Optimal Service-Based Procurement with Heterogeneous Suppliers

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Abstract

We investigate how a buyer can design an optimal mechanism to select suppliers, allocate demand and set procurement prices when both the buyer’s revenue and the suppliers’ costs depend on the service levels the suppliers provide. The suppliers differ in their service costs, production costs, and capacities. We consider three procurement mechanisms that buyers often utilize: take-it-or-leave-it contract, incentive contract, and competition. We characterize the optimal design for each mechanism from the buyer’s point of view and show that all three can achieve the maximum feasible profit for the channel and extract all profit for the buyer. However, this is true only when the buyer has full flexibility in setting all the contract and competition parameters and customizing them for each supplier. Through numerical examples, we show that when the buyer can decide on the value of procurement prices and incentive payments but cannot customize them for different suppliers, then generally incentive contracts perform better than the other two mechanisms. When the procurement prices are set exogenously, the preferred mechanism for the buyer depends on the value of the exogenously set procurement price. The buyer is better off using an incentive contract if the procurement price is relatively low, and better off using a take-it-or-leave-it contract or competition when the price is relatively high.

\textit{Keywords:} procurement and supplier selection, service quality, supplier competition, incentive contracts, take-it-or-leave contracts
1 Introduction

While some procurement decisions are still driven primarily by price, the service capability of suppliers is becoming an increasingly important factor for many manufacturing and service firms. A recent industry survey reveals that the most important factor in outsourcing decisions is process efficiency and quality while cost reduction is ranked third (Mazars Annual Outsourcing Survey 2010). Large retailers, such as Wal-Mart, and manufacturers, such as Dell, place a premium on the service levels their suppliers provide and use sophisticated supplier rating systems for tracking and rewarding supplier performance. Service quality features prominently in the supplier rating systems used by other firms as well. For example, Raytheon, a major aerospace and defense systems supplier, places the largest weight in its supplier selection score (35%) on responsiveness and schedule/delivery performance while assigning a relatively small weight (10%) to price. Saturn Electronics, a global supplier to original equipment manufacturers (OEMs), also uses a rating system that weighs on-time delivery (20%) and quality compliance (30%) more significantly than cost (15%).

This increased focus on supplier service level is driven, in part, by the availability in many industries of multiple qualified suppliers. The relatively weak bargaining position of these suppliers, particularly when the buying firm is large, allows the buying firm to set the price, with service level becoming a primary factor in differentiating between suppliers. Concern about service level is also driven by the operational policies adopted by many firms which emphasize on-demand production and on-time delivery. Such policies make firms particularly vulnerable to poor supplier performance because of the limited safety stocks and safety lead-times these firms maintain, with quality of service from the suppliers directly affecting their revenue. High supplier service level is, of course, critical to firms that have chosen to compete on the basis of customer service or that can extract a price premium for higher service levels. In some cases, the suppliers deal directly with the buying firm’s customers, such as in call centers. In those cases, the service level the suppliers provide directly affects the service levels end customers receive.

Service level is typically measured in terms of the availability of the demanded good or service at the time it is requested. For physical goods, typical measures of service quality include fill rate, expected order delay, the probability that order delay does not exceed a quoted lead-time, and the percentage of

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orders fulfilled within specification. For services, service level measures include expected customer waiting time, the probability that the customer receives service within the specified time window, and the probability that a customer does not renege before being served.

While the importance of having suppliers provide high service levels is clear, it is less clear how firms should go about inducing their suppliers to invest in service quality, particularly when suppliers vary in their capacity levels and cost structures. In settings where the buyer has also the power to set the procurement price, it is not clear how these prices should be set to entice high service levels without compromising profit. Similarly, when the buyer has the flexibility to allocate demand among more than one supplier, it is not clear how such allocation should be carried out to induce maximum service quality and how these allocations are affected by the procurement prices. One approach is to simply select those suppliers that promise to offer the highest service level and accept the lowest price. However, when suppliers vary in their capabilities, and these capabilities are common knowledge among all parties, there may not be enough incentive for the more capable suppliers to maintain their maximum feasible service levels.

In this paper, we investigate three mechanisms that could be used to select suppliers, allocate demand, and choose procurement prices when suppliers vary in their capacity and cost efficiency. Since our setting is one of complete information, we first consider “take-it-or-leave-it contracts” that specify the procurement prices, demand allocations, and required service levels. We show how the suppliers can be selected optimally and how the corresponding contract can be designed to achieve the maximum feasible profit for the buyer, including when the buyer’s profit depends directly on service levels. We also show that this solution is first-best in the sense that it maximizes total expected profit for the channel.

The second mechanism consists of offering “incentive contracts” to selected suppliers. These contracts specify the demand shares for each supplier and define associated procurement prices and incentive functions that determine financial transfers between the buyer and supplier based on the service level provided. In contrast to take-it-or-leave-it contracts, the service levels are not specified and are left to the suppliers to determine in response to the terms of the incentive contracts. We show how incentive contracts can be designed to match the performance of the “take-it-or-leave it” mechanism.

The third mechanism is a “supplier competition” where the buyer allows all suppliers to compete for a share of demand and specifies only procurement prices and a demand allocation function. The demand allocation function specifies shares of demand allocated to the suppliers based on the relative service
levels the suppliers commit to offer. We show how the buyer can choose the procurement prices and design the allocation function to again match the performance of the “take-it-or-leave it” mechanism.

While the three mechanisms differ in their form, our results show that each is capable of maximizing the buyer’s profit, making them equivalent in performance. However, this is the case only when the buyer has the flexibility of deciding on the amount of demand allocated and tailoring the procurement price or service incentive paid to each supplier. If the buyer does not have flexibility on either demand allocation or payments, the mechanisms can behave differently and result in different service levels and buyer profits. We provide insight into these trade-offs through a series of numerical examples.

An important contribution of this paper is in showing how allocation functions can be designed to induce suppliers to provide the maximum feasible service level. We do so regardless of whether or not there is flexibility in pricing and regardless of the heterogeneity in the cost structures of the suppliers and their capacities. Moreover, in settings where it is desirable to set the demand allocation in a specific way, we show that it is possible to design the allocation function to induce this desired allocation as an outcome of the competition.

The supplier competition we describe here is similar to the SA competition discussed in Benjaafar et al. (2007). However, that paper is not concerned with determining optimal allocation functions. Instead, the focus is on studying the behavior of suppliers who are engaged in a supplier competition orchestrated by a single buyer under a specific service-proportional allocation, exogenously determined. In that paper, the analysis is limited to identical suppliers with identical costs and revenue structures and with no constraints on capacity. Also in their case, procurement prices are exogenously determined and the buyer measures the performance of the procurement mechanism through (demand-weighted) average service level. In this paper we evaluate the performance of different mechanisms through a more direct measure: buyer’s profit.

In addition to Benjaafar et al. (2007), the other paper that is most related to our competition setting is Cachon and Zhang (2007). They consider a specific context where suppliers are modeled as single server queues and compete in terms of investment in service rates. Higher service rates translate into higher service levels in the form of lower queueing delays for the buyer. Similar to Benjaafar et al. (2007), they treat the case of homogeneous suppliers with identical revenue and cost structures. They compare different demand allocations and show that a linear allocation function leads suppliers to invest in the maximum feasible service rates for the fraction of demand they are allocated (see section 4 for further
discussion). However, because they consider only symmetric allocations (suppliers that provide the same service rates are allocated the same amount of demand), the proposed allocation does not necessarily maximize overall quality-of-service. As with Benjaafar et al. they do not consider other procurement mechanisms, capacity limits, and procurement price selection.

A general review of the literature on service-based supplier selection and procurement is included in Benjaafar et al. (2007). For the sake of brevity, we will not reproduce it here. For more recent papers see Jin and Ryan (2009), Xiaoyuan Lu et al (2009), and Zhou and Ren (2010). However, we should note that much of the existing literature has focused on schemes involving competition among identical suppliers or competition involving specific allocation functions, typically proportional allocation functions. Few results exist for settings with heterogeneous suppliers or suppliers with capacity constraints. We are not aware of any results on the joint optimization of demand allocation and procurement prices. We are also not aware of any results comparing procurement mechanisms involving competition with other non-competitive schemes, including take-or-leave-it or incentive contracts for settings where profit is affected by service levels.

There is related literature in economics on optimal mechanism design (reviews can be found for example in Klemperer 1999 and Kalra and Shi 2001). Papers in this area focus on finding the optimal method for a principal to elicit maximum effort or minimum price from a set of agents. This literature spans diverse application areas, including designing employee incentives, incentives for sales agents, contest rules among independent agents to elicit maximum effort, and auctions. In this literature, emphasis is placed on agents’ competition while there is information asymmetry regarding agent characteristics or effort. Our paper can be viewed as involving mechanism design. However, in our case, we assume complete information among all parties.

There is also related literature in economics on rent-seeking contests. In a rent-seeking contest, there are $N$ contestants who compete for a prize. The probability that a contestant wins the prize (the rent) increases with his expenditures and decreases in the expenditures of other contestants. A review of important results from this literature can be found in Mueller (2003, Ch. 15), Congleton et al. (2008), and Konrad (2009). A focus of this literature is on documenting the so-called inefficiency of rent-seeking contests. Rent-seeking is viewed as wasteful since the total expenditures by the contestants can equal the value of the prize itself, a phenomenon called rent dissipation. However, in systems with non-identical contestants, it has been shown that there may not be complete rent dissipation; see for example Hilman
and Reily (1989), Paul and Wilhite (1990), Nti (1999), and Dixit (1987). That is, heterogeneity in the contestants’ characteristics can diminish the intensity of the competition. In our research, we show that by carefully designing an allocation function (the equivalent of a selection probability in the rent seeking context), one may induce the contestants to exert the maximum feasible effort even when they have non-identical characteristics.

The rest of the paper is organized as follows. Section 2 describes our problem setting and characterizes the optimal take-it-or-leave-it contract. Section 3 focuses on incentive contracts and describes the structure of optimal incentive contracts. Section 4 provides similar analysis for the service competition mechanism. Section 5 compares the three mechanisms and highlights the impact of different market constraints on performance. Section 6 offers concluding remarks.

2. Preliminaries and the Take-it-or-Leave-it Contract

Our supply chain consists of a buyer and $N$ potential suppliers who differ in their production and service capabilities. The buyer wishes to allocate her expected demand quantity, $\lambda$, across these suppliers in a manner that maximizes expected profit. Our purpose in this section is to describe the revenue and cost functions that underlie the buyer’s problem and uncover the structure of the buyer’s optimal “take-it-or-leave-it” contract. Unlike previous studies, we assume the buyer’s revenue depends directly on the service level. The optimal “take-it-or-leave-it” contract serves as a benchmark for other contract mechanisms since, as we will show, it coordinates the channel while extracting all profits for the buyer. In other words, it provides a first-best solution to the buyer’s problem. However, it does so by having the buyer strictly dictate the demand allocation and service levels for each supplier and offering a procurement price that leaves no surplus to the supplier.

