2(1 - \frac{c}{w\alpha})\} \leq z_2 \leq \min\{\frac{d}{q_U}, z_0\}, \\
\min\{\frac{d}{q_U}, z_0\}, & \text{if } z_2 \geq \min\{\frac{d}{q_U}, z_0\}.
\end{cases}
\] (2.17)
The buyer’s optimal choice of \( z \) is found by comparing the objective function at the optimal \( z \) value in case (a)–(d). And finally, the optimal \( \gamma^* \) is found by the relation \( z^* = \frac{w(1-\alpha)q^*}{(1-\gamma^*)^\beta} \). This procedure gives us the optimal contract terms \( q^{(3)} \) and \( \gamma^{(3)} \) in Case 3 (i.e. when \( (1-\gamma)q_L \leq q \leq (1-\gamma)q_U \) is satisfied). Denote the optimal expected profit as \( \pi^{(3)}_B \). [End of analysis for Case 3]

Finally, the optimal subsidy contract is determined by comparing the buyer’s optimal expected profit in \( \pi^*_B \) (\( j = 1, 2, 3 \)).

**A.3. Comparison between the (1, \( \alpha \)) and (1, 0) model: an example**

In this example, the following parameter setting is used: \( r = 14, \beta = 20, w = 11, c = 3, \alpha = 0.38 \).

Figure 1(a) shows the buyer’s optimal order quantity \( q^* \), (b) shows the supplier’s optimal reliability level \( z^* \) induced by the buyer, (c) shows the supplier’s optimal profit, and (d) shows the buyer’s optimal profit.

![Figure 1](image_url)

**Figure 1:** Comparison between the (1, \( \alpha \)) and (1,0) model: an example (\( r = 14, \beta = 20, w = 11, c = 3, \alpha = 0.38 \))

As shown in Figure 1(b) and (d), there are two scenarios where the buyer is better off when sourcing from the (1,0) supplier: (i) when the (1, \( \alpha \)) supplier uses the production inflation strategy (hence sets \( z = 0 \) and guarantees full delivery by Proposition 3), and the (1,0) supplier uses either an...
imperfect yield process (e.g., when \( d \in [1.7, 1.8] \)) or a perfect yield process (e.g., when \( d \in [1.8, 2] \));
(ii) when both the \((1, \alpha)\) and \((1, 0)\) supplier use the investment strategy (e.g., when \( d \in [2.1, 2.9] \)).

In case (ii), the buyer is better off when sourcing from the \((1, 0)\) supplier because the supplier uses a perfect yield process, while a \((1, \alpha)\) supplier has less incentive to invest in reliability and uses an imperfect yield process. In case (i), there are situations where the buyer is better off with a \((1, 0)\) supplier even when the supplier uses an imperfect yield process, as for low values of demand, ensuring that the \((1, \alpha)\) supplier use a production inflation strategy (by ordering less than the demand, see Figure 1 (a)) is less profitable for the buyer.

We can also see from Figure 1(c) that the \((1,0)\) supplier can do better than the \((1, \alpha)\) supplier (e.g., when \( d \in [1.9, 2] \)). This happens as the \((1, \alpha)\) supplier prefers to use production inflation strategy for low values of demand and not invest in improving reliability.

As shown in the numerical example above, both results - the buyer preferring to source from the \((1, 0)\) supplier, and the \((1, 0)\) supplier making higher profits than the \((1, \alpha)\) supplier - are a direct consequence of the endogenous investment in reliability, which differentiates our paper from the existing literature.