

Carbon Footprint and the Management of Supply Chains: Insights from Simple Models

Saif Benjaafar¹ • Yanzhi Li² • Mark Daskin³

¹*Industrial and Systems Engineering, University of Minnesota,
Minneapolis, MN 55455, saif@umn.edu*

²*Department of Management Sciences, City University of Hong Kong,
Kowloon, Hong Kong, yanzhili@cityu.edu.hk*

³*Department of Industrial and Operations Engineering, University of Michigan,
Ann Arbor, MI 48106, msdaskin@umich.edu*

Abstract

Using relatively simple and widely used models, we illustrate how carbon emission concerns could be integrated into operational decision-making with regard to procurement, production, and inventory management. We show how, by associating carbon emission parameters with various decision variables, traditional models can be modified to support decision-making that accounts for both cost and carbon footprint. We examine how the values of these parameters as well as the parameters of regulatory emission control policies affect cost and emissions. We use the models to study the extent to which carbon reduction requirements can be addressed by operational adjustments, as an alternative (or a supplement) to costly investments in carbon-reducing technologies. We also use the models to investigate the impact of collaboration among firms within the same supply chain on their costs and carbon emissions and study the incentives firms might have in seeking such cooperation. We provide a series of insights that highlight the impact of operational decisions on carbon emissions and the importance of operational models in evaluating the impact of different regulatory policies and in assessing the benefits of investments in more carbon efficient technologies. In particular, we show that (1) it is possible (through operational adjustments only) to significantly reduce emissions without significantly increasing cost; (2) although different regulatory policies can achieve the same reduction in emission, the corresponding costs can be significantly different; (3) the cost of reducing emissions across the supply chain can be significantly lower if firms within the same supply chain collaborate (however, the cost and emission of individual firms may go up because of the collaboration).

Keywords: supply chains, carbon emissions, carbon footprint, climate control, environmental policy, supply collaboration and coordination, operations models

1. Introduction

There is growing consensus that carbon emissions (emissions of carbon dioxide and other greenhouse gases), if left unchecked, will lead to major changes in the earth's climate system. Governments are under growing pressure to enact legislation to curb the amount of these emissions. Firms worldwide, responding to the threat of such legislation or to concerns raised by their own consumers or shareholders, are undertaking initiatives to reduce their carbon footprint. However, these initiatives have focused for the most part on reducing emissions due to the physical processes involved. For example, firms are replacing energy inefficient equipment and facilities, redesigning products and packaging, finding less polluting sources of energy, or instituting energy savings programs¹. While there is clearly value in such efforts, they tend to overlook a potentially significant source of emissions, one that is driven by business practices and operational policies.

For example, determining how frequently supply deliveries are made could be as important in mitigating carbon emissions as the energy efficiency of the vehicles used to make these deliveries. In fact, one could argue that many of the popular business practices, such as *just-in-time* manufacturing and *lean* production, which favor frequent deliveries with less than truck-load shipments, small production runs, and multiple regional warehouses, could have as much of an impact on the carbon footprint of a firm as the energy efficiency of individual units deployed in production or distribution. Similarly, decisions that a firm makes regarding where to locate facilities, from which suppliers to source, and what mode of transportation to use can significantly affect its carbon footprint.

Moreover, a focus on emissions associated with physical processes could overlook important factors that emerge from the interaction among the multiple firms that constitute each supply chain. Multiple actors taking actions based on their own self-interests, and without coordination with others, are not likely to make decisions that minimize emissions for the entire supply chain. For example, if one firm requires shipments from its suppliers under short notice, then suppliers have little choice but to keep large inventories. For certain products, such as those requiring refrigeration, the associated carbon footprint can be significant. The need to respond quickly to suppliers may also require staging inventories in multiple locations that are close to the customers, further increasing the carbon footprint. The lack of coordination among multiple firms within the supply chain can also increase the overall carbon footprint. For example, coordinating production schedules among suppliers to the same customers could allow joint shipments, resulting in fewer emissions per delivery. However, acting on their own, the suppliers may have little

¹ See recent comprehensive reports on this subject from McKinsey (2009a, 2009b).

incentive to pursue such coordination. Clearly, efforts to reduce the carbon in a supply chain cannot afford to ignore the need to coordinate these efforts across the entire supply chain².