In determining the optimal demand allocation, procurement prices, and service levels for each supplier, the buyer must evaluate and compare the suppliers’ capabilities. Each supplier is uniquely

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3 Demand can be interpreted as either a single quantity covering one sales period or a demand-rate maintained over multiple periods.
4 The setting we consider is one where there is full information regarding the cost structure of the suppliers. This would be the case when the primary cost drivers of the suppliers are in the public domain (e.g., settings where costs are determined by regional factors such as labor costs; taxes and regulations; cost of materials and energy; type of production technology used; and transportation costs). The assumption of full information builds on assumptions made previously in the literature and serves as an upper bound on performance for the buyer and the supply chain. It also provides insight into the design of procurement mechanisms if the buyer is to leverage knowledge of cost and capacity differences among the supplier and to assess the benefit derived from this knowledge.
characterized by its capacity level, \( \omega \), unit operating cost, \( c_i \), and service related costs, \( f_i(s_i, \delta_i), i=1,..., N \). We assume that supplier \( i \)'s service related cost is a function of the proportion of demand allocated to the supplier, \( \delta_i \), as well as the supplier's service level, \( s_i \), both dictated by the buyer, where \( \sum_{i=1}^{N} \delta_i = 1 \) and \( s_i \geq 0 \). The actual demand allocated to supplier \( i \) will then be \( \delta_i \lambda \). We focus on a particular class of plausible service cost functions of the form

\[
 f_i(s_i, \delta_i) = k_\delta \delta_i s_i + v_i(s_i),
\]

where \( k_\delta \) is a positive constant and \( v_i(s_i) \) is a continuous, increasing and convex function in \( s_i \), with \( v_i(0) = 0 \), for \( i=1,..., N \). We also assume that \( v_i(s_i) \) is twice differentiable. This function generalizes service cost functions used in prior research (e.g., Benjaafar et al. 2007, Cachon and Zhang 2007) by accounting for supplier heterogeneity. The first term, \( k_\delta \lambda \delta_i \), is the demand-dependent service cost, which varies linearly with the demand allocated to the supplier. The second term, \( v_i(s_i) \), captures supplier-specific costs that increase only with the service level itself. This demand-independent cost is not affected by the amount of demand allocated\(^5\). Examples of service related costs that fit this model include investments in capacity, inventory, transportation, and/or continuous improvement efforts. We will elaborate on one specific example in section 5, as part of our numerical study.

Taking into account these unique aspects of her supply base, the buyer would like to set demand allocations, acquisition prices, and service levels in a manner that maximizes her expected profit. The buyer’s profit is determined by the revenue received from her customers minus the procurement prices paid to her suppliers to cover their production and service costs. We assume the buyer is directly rewarded for high service quality through the revenue she receives from her own customers. More specifically, we characterize the buyer’s revenue as the sum of increasing concave functions of the service levels provided by each supplier, \( h(s_i), i=1,\cdots, N \), weighted by the proportion of demand receiving that service. That is,

\[
 R(\delta, s) = \sum_{i=1}^{N} \delta_i \lambda h(s_i)
\]

where \( \delta = (\delta_1,\cdots,\delta_N) \) and \( s = (s_1,\cdots,s_N) \) are the demand allocations and service levels dictated by the buyer. Because the revenue function is concave in all elements of \( s \), it has the appealing property of

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\(^5\) A more general form of \( f_i(s_i, \lambda) \) can be defined in which the demand-dependent cost is also a function of the service level. That is, \( f_i(s_i, \lambda) = \lambda u_i(s_i) + v_i(s_i) \), in which \( u_i(s_i) \) is a continuous, non-decreasing, and convex function of \( s_i \). Most of our results also hold for this more general form of cost function.
decreasing returns to service. This revenue structure captures situations where the buyer’s customers observe the service level provided by the suppliers while the buyer is responsible for paying the penalty for poor service levels. Applications where the suppliers’ service level is observable by the end customer include outsourcing after-sales services, call centers, roadside assistance, or when the supplier directly ships to the end customer. In these situations, \( h(s_i) \) is simply the buyer’s original selling price minus any service penalty she pays the end customers\(^6\).

The buyer’s problem under a take-it-or-leave-it contract can now be written as follows,

\[
\max_{\mathbf{s}, \mathbf{\delta}, \mathbf{p}} \pi^B(\mathbf{s}, \mathbf{\delta}, \mathbf{p}) = R(s, \delta) - \sum_{i=1}^{N} \delta_i \lambda p_i
\]  

subject to:

\[
\pi^S_i(s_i, \delta_i, p_i) \geq 0, \quad i = 1, \ldots, N,
\]  

\[
\delta_i \lambda \leq \omega_i, \quad i = 1, \ldots, N,
\]  

\[
\sum_{i=1}^{N} \delta_i = 1,
\]  

\[
0 \leq \delta_i \leq 1, s_i \geq 0, \quad i = 1, \ldots, N.
\]  

where \( \pi^S_i(s_i, \delta_i, p_i) = \delta_i \lambda (p_i - c_i) - f_i(s_i, \delta_i) \) denotes the expected profit of supplier \( i = 1, \ldots, N \), and \( \mathbf{p} = (p_1, p_2, \ldots, p_N) \) is the vector of offered procurement prices. As mentioned earlier, the buyer’s profit function (3) is simply the revenue received from her customers minus the procurement charges paid to each supplier. Constraint (4) is the suppliers’ participation condition. It guarantees that the suppliers receive at least their reservation profit (assumed to be zero without loss of generality). Constraint (5) guarantees that no supplier receives more demand than his capacity can support. We also assume that \( \lambda \partial h(s_i) / \partial s_i \big|_{s_i = 0} > \partial v_i(s_i) / \partial s_i \big|_{s_i = 0} \) to avoid the trivial case where the supplier’s optimal service level is always zero. Moreover, we assume the amount of capacity available across suppliers is large enough to meet all demand, i.e. \( \sum_{i=1}^{N} \omega_i \geq \lambda \).

To characterize the structure of an optimal take-it-or-leave-it contract, we introduce the function \( \sigma_i(\delta_i) \), which is defined to be the maximum service level supplier \( i \) (for \( i = 1, \ldots, N \)) can provide when the demand share it receives is \( \delta_i \) and the profit he earns is not less than his reservation profit. It is easy to verify that

\[ \text{For example, when make-to-order suppliers directly ship to the buyer’s customers and service level is measured by the probability of meeting a quoted lead time, we have } h(s_i) = p_{0a} - a(1 - s_i), \text{ where } p_{0a} \text{ is the original buyer’s selling price per unit (the price that the buyer charges when the service level is as promised to her customers) and } a \text{ is the penalty paid for each unit delivered later than the quoted lead time.} \]
Proposition 1 describes the structure of the optimal take-it-or-leave-it contract that maximizes the buyer’s profit.

**Proposition 1.** The buyer problem in (3)-(7) always has a finite solution, \( \delta^* = (\delta_1^*, ..., \delta_N^*) \), \( p^* = (p_1^*, ..., p_N^*) \), and \( s^* = (s_1^*, ..., s_N^*) \). The optimal set of demand shares, \( \delta^* = (\delta_1^*, ..., \delta_N^*) \), is a solution to the following problem

\[
\max_{\delta} \pi^B(\sigma(\delta), \delta) = \lambda \sum_{i=1}^{N} \delta_i \left[ h(\sigma_i(\delta_i)) - (c_i + f_i(\sigma_i(\delta_i), \delta_i) / \lambda \delta_i) \right],
\]

subject to constraints (5)-(7), where \( \sigma(\delta) = (\sigma_1(\delta_1), ..., \sigma_N(\delta_N)) \). The optimal set of service levels and procurement prices can then be obtained as

\[
s_i^* = \sigma_i(\delta_i^*), \quad i = 1, ..., N \quad \text{and} \quad p_i^* = c_i + f_i(s_i^*, \delta_i^*) / \lambda \delta_i^*, \quad i = 1, ..., N.
\]

In proposition 1, \( \sigma_i(\delta_i) \) represents the service level that maximizes the buyer’s profit contribution from supplier \( i \) for a given demand allocation \( \delta_i \). Having this optimal service level, the buyer’s problem simplifies to (8), which is an optimization problem with only one set of decision variables, \( \delta \). Although the proposition does not provide a closed form solution for \( \delta^* \), it guarantees the existence of a finite solution which can be computed numerically using a standard multivariable optimization algorithm.

The buyer’s optimal take-it-or-leave-it contract also turns out to be efficient in the sense that it yields the maximum possible expected profit for the supply chain as a whole. In other words, it maximizes the combined profit of the buyer and suppliers.

**Proposition 2.** The solution to the buyer’s take-it-or-leave-it problem, as defined in proposition 1, maximizes channel profit for the supply chain and extracts all profits for the buyer.

As a result, the solution to the buyer’s take-it-or-leave-it problem serves as an upper bound on the expected profit level the buyer can achieve under any contract mechanism.

To provide more insight into the structure of the optimal solution, we examine a series of special cases where one or more aspects of the problem are simplified. In each case we determine the optimal demand allocation vector which, along with the results of proposition 1, defines the optimal take-it-or-leave-it contract. The following notion of supplier efficiency will help in our analysis. A supplier’s efficiency level \( e_i \) is defined as the maximum profit per unit demand that the supplier can generate for the buyer when the supplier is given the highest feasible demand allocation. This highest feasible demand
allocation depends on the supplier’s capacity and the total demand available; it is given by 
\( \delta_i = \min(\omega_i / \lambda, 1) \). The efficiency of supplier \( i \) can then be computed as
\[
e_i = h(\sigma_i(\delta_i)) - \left[ c_i + f_i(\sigma_i(\delta_i), \delta_i) / \lambda \delta_i \right],
\]
(9)
Without loss of generality, for the remainder of the paper we rename the suppliers in descending order of their efficiencies such that \( e_1 \geq e_2 \geq ... \geq e_N \). As we will see in the following cases, the buyer usually (but not always) allocates as much demand as possible to the most efficiency suppliers. Once the optimal allocation of demand shares is known, proposition 1 can be invoked to determine the service levels and procurement prices that maximize the buyer’s profit.

**Case 1: No Binding Capacity Constraint**

When the capacity of the most efficient supplier is at least as large as the buyer’s demand, it is optimal for the buyer to allocate all demand to this supplier. This result is stated in proposition 3.

**Proposition 3.** When the most efficient supplier’s capacity is larger than the buyer’s demand, the optimal set of demand shares is \( \delta_1^* = 1 \) and \( \delta_i^* = 0, i = 2, ..., N \).

If more than one supplier shares the highest efficiency, that is \( e_1 = e_2 = ... = e_M \), where \( 1 \leq M \leq N \), then the entire demand can be allocated to any of the first \( M \) suppliers. This result is consistent with intuition. Since the suppliers’ cost structure contains a demand-independent term, due to economy of scale, each supplier can provide a higher service level when receiving a higher demand share. Hence, when the most efficient supplier can process the entire buyer’s demand, the buyer allocates only to this supplier to gain the highest level of service.

**Case 2: Binding Capacity Constraint(s) and Homogenous Costs**

In this case, suppliers have identical cost structures, implying that
\[
e_i = c \quad \text{and} \quad f_i(s_i, \delta_i) = f(s_i, \delta_i) = k \delta_i + v(s_i) \quad \text{for} \quad i = 1, ..., N.
\]
(10)
This symmetric cost structure is typical, for example, of industries where the suppliers use similar processes and technologies. Although the suppliers have identical cost structures, they are still heterogeneous in terms of their capacities. The difference in the suppliers’ efficiencies is solely due to difference in the values of \( \delta_i \), which in turn depend only on the suppliers’ capacities, \( \omega_i \). Since \( \delta_i \) is
non-increasing in \( \omega_i \) it is easy to verify that the efficiency of each supplier is also non-decreasing in his capacity. In this case, ordering suppliers by decreasing efficiency levels is equivalent to ordering them by descending capacities.

The structure of the optimal solution in this case depends on whether the buyer’s profit function (8) is convex in \( \delta \). We focus first on the sub-case where this assumption holds. Proposition 4 characterizes the corresponding optimal set of demand shares.

**Proposition 4.** For suppliers with identical cost structures, when the buyer’s profit function (8) is convex in \( \delta \), the optimal set of demand shares which solves problem (3) subject to (4)-(7) is

\[
\hat{\delta}_i^* = \begin{cases} 
\omega_i & \text{for } i = 1, \ldots, \hat{N} - 1, \\
\lambda - \sum_{j=1}^{\hat{N}-1} \omega_j & \text{for } i = \hat{N}, \\
0 & \text{for } i > \hat{N}
\end{cases}, \quad i = 1, \ldots, N,
\]

where \( \hat{N} \) is the smallest integer such that \( \hat{N} \sum_{i=1}^{\hat{N}} \omega_i \).

We learned from proposition 3 that if capacity were unlimited, it would be optimal to allocate all demand to the most efficient supplier (supplier “1” in our ordering). However, because suppliers have limited capacities, the optimal solution in proposition 4 is to allocate demand to \( \hat{N} \) suppliers. More specifically, we should assign full capacity to the first \( \hat{N} - 1 \) most efficient suppliers and then assign the remaining demand to the \( \hat{N} \)th most efficient supplier.

The solution defined in (11) is fairly intuitive. We choose the most efficient supplier and allocate as much as possible to him. If there is still unallocated demand, we choose the second most efficient supplier and, again, allocate as much as possible to him. We continue this process until we allocate the entire demand. This process, however, is optimal only when we have a convex profit function with respect to \( \delta \). The convexity condition, which depends on both the service cost function and the buyer’s revenue function, holds for many practical applications, including the example application introduced in section 5.

When the buyer’s profit is not convex in \( \delta \), this structure may no longer hold. To provide more insight into how the solution may differ, consider the following example. Suppose the suppliers’ service cost function and the buyer’s revenue functions are \( v(s_i) = a - b(1 - s_i)^m \) for \( i = 1, \ldots, N \), and \( h(s_i) = p_s - t(1 - s_i)^m \), respectively. For the case of two suppliers, as long as \( m > n \) we have a convex buyer’s profit function and so the result of proposition 4 holds. However, if \( m < n \), then the buyer’s profit function become non-convex and a partial allocation is optimal. Figure 1 illustrates how the buyer’s profit function...
is impacted by changes in the demand allocation between two suppliers under a convex profit function \((m = 3, n = 2)\), and a non-convex profit function \((m = 2, n = 3)\). In both cases, the buyer’s demand is \(\lambda = 50\).

Now consider, for instance, the case where the capacities of the suppliers are \(w_1 = 40\) and \(w_2 = 30\). Figure 1 shows, under the convex profit function, the buyer optimizes her profit by allocating as much demand as possible to the more efficient supplier \((\delta_1 = 0.8, \text{or } \lambda \delta_1 = 40)\). Under the non-convex profit function the buyer is better off allocating half of its demand to each supplier \((\delta_1 = 0.5, \text{or } \lambda \delta_1 = \lambda \delta_2 = 25)\), which means partial allocations to both suppliers. Mathematically, when the objective function of our optimization problem is convex with respect to the decision variables, the maximum point happens on the boundaries of a convex domain. That is, each supplier either receives full allocation or no allocation, except for the supplier which receives the leftover to satisfy constraint (6), which is what we have in proposition 4. When the objective function is not convex it is not possible to derive general analytical conclusions, but our example shows that the maximum can happen in an interior point, which means all suppliers can receive partial allocation.

![Graph](image)

Figure 1 – The impact of revenue and service cost function on the Buyer’s profit

\[ k(s_i) = p_s - t(1 - s_i)^m, \quad v(s_i) = a - b\left(1 - s_i\right)^n, \quad p_s = 1900, \quad t = 1000 \quad \text{and} \quad \delta_1 \lambda + \delta_2 \lambda = \lambda = 50 \]

Case 3: Binding Capacity Constraint(s), Homogeneous Capacity and Heterogeneous Costs

In this case, suppliers have identical capacity, or the buyer is restricted not to allocate more than a given value to each supplier, implying \(w_i = w < \lambda\) and \(\overline{\delta}_i = \overline{\delta} = w / \lambda\) for \(i = 1, \ldots, N\). The efficiency of supplier \(i\) then becomes
\[ e_i = h(\frac{\sigma_i}{\lambda}) - \left[ c_i + f_i(\frac{\sigma_i}{\lambda}) \right] / w \]  \hspace{1cm} (12)

for \( i = 1, \ldots, N \). Intuitively, the efficiency measure is evaluated at the same maximum level of capacity for each supplier. In this case, when we order suppliers according to their efficiency levels we are simply ordering them by increasing order of their costs.

The structure of the optimal solution is the same as that described in Case 2, expect that efficiency is now defined in terms of costs rather than capacities. Specifically, the structure stated in proposition 4 holds when the buyer’s profit is convex in \( \delta \), but the optimal allocation scheme may contain partial allocations to suppliers otherwise.

### 3. Incentive Contracts

We now consider an alternative contract mechanism, which is less restrictive in terms of what is dictated to the suppliers. A take-it-or-leave-it contract simultaneously dictates the demand allocation, service levels, and price. An incentive contract, on the other hand, only dictates the demand allocation and price while leaving the decision about the service level to the supplier. This type of contract may be more attractive to suppliers who desire some decision freedom. It is also less burdensome for the buyer since she only has to determine demand allocations and prices. Because the buyer cannot dictate service levels directly, she must use an incentive to encourage the suppliers to invest in service quality.

Let \( D_i(s_i) \) denote the financial incentive that the buyer agrees to pay supplier \( i \) if the supplier promises to provide service level \( s_i \), where \( D_i(s_i) \) is increasing in \( s_i \). We will focus on the simple linear case,

\[ D_i(s_i) = d_i s_i, \]  \hspace{1cm} (13)

where \( d_i > 0 \), is a scalar and \( \mathbf{d} = (d_1, d_2, \ldots, d_N) \). As we will see shortly, this simple incentive scheme is sufficient to provide an optimal solution for the buyer and the channel as a whole.

The buyer’s problem under an incentive contract can now be written as

\[ \max_{\mathbf{d}, \delta, \mathbf{p}} \pi^B(\mathbf{d}, \delta, \mathbf{p}) = R(\mathbf{d}, \delta) - \sum_{i=1}^{N} (\delta_i \lambda p_i + d_i s_i) \]  \hspace{1cm} (14)

subject to:

\[ s_i = \arg \max_{x} \left( \delta_i \lambda (p_i - c_i - k_i) - v_i(x) + d_i x \right), \ \ i = 1, \ldots, N, \]  \hspace{1cm} (15)

\[ \pi^B_i(s_i) = \delta_i \lambda (p_i - c_i - k_i) - v_i(s_i) + d_i s_i \geq 0, \ \ i = 1, \ldots, N, \]  \hspace{1cm} (16)

\[ \delta_i \lambda \leq \omega_i, \ \ i = 1, \ldots, N, \]  \hspace{1cm} (17)
\[ \sum_{i=1}^{N} \delta_i = 1, \]  
\[ 0 \leq \delta_i \leq 1, \quad i = 1, \ldots, N. \]  

The buyer’s problem under an incentive contract has one additional constraint compared with the buyer’s profit under a take-it-or-leave-it contract. The new constraint (15) captures the suppliers’ decisions where each supplier chooses a service level to maximize his own profit.

The following proposition characterizes the buyer’s optimal solution under this incentive contract.

**Proposition 5.** When the buyer outsources \( \lambda \delta_i \leq \omega_i \) to supplier \( i \), the optimal incentive contract that maximizes the buyer’s profit is \( (d_i^*, \delta_i^*, p_i^*) \), in which

\[ d_i^* = \partial v(s_i) / \partial s_i \bigg|_{s_i = s_i^*} \quad \text{and} \quad p_i^* = c_i + f_i(s_i^*, \delta_i^*) - d_i s_i^*, \]

where, \( s_i^* = \arg \max_s \left[ h(x) - c_i - f_i(x, \delta_i) / \delta_i \right] \) is the optimal service level provided by the supplier.  
Moreover, this contract extracts all profits for the buyer, \( \pi_i^* = 0 \).

Proposition 5 characterizes the optimal incentive term and procurement price for the buyer when dealing with a single supplier \( i \) who is given an arbitrary allocation \( \delta_i \leq \omega_i / \lambda \). To fully define the buyer’s optimal solution, we need to also characterize the optimal allocation of demand across the supply base. The optimal allocation in this case is identical to the optimal allocation defined for the take-it-or-leave-it contract. Proposition 6 states this results.

**Proposition 6.** The solution to the buyer’s problem under an incentive contract (14)-(19), is achieved when the incentive terms and prices are set as in proposition 5, and demand shares are set as in proposition 1. This solution provides the maximum feasible profit for the buyer which is the same as the buyer’s profit under an optimal take-it-or-leave-it contract.

Proposition 6 asserts that an optimal set of incentive contracts can achieve the maximum feasible channel profit which is the same as the profit of an optimal take-it-or-leave-it contract. The suppliers’ actual procurement prices, however, are lower under the optimal incentive contract because of the additional incentive term paid by the buyer.