Although a lot has been written about the carbon footprint of supply chains in the popular press and in trade magazines (see for example Lee 2010, Brody and Ben-Hamida 2008, Parry et al. 2007, Plambeck 2007, Lash and Wellington 2007, Carbon Trust 2006, among many others), and although numerous websites, non-profit organizations, trade groups, and government bodies have been dedicated to the issue (see Benjaafar and Li 2009 and the references therein), the research community in Operations Management (OM) and Operations Research (OR) has only recently begun to pay attention to this area³. There is of course significant literature on sustainability and operations in general; see Kleindorfer et al. (2005), Linton et al. (2007), Srivastava (2007) and Corbett and Klassen (2006) for reviews. However, the concern in that literature tends to be more focused on product recycling or reuse (e.g., Flapper 2005, Guide and Van Wassenhove 2006a, 2006b) or product life cycle assessment (Matos and Hall 2007, Guide and Van Wassenhove 2009).

In contrast, there is extensive literature in economics, dating back to at least the 1970's, that incorporates environmental concerns. An important stream from this literature examines the impact of different policy instruments; see Weitzman (1974) for an early reference and Cropper and Oates (1992) and Hepburn (2006) for a recent review. These policy instruments can be classified as being either price-based (e.g., imposing a tax on carbon emissions) or quantity-based (e.g., imposing a cap on emissions and allowing firms to trade emission permits among each other). Numerous variations on these two types of instruments have been studied; see for example Jacoby and Ellerman (2004), Webster et al. (2010), and Burtraw et al. (2010). Another stream in this literature focuses on the design of markets for emissions and the trading of emission permits (Montgomery, 1972; Laffont and Tirole, 1996a, 1996b; Tietenberg 2006; Fankhauser and Hepburn 2010a, 2010b; Grubb and Neuhoff, 2006; Subramanian et al., 2007) and the references therein. Comprehensive reviews of the economics and politics of carbon emissions and climate change can be found in Ellerman et al. (2010), Helm and Hepburn (2010), Stern (2007), and Nordhaus (2008). In general, this literature does not deal with operational issues and the corresponding models are not typically used to optimize operational decisions.

² Industry is increasingly recognizing the impact of a firm's actions on the carbon emissions of its supply chain. Standards for measuring a firm's carbon footprint are being developed to account for supply chain emissions. Supply chain emissions are often referred to as Scope 3 emissions, in contrast to Scope 1 and Scope 2 emissions, which refer respectively to direct emissions by the firm at its installations and to indirect emissions by the firm via electricity usage. A prominent example of such a standard is the GHG Protocol developed by the World Resource Institute and the World Business Council for Sustainable Development; see WRI (2009). The importance of supply chain emissions has also been recognized in the industrial ecology literature; see for example Matthews et al. 2008 and Huang et al. 2009 and the extensive literature therein.

³ See for example the presentations made at the recently held NSF Symposium on Low Carbon Footprint Supply Chain (www.isye.umn.edu/NSFsymposium).

More relevant to operational concerns is the growing literature from Industrial Ecology and related fields on the measurement of carbon emissions of products and processes. This literature uses the methodology of life cycle assessment (LCA) to document carbon emissions that can be attributed to a product at various stages of its production, distribution, consumption, and end-of life disposal. Examples include Lenzen (2000), Williams and Tagami (2003), Tan and Khoo (2005), Matthews et al. (2008) and Wakeley et al. (2009), among many others. A review of this literature can be found in Suh and Huppes (2005), Tukker et al. (2006), and Browne et al. (2005). An important stream in this literature also uses economic input-output (EIO) modeling to account for carbon emissions at a sector level in the economy and to study the impact of one sector on the carbon emissions of other sectors; see for example Huang et al. (2009a, 2009b), Minx et al. (2009), Wiedmann (2009) and Wiedmann et al. (2009). Results from this literature are playing an important role in defining the emerging standards for measuring carbon footprint (Minx et al. 2009). This literature does not typically account for differences in operational practices among firms within the same industry and does not consider operational costs. In the case of the literature on EIO modeling, the level of emission data aggregation involved does not generally allow for a detailed analysis for an individual firm and its suppliers and customers (there are however growing efforts at building hybrid approaches that combine EIO analysis with process data from LCA studies).