4. **Service-Based Competition**

The final mechanism we consider is the least restrictive. The buyer dictates neither the demand allocations nor the service levels. Rather, the buyer allows the suppliers to compete with each other for
demand through their service level decisions. We examine a general form for this service-based competition where \( \alpha_i(s_i, s_{-i}) \) denotes a function specifying the fraction of demand allocated to supplier \( i \) given the supplier’s service level \( s_i \) and the service levels \( s_{-i} = (s_1, ..., s_{i-1}, s_{i+1}, ..., s_N) \) offered by her competitors. We assume \( \alpha_i(s_i, s_{-i}) \) is non-decreasing in \( s_i \), equal to zero when \( s_i = 0 \), and \( 0 \leq \alpha_i(s_i, s_{-i}) \leq 1 \), for \( i = 1, ..., N \).

The competition begins with the buyer, who announces the competition terms \( (\alpha(s), p) \), where \( \alpha = (\alpha_1, ..., \alpha_N) \) denotes a vector of allocation functions and \( p \) is the vector of unit procurement prices defined previously. Each supplier \( i \) responds to the buyer’s announcement by choosing a service level that maximizes her expected profit subject to the behavior of other suppliers.

The buyer’s problem under this competition setting can be stated as

\[
\max_{(\alpha(s), p) \in \mathbb{R}^N} \pi^{BC}(s, \alpha(s), p) = R(s, \alpha(s)) - \sum_{i=1}^{N} (\alpha_i(s) \lambda p_i),
\]

subject to:

\[
s_i = \arg \max_x \left( \alpha_i(x, s_{-i}) \lambda [p_i - c_i - k_i] - v_i(x) \right), \quad i = 1, ..., N
\]

\[
\pi^{NC}_i(s_i, s_{-i}) = \alpha_i(s_i, s_{-i}) \lambda \left[ p_i - c_i - k_i \right] - v_i(s_i) \geq 0, \quad i = 1, ..., N,
\]

\[
\alpha_i(s) \lambda \leq \omega_i, \quad i = 1, ..., N
\]

where \( \mathbb{S}^N = \{ (\alpha_1(x), ..., \alpha_N(x)) \} \) is the set of all \( N \)-dimensional vectors of functions with \( \alpha_i : \mathbb{R}^N \mapsto [0, 1] \) and \( \sum_{i=1}^{N} \alpha_i(x) = 1 \). Unlike the previous buyer problems, the optimization is carried out over all vectors of functions in \( \mathbb{S}^N \), in addition to all vectors of positive real numbers in \( \mathbb{R}^{N^+} \). The first set of new constraints (20) reflect the suppliers’ subgame, where each supplier chooses a service level to maximize her own profit for any given set of service levels chosen by her competitors. Each supplier’s decision in this subgame is affected not only by his competitors’ decisions, but also by the form of allocation function set by the buyer. Constraint (22) guarantees non-negative profits for suppliers. The other new constraints (23) define capacity limits on the allocation function values.

As we mentioned above, in this procurement mechanism, the buyer needs to find the best form of allocation function as well as the optimal set of procurement prices. We will show below that this can be done in a two step process. In the first step, the buyer designs allocation functions which induce maximum feasible service levels through supplier competition for any given set of procurement prices and target demand shares. In the second step, the buyer seeks the optimal procurement prices as well as the optimal set of target demand shares which she wishes to induce in the suppliers’ competition.
Step 1: Finding a Service-Maximizing Allocation Function

Finding an allocation function that solves the buyer’s problem (20) involves searching for optimal functions (not optimal quantities) and so is difficult to solve directly. Our goal, in this step, is to characterize a family of allocation functions that maximizes the buyer’s profit for a given set of procurement prices $p$ and targeted demand shares $\delta$. So, for now we assume that $p$ and $\delta$ are pre-specified and therefore fixed. Throughout our analysis, we assume that the given target demand shares and procurement prices are feasible, implying that $\sum_{i=1}^{N} \delta_i = 1$, $\delta_i \leq \omega_i$ and $p_i \geq c_i + k_i$ for $i = 1, \ldots, N$.

We begin by defining a specific type of allocation function, which we refer to as service-maximizing.

When $(p, \delta)$ is fixed, the buyer’s profit increases in each supplier’s service level and so the buyer’s problem reduces to finding an allocation function which induces the maximum feasible service levels. In other words, we are looking for an allocation function that intensifies the competition to a level where each supplier applies all the income gained from his associated revenue (i.e., $\delta_i \lambda p_i$) to cover his production cost and maximize service. Let $s_i^{\text{max}}(\delta_i, p_i)$ denote the maximum service level associated with $(\delta_i, p_i)$, which is the solution to $\delta_i \lambda [p_i - c_i - f_i(s_i)/\delta_i \lambda] = 0$ for $i = 1, \ldots, N$. We are now ready to define our specific type of allocation function.

**Definition 1:** An allocation function $\alpha_i(s_i, s_{-i})$ is service-maximizing, with respect to $(\delta, p)$, if it induces a Nash equilibrium service level vector $s^* = (s_1^*, \ldots, s_N^*)$ for which $s_i^* = s_i^{\text{max}}(\delta_i, p_i)$ and $\alpha_i(s_i^*, s_{-i}^*) = \delta_i$, $i = 1, \ldots, N$.

This property is important because if an allocation function is shown to be service-maximizing, this ensures that it maximizes the buyer’s problem when $(\delta, p)$ is given. It is a sufficient condition for optimality in this special case.

In our search for a service maximizing allocation function, we focus on proportional allocation functions, since they are commonly used in the literature. In particular, we consider the following general characterization of proportional allocation functions that allows for heterogeneity across suppliers:

$$\alpha_i(s_i, s_{-i}) = \frac{g_i(s_i)}{\sum_{j=1}^{N} g_j(s_j)},$$

(24)

for $i = 1, \ldots, N$, where $g_i(s_i)$ is a non-decreasing function of $s_i$ with $g_i(0) = 0$. Unlike prior literature which focuses almost exclusively on symmetric functions\(^7\) (e.g., Benjaafar et al. 2007, Cachon and

\(^7\) Symmetric functions imply that if two or more suppliers choose the same service investment, they will receive the same proportion of demand. This type of allocation function is known to be service-maximizing in some special cases when suppliers
Zhang 2007, Allon and Federgruen 2005, and the references therein), we allow the parameters of the allocation function to differ by supplier. Symmetric functions are a subset of this family, with the restriction that \( g_i(s_i) = g(s_i) \) for \( i = 1, ..., N \).

We know from prior research that a proportional allocation function does not always guarantee a unique Nash equilibrium solution. For example, Benjaafar et al. (2007) show that a symmetric proportional allocation function in a system with identical suppliers does not guarantee uniqueness of the Nash equilibrium when \( g(s_i) \) is not concave (see also Cachon and Zhang (2007) for a similar result). It follows that in our more general case of heterogeneous allocation functions and heterogeneous suppliers, a unique Nash equilibrium will not be possible for all forms of \( g_i(s_i), i = 1, ..., N \).

The following proposition defines a specific form for \( g_i(s_i) \) that guarantees both a service-maximizing allocation and a Nash equilibrium. The function is parameterized for each supplier \( i \) by \( \theta_i(s_i) = v_i(s_i) / \delta_i \lambda(p_i - c_i - k_i) \), which is the ratio of a supplier’s demand independent cost to the supplier’s revenue. This function can be interpreted as a measure of the relative impact of service level on the supplier’s revenue. Here, we define \( \theta_i(s_i) \) as a function of only \( s_i \) since we are currently holding \( p_i \) and \( \delta_i \) fixed.

**Proposition 7.** The proportional allocation function defined in (24) is service-maximizing for a given \((p, \delta)\) when \( g_i(s_i) = \delta_i \theta_i(s_i)^{1/(1-\delta_i)}, 0 < \delta_i < 1, \ i = 1, ..., N \). Furthermore, if this allocation function is used in the buyer’s problem for a given \((p, \delta)\) then the following properties hold.

(a) A Nash equilibrium exists, with the suppliers’ service levels and profits given by \( s_i^* = s_i^{\max}(\delta_i \lambda) \) and \( \pi_i(s^*) = 0 \) for \( i = 1, ..., N \).

(b) Allocation levels at this equilibrium are given by \( \alpha_i(s^*) = \delta_i \) for \( i = 1, ..., N \).

(c) This Nash equilibrium is unique if suppliers are constrained to provide a strictly positive service level (i.e., \( s_i > 0 \) for \( i = 1, ..., N \)).

Unlike previous research involving symmetric allocation functions, it is worth noticing that proposition 7 does not require convexity of the cost functions. To ensure that \( s_i^{\max}(\delta_i \lambda) \) is finite, we only need to restrict our analysis to the non-trivial case where \( \sup_{x > 0} v_i(x) \) is strictly greater than \( (p_i - c_i - k_i) \lambda \). Otherwise, the supplier’s service cost would never exceed his potential revenue and the

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are identical (Benjaafar et al. 2007, Cachon and Zhang 2007). However, a symmetric function is not service-maximizing when suppliers are heterogeneous.
supplier would increase his service level unlimitedly. This condition could prevent us from achieving a finite Nash equilibrium.

The implications of proposition 6 are rather remarkable. By simply manipulating the parameters of the allocation function, the buyer can orchestrate the competition so that each supplier, regardless of his efficiency, has the incentive to spend all his revenue to provide the maximum feasible service level. In addition, the allocation function \( \alpha_i(s^\tau) \) results in the predefined demand share \( \delta_i \) at the Nash equilibrium.

**Step 2: Characterizing the Optimal Competition Mechanism**

We now turn to the more general problem where \( p \) and \( \delta \) are no longer fixed, but are decision variables for the buyer. Since for any given set of \( p \) and \( \delta \), the buyer’s profit is increasing in all \( s_i, i = 1, \ldots, N \), we can conclude that it is always optimal for the buyer to use a service maximizing allocation function. Using a service maximizing allocation function, the buyer can then induce equilibrium service levels \( s_i^{\max} (\delta_i, p_i) \) which zero-out suppliers’ profit. That is,

\[
\pi_i^{SC} (s_i^{\max} (\delta_i, p_i)) = \delta_i \lambda \left[ p_i - c_i - f_i \left( s_i^{\max} (\delta_i, p_i) \right) / \lambda \delta_i \right] = 0,
\]

or equivalently

\[
p_i = c_i + f_i \left( s_i^{\max} (\delta_i, p_i) \right) / \lambda \delta_i. \tag{25}
\]

Therefore, when the buyer is using a service maximizing allocation function to induce the targeted set of demand shares \( \delta \), we can rewrite the buyer’s profit function as

\[
\pi^{BC}(\delta, p) = \lambda \sum_{i=1}^N \delta_i \left[ h \left( s_i^{\max} (\delta_i, p_i) \right) - \left[ c_i + f_i \left( s_i^{\max} (\delta_i, p_i) \right) \right] / \lambda \delta_i \right]. \tag{26}
\]

This profit function has a similar form to the profit function of the take-it-or-leave-it contract (8), with \( s_i^{\max} (\delta_i, p_i) \) substituted for \( \sigma_i (\delta_i) \). Note that we can set the value of \( s_i^{\max} (\delta_i, p_i) \) by choosing the proper value of \( p_i \). Proposition 8 uses this similarity to characterize the optimal set of \( p \) and \( \delta \).

**Proposition 8:** The buyer’s problem under a competition contract (20)-(23) can be solved by using a service maximizing allocation function along with optimal sets of procurement prices, \( p^* = (p_1^*, \ldots, p_N^*) \), and demand shares, \( \delta^* = (\delta_1^*, \ldots, \delta_N^*) \), where \( p_i^* = c_i + f_i \left( s_i^* (\delta_i^*), \delta_i^* \right) / \delta_i^* \lambda \), for \( i = 1, \ldots, N \), and \( \delta^* \) and \( s^* \) are as defined in proposition 1. Moreover, \( s_i^{\max} (\delta_i^*, p_i^*) = s_i^* \), for \( i = 1, \ldots, N \).

This immediately implies the following corollary.

**Corollary 1:** The solution to the buyer’s competition problem, as defined in proposition 7, extracts all profits for the buyer, \( \pi_i^{SC} (s_i^*) = 0, i = 1, \ldots, N \). Furthermore, when the optimal demand shares result in an allocation to more than one supplier, the solution maximizes channel profit.
When the capacity of the most efficient supplier is sufficient to fulfill the buyer’s entire demand, we cannot directly identify a service-maximizing allocation function that can induce $\delta^*_i = 1, \delta^*_i = 0, i = 2, \ldots, N$, since this is inconsistent with the requirement of $\delta_i > 0$ for the service-maximizing allocation functions introduced in proposition 7.

5. Systems with Restrictions on Procurement Prices and Allocation Functions

We have shown that the take-it-or-leave-it, incentive, and competition contract mechanisms can each be designed to achieve the maximum feasible profit for the buyer and the channel as a whole. These results require that the buyer has full power to set the allocation rule, procurement prices, and incentive terms and, if necessary, to make these supplier-specific. In practice, this is not always the case. Regulated markets might prevent the buyer from discriminative treatments of different suppliers. In this case, the buyer would have to offer the same terms to all suppliers regardless of their capabilities (e.g., regardless of differences in cost efficiencies or capacity levels). In some industries, the buyer might lack the power to set the procurement prices (e.g., procurement prices are set exogenously by the market or by a handful of powerful suppliers). In some settings, simple allocation schemes might also be the accepted norm because of their simplicity, widespread use, or perceived fairness.

In this section, we investigate how such restrictions would change the buyer’s optimal decision under the three contract mechanisms. We do so in the context of a specific application setting which we describe below. This is an extension of an example originally introduced in Benjaafar et al. (2007), but where we now allow suppliers to be heterogeneous in cost and capacity. Although the numerical results we present are specific to this example application, the general insights apply more broadly to other applications that fit our model assumptions.

Example description: Consider a system with one buyer and two suppliers. Demand arrives according to a Poisson process with mean $\lambda$ and the processing time at supplier $i$ is exponential with mean $1/\mu_i$. Each supplier’s service level is defined by his probability of meeting a given lead-time target $\tau$, i.e., $s_i = \Pr(W_i \leq \tau)$ where $W_i$ is a random variable representing lead-time for supplier $i$. Suppliers have the ability to increase their service level by investing in capacity at an amortized capacity cost of $k_i$ per unit of
service rate. In addition, supplier $i$ incurs production cost $c_i$ for each item produced. The above implies that the suppliers’ service cost functions are given by

$$f_i(s_i, \delta_i) = k_i \lambda \delta_i + v_i(s_i) = k_i \lambda \delta_i - k_i \ln[1/(1 - s_i)]/\tau, \ i = 1,\ldots, N,$$

where $s_i = \Pr(W_i \leq \tau) = 1 - e^{-(\mu - \lambda) \tau}$, see Benjaafar et al. (2007) for further details and motivation. The buyer’s revenue function is given by the following relatively general quadratic concave function:

$$h(s_i) = p_i - t(1 - s_i)^2,$$

where $p_i$ and $t$ are positive constants.

Scenario A: Non-Differentiated Procurement Prices and Incentives

When the buyer has the power to set the procurement prices and incentives, but cannot tailor them to specific suppliers, the buyer’s decision vectors for each mechanism (in terms of procurement prices and incentive terms) collapse to single variables. The impact of this restriction on the performance of each mechanism depends on the level and type of heterogeneity across the supply base. Consider the cases outlined in section 2. If there are no binding capacity constraints (case 1), only the most efficient suppliers are relevant. In this case both incentive and take-it-or-leave-it contracts can still provide an optimal solution. However, we know from Corollary 1 that the competition mechanism will achieve a lower profit level in this case.

When the optimal solution contains binding capacity constraint(s), the performance of the mechanisms depends on the type and level of supplier heterogeneity. Figure 2 illustrates how the mechanisms compare when the suppliers have different capacity levels but homogeneous costs (case 2). For this example, we assume $\tau = 1, \ p_i = 1900, \ t = 1000, \ c_1 = c_2 = 450, \ k_1 = k_2 = 500$, and vary the ratio of capacity levels while keeping the total capacity available equal to demand, $w_1 + w_2 = \lambda = 50$. In general, performance degrades with higher levels of heterogeneity for all three mechanisms. However, the impact is less severe under the incentive contract.
The optimal solution in figure 2 reflects the solution achieved when the buyer has full flexibility in customizing the contract terms by supplier (differentiated $p$ and $d$). This serves as an upper bound on performance and a benchmark for measuring the impact of using non-differentiated contract terms. The other lines on the graph illustrate the performance of our three mechanisms under non-differentiated terms, with the terms including either identical procurement prices or both identical procurement prices and incentive terms (in the case of the incentive contract). The incentive contract clearly outperforms the other two mechanisms for all capacity ratios. This holds true regardless of whether the incentive terms are also forced to be identical for all suppliers. While performance for the incentive contract does degrade with the capacity ratio, the impact is fairly minor (a 0.02% reduction in performance when capacities vary by a factor of 4 in this example). In contrast, the performance of the competition and take-it-or-leave-it contract degrade at a faster rate with the capacity ratio (a performance reduction of 1% for a capacity ratio of 4 in this example). The reason an incentive contract yields higher buyer profit is in that this contract includes an extra payment transfer term, the incentive payment, which gives the buyer more power to induce service levels. Since the form of heterogeneity is in capacity rather than cost, and the fact that the buyer can directly dictate the demand shares, the optimal parameter values of the incentive contract do not vary significantly by supplier.
The competition and take-it-or-leave it contracts, in contrast, significantly degrade in performance as the difference in the suppliers’ capacity levels increase. Both mechanisms continue to induce the suppliers to fully invest in service level, resulting in zero profit for the suppliers. However, there is not sufficient leverage to optimally differentiate service levels since the suppliers cannot be rewarded based on their relative capacity levels (and thus differences in economies of scale).

We see a similar phenomenon when suppliers are heterogeneous in their cost structures rather than their capacity levels (case 3). Figure 3 illustrates how the previous example changes when \( w_1 = w_2 = 25 \), \( c_1 = \$450 \), and the ratio of service cost parameters \( c_2 / c_1 \) varies. Once again, the incentive contract with equal procurement prices performs reasonably well, while the gap in performance for the competition and take-it-or-leave-it contract mechanisms increases as the cost differential between suppliers grows. Surprisingly, the incentive contract with both identical procurement prices and incentive terms now performs much worse than when only the procurement prices are constrained. Its performance is now similar to the competition and take-it-or-leave it mechanisms. This degradation in performance occurs because the incentive contract now needs the flexibility of setting different incentive terms among the suppliers to counter the effect of the suppliers’ different production costs. The buyer cannot set the single incentive term high enough to extract all of the most efficient supplier’s profit, while also allowing the

![Figure 3 – Performance of mechanisms: heterogeneous costs and homogeneous capacities (case 3)](image-url)
less cost efficient supplier to maintain his reservation profit. This results in lower profit for the buyer but positive profit for the more efficient supplier. In fact, this is the only example (of all the examples illustrated in figures 2 and 3) where the buyer does not extract all the supply chain profit.

Scenario B: Exogenous Procurement Prices

Now suppose that rather than constraining the buyer to choose identical contract parameters for all suppliers, some subset of these parameters is dictated by the market. For example, in some settings the procurement prices may be set exogenously by market mechanisms, while other terms of the contract are more flexible. Figure 4 shows how the buyer’s expected profit varies across the three contract mechanisms for different exogenously determined procurement prices when suppliers are identical (i.e., \( w_1 = w_2 = 25, c_1 = c_2 = 450, k_1 = k_2 = 500 \)). Each point in the figure represents the optimal solution for the buyer given the mechanism and dictated procurement price.

We know from the results in the previous sections that there exists an optimal procurement price for each mechanism where the resulting profit for the buyer and supply chain is maximized. These prices are indicated in the figure. The optimal procurement price for the incentive contract is lower than the others since the incentive contract provides the suppliers with both a revenue payment (through the procurement price) and an incentive payment, while the competition and take-it-or-leave-it mechanisms only provide a revenue payment.

More interesting is how the buyer’s profit changes as the prices move away from these optimal values. Under the incentive contract, performance is impacted more severely if the procurement prices are set too high rather than too low. Figure 4 shows that the buyer is better off under the incentive contract when procurement prices are low. However, if the price sufficiently increases, the competition and take-it-or-leave-it mechanisms provide higher profits for the buyer. For the competition and take-it-or-leave-it mechanisms, there is also a minimum procurement price \( p = c + k \) required for the suppliers to engage in these mechanisms. If the procurement price is set below this minimum, the suppliers will earn negative profit and so will not accept the buyer’s contract.

Figure 5 illustrates how the comparison differs when suppliers are heterogeneous in cost. In this example, the ratio of costs is 500/450 = 1.11. As noted previously, the competition and take-it-or-leave-it mechanisms are less effective in this environment; an optimal solution for the buyer is no longer possible for any procurement price. We also know from previous discussion that the effectiveness of the incentive
contract under heterogeneous costs is impacted more by constraints on the incentive term than by constraints on the procurement prices. So, it is not surprising that the performance of the incentive contract is not impacted substantially. However, if the incentive term were also determined exogenously, the impact would be more significant.

Scenario C: Proportional Allocation Rule
The buyer might also have constraints on the choice of allocation function in a competition mechanism. The optimal allocation defined by proposition 7 is an asymmetric function which discriminates between suppliers in the way it assigns demand shares in order to maximize the intensity of the competition. This discrimination is based on the suppliers’ cost structures and capacity differences. While this approach is optimal for the buyer and the channel as a whole, it may be resisted by the suppliers either because of its complexity or its ability to discriminate. In this section, we consider the impact of using a much simpler service-proportional allocation function that has been widely studied in the literature (see Cachon 2003 for a review).