Given the potential impact of operational decisions on carbon emissions, there is clearly a need for Operations Management research that incorporates carbon emission concerns that would complement (and benefit) from the body of knowledge in other disciplines. In particular, there is a need for *model-based* research that extends quantitative models, which typically focus on either minimizing cost or maximizing profit, to include carbon footprint. These models could then be used to understand how accounting for carbon emissions (either as a constraint or as a decision criterion) might affect operational decisions. They could also be used to inform operations managers on how policies, such as mandatory emission caps, taxes on carbon emissions, and emission cap and trade, among others, ought to affect operational decision-making. Moreover, the models could be used to study how the specifics of these policies (e.g., the scope of carbon emission responsibilities and how these responsibilities are allocated among members of the same supply chain) would affect the costs and emissions of various firms.

This paper is a step in this direction. Our objective in this paper is to draw attention to the strong connection between operational decisions across the supply chain and the carbon footprint of these supply chains and the extent to which concerns about carbon emissions can be addressed by adjusting operational decisions and improving collaboration among supply chain partners. Using relatively simple and widely used models, we illustrate how carbon emission concerns could be integrated into operational decision-making with regard to procurement, production, and inventory management. We show how, by associating carbon emission parameters with various decision variables, traditional models can be

modified to support decision-making that accounts for both cost and carbon footprint. We examine how the values of these parameters as well as the parameters of regulatory emission control policies affect cost and emissions. We use the models to study the extent to which carbon reduction requirements can be addressed by operational adjustments alone, as an alternative to costly investments in carbon-reducing technologies. We also use the models to investigate the impact of collaboration among firms within the same supply chain on their costs and carbon emissions and study the incentives firms might have in seeking such cooperation. A contribution of this paper is a set of insights, some of which would be difficult to obtain without the support of *operations* models such as the ones we consider here. A few of these insights are also surprising and point to important factors of which managers and other decision makers should be aware. The following is a highlight of the main insights:

- Although lower emissions are generally associated with higher costs, it is possible (through adjustments in the ordering decisions) to significantly reduce emissions without significantly increasing cost, a result due to the fact that cost tends to be flat around the *cost-optimal* solution. This also means that relatively modest carbon prices/taxes are sufficient to yield significant reductions in emissions.
- The ability to reduce emission by only adjusting the ordering decision requires that the ratio of the fixed cost to the inventory holding cost parameters is different from the ratio of the fixed emission to the inventory holding emission parameters. This implies that operational adjustments are effective in reducing emissions as long as the emission and cost parameters are not strongly correlated (otherwise, the *cost-optimal* solution is also *emission-optimal*).
- All the emission regulatory policies we consider can achieve the same reduction in emission levels with an appropriate choice of policy parameters. However, the corresponding cost (and revenue) to firms can be significantly different. For example, there is an opportunity for firms to leverage differences between their emission reduction costs and the market carbon price to earn additional revenue under cap-and-trade.
- The cost of reducing emissions can be significantly lower if firms within the same supply chain collaborate and carry out their ordering decisions jointly so as to minimize total supply chain cost instead of doing it independently to minimize individual costs.
- Although collaboration always leads to lower total supply chain cost, it may not, depending on the emission regulatory policy, always lead to lower total emissions for either the individual firms or for the entire supply chain. Collaboration may also lead to a significant shift in responsibilities within the supply (e.g., the location of inventory within the supply chain).
- Cost savings derived from collaboration can be significantly affected by the type of regulatory policy that is in place, with some policies providing greater benefits from collaborations than others. The

mechanisms through which these cost savings are obtained can also be quite different. For example, under a policy of strict emission caps, collaboration leads to lower costs by allowing firms to relax their individual caps. On the other hand, under cap-and-trade, collaboration allows firms to jointly optimize the amount of carbon traded to take advantage of carbon price fluctuations and adjust ordering decisions accordingly.