Figure 4 – The impact of exogenous procurement prices on buyer’s profit: homogeneous costs
The service proportional allocation function is defined as follows:

\[ \alpha_i(s_i, s_{-i}) = s_i / \sum_{j=1}^{N} s_j. \]

This function is simple to communicate and does not discriminate by a supplier’s relative costs or capacity level. The function is symmetric in the sense that each supplier receives a demand share proportional to the service level the supplier provides.

Figure 5 compares the performance of optimal versus service proportional allocation functions under different levels of supplier heterogeneity in production cost when \( k_1 = k_2 = 500, \ w_1 = w_2 = 40, \ \lambda = 50, \ \tau = 1, \ p_s = 2980, \ t = 2980 \) and \( c_1 = 450 \). The benefit of using an optimal allocation can be substantial and increases with the difference in production costs across the two suppliers. Using a service proportional allocation function prevents the buyer from extracting all the channel profit, leaving suppliers with positive profits. In this case, the total profit of the channel is also suboptimal. Since the service proportional allocation function is a symmetric allocation function, it cannot take into account differences in the suppliers’ cost structures. As a result, this allocation function induces a lower level of competition intensity as the suppliers’ costs diverge.
6. Conclusion

Our goal in this paper was to analyze the performance of three common procurement mechanisms in an environment where suppliers are heterogeneous in their cost structures and capacity levels, and where the buyer’s revenue is dependent on the services levels provided by the chosen suppliers. While the three mechanisms differ in their form, we showed that each is capable of maximizing the buyer’s profit, making them equivalent in performance. How this solution is obtained varies across the mechanisms. We outlined specific procedures for setting the optimal contract parameters and allocation functions when the buyer has authority over these decisions.

These results extend prior research on service-base procurement contracts by offering a more general setting for comparing between mechanisms. Our focus on heterogeneous suppliers led to the development of a new measure of supplier efficiency, reflecting cost and capacity capabilities, which can be used to rank and select suppliers. Also, by formulating the problem as one of optimizing profit, rather than optimizing indirect measures such as service level (as is typical in service-based contract studies, e.g., Benjaafar et al. (2007) and Cachon and Zhang (2007), the benefits of higher service levels are more naturally balanced against higher costs. This approach allowed us to define optimal forms for the three mechanisms where both the buyer and channel profit is optimized.
For the supplier competition mechanism, we showed how allocation functions can be designed to induce suppliers to provide the maximum feasible service level regardless of whether or not there is flexibility in pricing and regardless of the heterogeneity in the cost structures of the suppliers and their capacities. Moreover, in settings where it is desirable to set the demand allocation in a specific way, we showed that it is possible to design the allocation function to induce this desired allocation as an outcome of the competition.

Our general approach also allowed us to capture important insights into how performance of the mechanisms changes when the buyer does not have flexibility of setting either demand allocation or payments. Through numerical examples, we showed that when the buyer can decide on the value of procurement prices and incentive payments but cannot customize them for different suppliers, then incentive contracts generally perform better than the other two mechanisms. When the procurement prices are set exogenously, the mechanism which provides higher buyer profit depends on the exogenously set procurement price. The buyer is better off using an incentive contract if the procurement price is relatively low, while she is better off using a take-it-or-leave-it contract or competition when the price is relatively high.

There are several possible avenues for future research. We know from the results of this study that the ability to differentiate contract terms is critical in some settings. However, as discussed in section 5, full differentiation of the form implied by our optimal solutions is not always feasible. A possible alternative might be to divide the suppliers into groups and offer the same contract terms within a group (e.g., suppliers located in the same region or country). A group-based strategy raises new questions for how the buyer should design each procurement scheme, how the performance of each scheme is affected by varying levels of grouping, and how degrees of similarity or dissimilarity in cost and capacity between groups (and between members of each group) affect buyer and suppliers’ profit.

Our analysis currently assumes that the buyer’s revenue increases with service quality while the demand rate is fixed. If demand were also an increasing function of service, the suppliers’ incentive to invest in service would change since an increase in service could increase the supplier’s allocation as well as the overall demand level available to all suppliers. It would be interesting to study how this relationship between customer demand and service, including the possibility of free riding by suppliers, might change the relative performance of the three mechanisms.
Finally, although the three mechanisms we studied are equivalent when there is full flexibility, there is arguably a practical advantage to achieving optimality via a competition that induces suppliers to voluntarily provide the maximum service level and allows the optimal demand allocations to arise naturally as an outcome of the competition. There is perhaps also an advantage to carrying out this competition via a demand allocation function that, once announced, does not require further intervention on the part of the buyer. In contrast to take-it-or-leave-it or incentive contracts, the competition does not directly dictate how much demand is allocated to each supplier but rather achieves a desired allocation through the suppliers’ own decisions. In this sense, competition provides the suppliers with more transparency into how demand allocation decisions are made. In practice, take-it-or-leave-it or incentive contracts may be more susceptible to bargaining on the part of the suppliers, leading to the renegotiating of the procurement terms to the detriment of the buyer. It would be interesting to empirically test these hypotheses by identifying settings in practice where the result may or may not hold true. These hypotheses could also be tested through a series of controlled experiments with human subjects taking on the role of suppliers making service investment decisions under the different mechanisms.

References


Appendix

**Proof of Proposition 1.**

We can rewrite the buyer’s profit function as

\[ \pi^B(s, \delta, p) = \sum_{i=1}^{N} \delta_i \lambda \left[ h(s_i) - p_i \right] \]  \hspace{1cm} (A1)

It is easy to see that for any set of \( \delta \) and \( s \), the maximum buyer profit can be achieved by reducing the procurement prices to a level which results in reservation profits for suppliers (zero in our discussion). That is the optimal procurement prices for any set of \( \delta \) and \( s \) is

\[ p_i^*(\delta, s) = (c_i + k_i) + v_i(s_i) / \delta_i \lambda . \]  \hspace{1cm} (A2)

By replacing (A2) into (A1), the buyer’s problem can be rewritten as

\[ \max_{s, \delta} \pi^B(s, \delta) = \sum_{i=1}^{N} \delta_i \lambda \left[ h(s_i) - c_i - k_i - v_i(s_i) / \delta_i \lambda \right] \]  \hspace{1cm} (A3)

For any given set of demand allocations \( \delta \), each term of the summation in (A3) will be maximized if each supplier uses a unique optimal service level, \( \sigma_i(\delta_i) \), for the allocated demand to that supplier. This optimal service level is

\[ \sigma_i(\delta_i) = \begin{cases} \arg \max_x \left[ h(x) - (c_i + k_i + v_i(x) / \delta_i \lambda) \right] & \text{if } \delta_i > 0 \\ 0 & \text{otherwise} \end{cases}, \quad i = 1, \ldots, N \]  \hspace{1cm} (A4)

\( \sigma_i(\delta_i) \) returns a unique solution since \( h(x) - (c_i + k_i + v_i(x) / \delta_i \lambda) \) is convex in \( x \). Therefore, the buyer’s problem can be simplified to

\[ \max_{\delta} \pi^C(\sigma(\delta), \delta) = \lambda \sum_{i=1}^{N} \delta_i \left[ h(\sigma_i(\delta_i)) - (c_i + k_i) \right] - v_i(\sigma_i(\delta_i)), \]  \hspace{1cm} (A5)

subject to constraints (5)-(7). This objective function is finite over the compact domain which is defined in the constraints. Therefore, we always have a finite solution for the problem. If a solution to this problem is \( \delta^* = (\delta_1^*, \ldots, \delta_N^*) \), then the optimal service levels and procurement prices will naturally be \( s_i^* = \sigma_i(\delta_i^*) \) and \( p_i^* = c_i + f_i(s_i^*, \delta_i^*) / \lambda \delta_i^*, \quad i = 1, \ldots, N \).

**Proof of Proposition 2.**

In the proof of proposition 1, we showed that the optimal procurement price guarantees zero profit for each supplier. On the other hand, it is easy to verify that the channel profit function will be the same as the profit function in (A3). Therefore, the optimal channel profit and the optimal take-it-or-leave-it contract will have result in the same profit level.
Proof of Proposition 3

We can rewrite the profit function in (8) as

\[ \pi^B(\sigma(\delta), \delta) = \lambda \sum_{i=1}^{N} \delta_i \left[ h(\sigma_i(\delta_i)) - (c_i + k_i + v_i(\sigma_i(\delta_i))/\delta_i\lambda) \right] \]

\[ \leq \lambda \sum_{i=1}^{N} \delta_i \left[ h(\sigma_i(\delta_i)) - (c_i + k_i + v_i(\sigma_i(\delta_i))/\delta_i\lambda) \right] = \lambda \sum_{i=1}^{N} \delta_i e_i \leq \lambda e_i \quad (A6) \]

The first inequality is achieved by virtue of lemma 1 below. The second inequality is due to the fact that any weighted average of a set of numbers is always less than or equal to the largest number in that set. Since the term \( \sum_{i=1}^{N} \delta_i e_i \) is in fact the weighted average of suppliers’ efficiencies, it is always smaller than the efficiency of the most efficient supplier, i.e. supplier 1. This means that the maximum feasible buyer’s profit can be achieved when the whole demand is allocated to the most efficient supplier with a capacity higher than the buyer’s demand.

Lemma 1. Let \( \phi(s_i, \delta_i) = h(s_i) - (c_i + k_i + v_i(s_i)/\delta_i\lambda) \) and let \( s_i = \sigma_i(\delta_i) \) be the value of \( s_i \) which maximizes \( \phi(s_i, \delta_i) \) for any value of \( \delta_i \), that is \( \sigma_i(\delta_i) = \arg\max_x \left[ h(x) - (c_i + k_i + v_i(x)/\delta_i\lambda) \right] \) for \( \delta_i > 0 \). Then \( \phi(\sigma_i(\delta_i), \delta_i) \leq \phi(\sigma_i(\delta_i), \delta_i) \).

Proof of Lemma 1.

We know \( \delta_i \leq \delta_i \). Therefore, \( \phi(s_i, \delta_i) \leq \phi(s_i, \delta_i) \) for any \( s_i \). As a result,

\[ \phi(\sigma_i(\delta_i), \delta_i) \leq \phi(\sigma_i(\delta_i), \delta_i) \quad \text{(A7)} \]

We also have

\[ \frac{\partial \phi(s_i, \delta_i)}{\partial s_i} = \frac{\partial h(s_i)}{\partial s_i} - \frac{1}{\delta_i\lambda} \frac{\partial v_i(s_i)}{\partial s_i} \]

Which means \( \partial \phi(s_i, \delta_i)/\partial s_i \) is increasing in \( \delta_i \). Hence for any \( s_i \),

\[ \frac{\partial \phi(s_i, \delta_i)}{\partial s_i} \leq \frac{\partial \phi(s_i, \delta_i)}{\partial s_i} \quad \text{(A8)} \]

Therefore,

\[ \frac{\partial \phi(s_i, \delta_i)}{\partial s_i} \bigg|_{s_i = \sigma_i(\delta_i)} \leq \frac{\partial \phi(s_i, \delta_i)}{\partial s_i} \bigg|_{s_i = \sigma_i(\delta_i)} \quad \text{(A9)} \]

However, since \( s_i = \sigma_i(\delta_i) \) maximizes the concave function \( \phi(s_i, \delta_i) \), we have \( \frac{\partial \phi(s_i, \delta_i)}{\partial s_i} \bigg|_{s_i = \sigma_i(\delta_i)} = 0 \) and therefore, \( \frac{\partial \phi(s_i, \delta_i)}{\partial s_i} \bigg|_{s_i = \sigma_i(\delta_i)} \geq 0 \). In other words, \( \phi(s_i, \delta_i) \) is increasing at \( s_i = \sigma_i(\delta_i) \). Again, since \( \phi(s_i, \delta_i) \) is unimodal and concave in \( s_i \), we can conclude that the value of \( s_i \) which maximizes \( \phi(s_i, \delta_i) \) is larger than \( \sigma_i(\delta_i) \). That is,

\[ \sigma_i(\delta_i) \geq \sigma_i(\delta_i) \quad \text{(A9)} \]

From (A7), (A8), (A9) we can conclude that \( \phi(\sigma_i(\delta_i), \delta_i) \leq \phi(\sigma_i(\delta_i), \delta_i) \). Figure (A1) helps to visualizes this proof.
Proof of Proposition 4.

When $\omega \geq \lambda$, by virtue of proposition 3, the optimal allocation is

$$\delta^*_i = \begin{cases} \omega_i & \text{for } i = 1, \ldots, N, \\ 0 & \text{otherwise} \end{cases}$$

which is consistent with (11). Here we present the proof for the case where $\omega < \lambda$. Since the suppliers have identical cost structures, the difference in suppliers’ efficiencies is solely due to difference in their capacities. Therefore, when we rename suppliers according to their descending order of their efficiencies, in fact, it is according to their descending order of their capacities. Hence we have $\delta_1 \geq \delta_2 \geq \ldots \geq \delta_N$.

Then we can rewrite (11) as

$$\delta^*_i = \begin{cases} \delta_i & \text{for } i = 1, \ldots, \hat{N} - 1 \\ 1 - \sum_{i=1}^{\hat{N}-1} \delta_i & \text{for } i = \hat{N}, \ldots, N \\ 0 & \text{for } i > \hat{N} \end{cases} \quad (A10)$$

Let $\psi(\delta_i) = \delta_i \lambda [h(\sigma(\delta_i)) - c - \mu] - \nu(\sigma(\delta_i))$. Hence we have $\bar{\pi}^B (\hat{\delta}) = \pi^B (\sigma(\delta), \hat{\delta}) = \sum_{i=1}^{N} \psi(\delta_i)$. Notice that $\bar{\pi}^B (\hat{\delta})$ is convex in $\delta$ if and only if $\psi(\delta_i)$ is convex in $\delta_i$. Here, we propose a procedure through which we can build $\delta^* = (\delta_1^*, \ldots, \delta_N^*)$ from any feasible initial set of $\delta_1, \ldots, \delta_N$. We then show that we always have $\bar{\pi}^B (\hat{\delta}^*) \geq \bar{\pi}^B (\hat{\delta})$, which proofs the proposition. At each round of this procedure, we modify only two elements of allocation vector until we achieve $\delta^*$.

Step 1: $k = 0; \quad i = 1; \quad j = N$

Step 2: $\delta^k_i = \delta^*_i \quad \text{for } t = 1, \ldots, N$

Step 3: $\Delta = \min(\delta^k_i, \delta^*_i - \delta^k_j)$

Step 4: $\delta^{k+1}_i = \delta^k_i + \Delta; \quad \delta^{k+1}_j = \delta^k_j - \Delta; \quad \delta^{k+1}_i = \delta^k_i \quad \text{for } t \neq i, j$
Note: By virtue of Lemma 2 below, we have \( \tilde{\pi}^B(\delta^{k+1}) \geq \tilde{\pi}^B(\delta^k) \), where \( \delta^k = (\delta_1^k, \ldots, \delta_N^k) \).

Step 5: if \( \delta_i^{k+1} = \tilde{\delta}_i \) then \( i = i + 1 \)

Step 6: if \( \delta_i^{k+1} = 0 \) then \( j = j - 1 \);

Step 7: if \( i < j \) then \( k = k + 1 \) and return to step 3

else \( \delta_i^* = \delta_i^{k+1} \) for \( t = 1, \ldots, N \); end of procedure;

In any round of this procedure we remove all or part of the allocation from a low capacity supplier and add the same amount of allocation to a higher capacity supplier. By virtue of Lemma 2, this reallocation results in higher centralized profit. We repeat this procedure until we cannot reallocate demand from a lower capacity supplier to a higher capacity supplier. It is not difficult to see that at the end of this procedure we have the allocation stated in (11). Since \( \tilde{\pi}^B(\delta^*) \geq \tilde{\pi}^B(\delta) \) for any \( \delta \), we can conclude that \( \delta^* \) is the optimal allocation. This concludes the proposition.

**Lemma 2:** If \( \tilde{\delta}_i \geq \tilde{\delta}_j \) then \( \psi(\delta_i^*) + \psi(\delta_j^*) \leq \psi(\delta_i + \Delta) + \psi(\delta_j - \Delta) \) where \( \Delta = \min(\delta_j^*, \tilde{\delta}_i - \delta_i^*) \).

**Proof of Lemma 2.**

From the definition of \( \Delta \) we have:

\[
\delta_i + \Delta = \min(\tilde{\delta}_j + \delta_i, \tilde{\delta}_i) \quad \text{and} \quad \delta_j - \Delta = \max(0, \delta_j - (\tilde{\delta}_i - \delta_i)).
\]

Since \( \tilde{\delta}_i \geq \tilde{\delta}_j \geq \delta_j \geq 0 \) we have \( \delta_i + \Delta = \delta_i \) and \( \delta_j - \Delta = \delta_j \).

To prove this lemma we consider the following two possible cases:

1. \( \tilde{\delta}_i \geq \tilde{\delta}_j \)

   \[
   \Delta^* = (\delta_i + \Delta) - \delta_i = \delta_j - (\delta_i - \Delta) = \Delta
   \]

   In this case let \( d_1 = \delta_j - \Delta; \quad d_2 = \tilde{\delta}_j; \quad d_3 = \delta_i; \quad d_4 = \tilde{\delta}_i + \Delta. \)

2. \( \tilde{\delta}_i < \tilde{\delta}_j \)

   \[
   \Delta^* = (\delta_i + \Delta) - \delta_j = \tilde{\delta}_i - (\delta_j - \Delta)
   \]

   In this case let \( d_1 = \delta_j - \Delta; \quad d_2 = \delta_i; \quad d_3 = \tilde{\delta}_j; \quad d_4 = \tilde{\delta}_i + \Delta. \)

Since \( \psi(.) \) is a convex function, we have \( \psi'(d_1) \leq \psi'(d_2) \leq \psi'(d_3) \leq \psi'(d_4) \) where \( \psi'(x) = \frac{\partial \psi(x)}{\partial x} \).

Again from the convexity of \( \psi(.) \) we can conclude that

\[
\frac{\frac{\psi(d_2) - \psi(d_1)}{\Delta^*}}{\psi'(d_2)} \leq \psi'(d_3) \leq \frac{\frac{\psi(d_4) - \psi(d_3)}{\Delta^*}}{\psi'(d_4)}
\]
Therefore,
\[ \psi(d_2) - \psi(d_1) \leq \psi(d_4) - \psi(d_3) \implies \psi(d_2) + \psi(d_3) \leq \psi(d_1) + \psi(d_4) \quad \text{(A11)} \]

It is easy to verify that, for both cases (1) and (2), the inequality in (A11) is equivalent to
\[ \psi(\delta_i) + \psi(\delta_j) \leq \psi(\delta_i +\Delta) + \psi(\delta_j - \Delta) \]

This concludes the proof of the lemma.

**Proof of Proposition 5**

For any \( d_i \) and \( \delta_i \) chosen by the buyer and any \( s_i \) chosen by the supplier \( i \), the buyer can maximize his profit by decreasing the procurement price \( p_i \) to a level which leaves the supplier with its reservation profit (zero in our case). That is,
\[ p_i^*(s_i, \delta_i) = c_i + k_i + \frac{v_i(s_i) - d_is_i}{\delta_i} \cdot \]

Therefore, the buyer’s profit from supplier \( i \) can be rewritten as
\[ \pi_i^{SI}(s_i, \delta_i) = \delta_i \lambda [h(s_i) - c_i - k_i - v_i(s_i) / \delta_i \lambda] \]

In this form, the buyer’s profit depends only on \( \delta_i \) and the service level chosen by the supplier. The value of supplier’s service level which maximizes the buyer’s profit then will be
\[ s_i^* = \arg \max_x [h(x) - c_i - k_i - v_i(x) / \delta_i \lambda] \]

Now, the buyer has to choose the value of \( d_i \) in a way that gives the supplier the proper incentive to provide a service level equal to \( s_i^* \).

On the other hand the supplier always chooses his service level to maximize his own expected profit, which can be written as
\[ \pi_i^{SL}(s_i) = \delta_i \lambda [p_i - c_i - k_i - v_i(s_i) / \delta_i \lambda] + d_is_i \]

Knowing the supplier’s profit function is concave in \( s_i \), the value of service level which maximizes this profit function can found as follows
\[ \frac{\partial \pi_i^{SI}(s_i)}{\partial s_i} = -\frac{\partial v_i(s_i)}{\partial s_i} + d_i = 0 \implies d_i = \frac{\partial v_i(s_i)}{\partial s_i} \]

Therefore, if the buyer chooses \( d_i^* = \left. \frac{\partial v_i(s_i)}{\partial s_i} \right|_{s_i = s_i^*} \), then the supplier provides \( s_i = s_i^* \) which in turn maximizes the buyer’s profit. To summarize our proof, the buyer achieves the maximum profit by choosing the procurement price and incentive term as follows.
\[ d_i^* = \left. \frac{\partial v_i(s_i)}{\partial s_i} \right|_{s_i = s_i^*} \quad \text{and} \quad p_i^* = c_i + \frac{f_i(s_i^*, \delta_i) - d_i^*s_i^*}{\lambda \delta_i} \]
In which \( s_i^* = \arg \max_x \left[ h(x) - c_i - k_i - v_i(x) / \delta_i \lambda \right] \). These values induce a service level equal to \( s_i^* \) at the supplier and results in zero profit for him. This concludes the proof of proposition 5.

**Proof of Proposition 6**

We know from proposition 5 that for any chosen \( \delta_i > 0 \) the optimal values \( d_i^* \) and \( p_i^* \) induces a service level \( s_i^* = \arg \max_x \left[ h(x) - c_i - f_i(x, \delta_i) / \delta_i \lambda \right] \), which in turn results in \( \pi_i^*(s_i^*) = 0 \). Therefore,

\[
\pi_i^*(s_i) = \delta_i \lambda (p_i - c_i - k_i) - v_i(s_i) + d_i s_i = 0 \Rightarrow \delta_i \lambda p_i + d_i s_i = \delta_i \lambda c_i + f_i(s_i, \delta_i).
\]

By replacing this equation into buyer’s profit function, we have

\[
\pi^{bi}(d^*, \delta, p^*) = R(d^*, \delta) - \sum_{i=1}^N (\delta_i \lambda c_i + f_i(s_i^*, \delta_i)) = \lambda \sum_{i=1}^N \left[ h(s_i^*) - (c_i + f_i(s_i^*, \delta_i) / \delta_i \lambda) \right].
\]

Noticing that \( s_i^* \) is basically equal to \( \sigma_i(\delta_i) \) as defined in section 2, we can see that the above buyer’s profit function is equivalent to the buyer’s profit function in proposition 1. Therefore, the optimal set of demand share which maximizes the buyer’s profit will also be the same as optimal demand shares in proposition 1. This means that the optimal incentive contract provides the maximum feasible profit for the buyer which is the same as the buyer’s profit under an optimal take-it-or-leave-it contract.

**Proof of Proposition 7**

We first prove part (c) of the theorem. Let us write the profit functions of the suppliers in terms of \( y_i = g_i(s_i) = \delta_i \theta_i(s_i)^{1/(1 - \delta_i)} \). That is,

\[
\pi_i(y_i, y_{-i}) = \frac{y_i}{G} \lambda r_i \lambda r_i \delta_i y_i^{(1 - \delta_i)} \quad \text{(A12)}
\]

Where \( G = \sum_{k=1}^N y_k \) and \( r_i = p_i - c_i - k_i \). The constraint that all suppliers provide positive service levels is equivalent to \( y_i > 0, i = 1, \ldots, N \). Since \( y_i > 0 \), a Nash equilibrium point must satisfy the first order optimality condition. That is, in order to prove part (c) of the theorem, we need to show that the following system of \((N+1)\) equations with unknowns \( y_i, i = 1, \ldots, N \) and \( G \) has a unique solution that maximizes (A12).

\[
\frac{\partial \pi_i(y_i, y_{-i})}{\partial y_i} = \frac{G - y_i}{G^2} \lambda r_i \lambda r_i \delta_i y_i^{1 - \delta_i} - \delta_i y_i^{1 - \delta_i} = 0, \quad i = 1, \ldots, N, \quad \text{and} \quad G = \sum_{k=1}^N y_k, \quad \text{(A13)}
\]

or, equivalently,

\[
Y_i^{\delta_i} (1 - Y_i) - \delta_i \delta_i (1 - \delta_i) G^{1 - \delta_i} = 0, \quad i = 1, \ldots, N, \quad \text{and} \quad \sum_{k=1}^N Y_k = 1, \quad \text{(A14)}
\]

where \( Y_i = y_i / G \). We can rewrite the first order optimality conditions as

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\[ D(Y_i) = \delta_i^4 (1 - \delta_i) G^{1 - \delta_i}, \quad i = 1, \ldots, N, \quad \sum_{k=1}^N Y_k = 1, \]  \tag{A16}

where \( D(Y_i) \equiv Y_i^\delta (1 - Y_i) \). We can see that \( D(0) = D(1) = 0, \ D(Y_i) > 0 \) for \( 0 < Y_i < 1 \), and \( D(Y_i) < 0 \) for \( Y_i > 1 \). Also, \( D(Y_i) \) has a maximum at \( \delta_i / (1 + \delta_i) \) which is equal to \( \delta_i^4 / (1 + \delta_i)^{1 + \delta_i} \). Hence, for any \( G < G_{\text{max}} = \left( 1 / (1 - \delta_i)(1 + \delta_i)^{1 + \delta_i} \right)^{1 - \delta_i} \), equation (A16) has two solutions \( 0 < Y_{i,1}, Y_{i,2} < 1 \). We want to argue that \( Y_{i,1} \) corresponds to a local minimum for supplier \( i \)’s profit function, (A12). We observe that the sign of the derivative of the profit function of supplier \( i \), equation (A14), changes from negative to positive when we increase \( Y_i \) from values smaller than \( Y_{i,1} \) to values bigger than \( Y_{i,1} \). We also observe that, for fixed decisions of the other suppliers, \( Y_i = y_i / \left( y_i + \sum_{j \neq i} y_j \right) \) is increasing in \( y_i \). Therefore, when we increase \( y_i \), the sign of equation (A14) changes from negative to positive at \( y_i = \left( Y_{i,1} / (1 - Y_{i,1}) \right) \sum_{j \neq i} y_j \), which in turn means that \( Y_{i,1} \) corresponds to a local minimum of supplier \( i \)’s profit function. Thus, \( Y_{i,1} \) cannot be a feasible solution. Similarly, we can show that \( Y_{i,2} \) corresponds to a local maximum of supplier \( i \)’s profit function. Therefore, \( Y_{i,2}, \quad i = 1, \ldots, N \) is the unique solution to the system of equations (A16) which corresponds to the values of \( y_i \) that maximize the profit functions of the suppliers. Figure A2 graphically illustrates the above argument.

Lemma A2 below shows that \( G_{\text{max}} > 1 \) for any \( 0 < \delta_i < 1 \). For \( G = 1 \), the only solution for equations in (A16) that maximizes profit function (A12) is \( Y_{i,2} = \delta_i \). It is easy to see that \( Y_{i,2} \) is decreasing in \( G \) (see figure A2). Therefore, for \( G < 1 \), we have \( Y_{i,2} > \delta_i \) or equivalently \( \sum_{i=1}^N Y_{i,2} > 1 \), which is not a feasible solution. Also, for \( G > 1 \), we have \( Y_{i,2} < \delta_i \) or equivalently \( \sum_{i=1}^N Y_{i,2} < 1 \), which is not a feasible solution as well. Therefore, \( Y_i = \delta_i \) and \( G = 1 \) (or equivalently \( y_i^* = \delta_i \)) is the unique solution of the system of equations (A14)-(A15) which maximizes each profit function in (A12), given all suppliers provide a positive service level.

![Figure A2 – The solution to first order optimality condition (A16)](image)

We now prove part (a) and (b) of the theorem. We showed that \( y_i^* = \delta_i \) satisfies the first order optimality condition (A13). If we release the positive service level constraint, \( y_i^* = \delta_i, \quad i = 1, \ldots, N \) is still a
Nash equilibrium since any supplier \( j \) cannot increase his profit by choosing \( y_j = 0 \) while other suppliers choose \( y_i^* = \delta_i \), \( i \neq j \). We can easily verify that \( y_i^* = \delta_i \) results in \( s_i^* = s_i^{\text{max}}(\delta_i, \lambda) \), \( \pi_i(s^*) = 0 \), and \( \alpha_i(s^*) = \delta_i \).

**Lemma 3:** For any \( 0 < x < 1 \), we have \( \left( \frac{1}{(1-x)(1+x)^{1+x}} \right)^{\frac{1}{(1-x)}} > 1 \).

**Proof of Lemma 3:** It is enough to prove that \( Z(x) = (1-x)(1+x)^{1+x} < 1 \) for any \( 0 < x < 1 \). We can see that \( Z(0) = 1 \) and \( Z(1) = 0 \). Therefore, it is enough to show that \( Z(x) \) is decreasing in the interval \((0, 1)\). The derivative of \( Z(x) \) can be written as

\[
\frac{dZ(x)}{dx} = (1+x)^{1+x} \left[ (1-x)(\ln(1+x)+1)-1 \right] = (1+x)^{1+x} \left[ X(x) - 1 \right],
\]

where \( X(x) = (1-x)(\ln(1+x)+1) \). Noticing that \( X(0) = 1 \), \( X(1) = 0 \), and

\[
\frac{dX(x)}{dx} = -(\ln(1+x)+1) + \frac{1-x}{1+x} < 0,
\]

we can conclude that \( X(x) \) is decreasing and less than 1 in \((0, 1)\). Hence, \( Z(x) \) is decreasing and less than 1 in \((0,1)\). This completes the proof of the lemma.

**Proof of Proposition 8**

We have already argued that for any given set of \( \delta \) and \( p \), using a service maximizing allocation function maximizes the buyer’s profit. A service maximizing allocation function always provides competitive incentive for each supplier to increase his service level to \( s_i^* = s_i^{\text{max}}(\delta_i, \lambda, p_i) \) which leaves him with zero profit. Therefore, the following relationship always holds between the procurement price and the demand allocation at a Nash equilibrium of a competition based on a service maximizing allocation function.

\[
\pi^SC(s_i^*, s_j^*) = \delta_i \lambda (p_i - c_i - k_i) - v_i(s_i^*) = 0 \quad \Rightarrow \quad p_i = c_i + k_i + v_i(s_i^*) / \delta_i \lambda \quad \text{for } i = 1, \ldots, N
\]

Replacing \( p_i \) in the buyer’s profit function from this relationship, we will have

\[
\pi^{bc}(\delta, p) = \sum_{i=1}^{N} \delta_i \lambda \left[ h(s_i^*) - c_i - k_i - v_i(s_i^*) / \delta_i \lambda \right].
\]

To maximize her profit, the buyer has to induce each supplier to provide a service level that maximizes \( h(s_i^*) - c_i - k_i - v_i(s_i^*) / \delta_i \lambda \) for the given \( \delta_i \). Therefore, the buyer should choose the procurement price such that \( s_i^* \) which is equal to \( s_i^{\text{max}}(\delta_i, \lambda, p_i) \), attain a value equal to

\[
\sigma_i(\delta_i) = \arg \max_x \left[ h(x) - c_i - k_i - v_i(x) / \delta_i \lambda \right].
\]

This means \( p_i^* = c_i + k_i + v_i(\sigma_i(\delta_i)) / \delta_i \lambda \) for \( i = 1, \ldots, N \). For a service maximizing allocation function, at the Nash equilibrium \( \alpha_i(s^*) = \delta_i \). Therefore, the buyer’s problem can be rewritten as
\[
\max_{\delta} \lambda \sum_{i=1}^{N} \delta_i \left[ h(\sigma_i(\delta_i)) - (c_i + k_i + v_i(\sigma_i(\delta_i)) / \delta_i, \lambda) \right],
\]
which is the same as the buyer’s optimal profit in a take-it-or-leave-it contract. Therefore, the optimal demand allocation will also be the same as the optimal demand allocation in the take-it-or-leave-it contract. This concludes the proof of the proposition.