Although our model formulations and our analysis build on specific models for supply chain planning, namely single and multi-stage *lot-sizing* models, similar treatment could be extended to other common operations management models, including multi-location news-vendor models, economic order quantity models, multi-period stochastic inventory models, and supply chain coordination and contracting models, among many others (see Section 4 and Appendix 2 for further discussion). Moreover, throughout the paper, we make various assumptions when formulating the models and when carrying out numerical experiments. We do so mostly for the sake of illustrating how such models could be constructed and how useful insights could be derived. We believe that similar analysis and results could be obtained under alternative assumptions.

The rest of the paper is organized as follows. In Section 2, we introduce various formulations of production planning and procurement models for single and multiple firms that incorporate carbon emissions under varying regulatory policy assumptions. In Section 3, we use these models to obtain insights from numerical experiments and discuss the implications of these insights to management practice and to public policy making. In Section 4, we summarize key findings and offer ideas for future research.

2. Model Formulations

In this section, we present a series of model formulations that illustrate how carbon emissions considerations can be incorporated into operations management models. The models we formulate build on classic lot-sizing models for single and multiple firms. Our choice of models is motivated by (1) the widespread use of such models in practice, as they form the building block for many commercial supply chain planning applications, (2) the availability of standard tools for solving such models, and (3) the ability of these models to accommodate many of the relevant concerns associated with carbon emissions, including different regulatory policies. However, as mentioned earlier, it is possible to develop models

that are rooted in other operations management models and we view our treatment in this paper as providing a potential template for doing so⁴.

To incorporate carbon emission concerns, we consider several regulatory policy settings, including settings where (a) firms are subject to mandatory caps on the amount of carbon they emit, (b) firms are taxed on the amount of emissions they emit, (c) firms are subject to carbon caps but are rewarded (penalized) for emitting less (more) than their caps, and (d) firms can invest in carbon offsets to mitigate carbon caps. We consider systems involving a single firm as well as systems with multiple firms that operate either independently or coordinate their operations and carbon emissions. We consider variants of these systems with different assumptions regarding how carbon emissions are accounted for over time and how emissions are allocated among members of the same supply chain.

Model I: A Single Firm with Strict Carbon Caps

Consider the problem faced by a firm that must determine, over a specified planning horizon consisting of multiple periods with known demand, when and how much to order or when and how much to produce. In the absence of carbon emission considerations, the firm makes ordering decisions to minimize the sum of its fixed and variable ordering or production costs, inventory holding costs, and inventory shortage costs. Fixed ordering costs may correspond to transactions costs associated with placing an order with an outside supplier, such as transportation costs, or with initiating production internally, such as process setup costs. Variable costs may correspond to either unit purchasing or unit production costs. Inventory shortage costs are costs incurred if demand in one period cannot be fulfilled from inventory in that period, and can be in the form of either backorder costs or lost sales costs. In the presence of carbon emission considerations, the firm must account for the emissions associated with various decisions regarding ordering, production, and inventory holding. In particular, there may be emissions associated with placing an order with an outside supplier (e.g., emissions due to transportation) or with initiating production (e.g., emissions due to process setup). There may also be variable emissions associated with each unit ordered or produced (e.g., emissions due to the handling or the production of each unit) and emissions associated with the storage of each unit held in inventory in each period.

⁴ In a companion note (Benjaafar and Chen 2010), we use the EOQ model to show how the formulations and the results obtained can be extended to that setting. In that case, the numerical results we describe in Section 3 can be validated analytically (albeit under the simpler assumptions of the EOQ model); see Appendix 2 for further discussion.

We use cost parameters f_t , c_t , h_t , and b_t , where for each period t , $t = 1, \dots, T$, f_t denotes the fixed cost per order, c_t the variable cost per unit, h_t the cost per unit for inventory carried over from one period to the next, and b_t the cost per unit backordered in each period and T is the number of periods in the planning horizon. To account for carbon emissions, we introduce carbon emission parameters \hat{f}_t , \hat{c}_t , and \hat{h}_t , where, for each period t , \hat{f}_t denotes the amount of fixed carbon emissions associated with each order (e.g., transportation emissions or emissions associated with production), \hat{c}_t the variable amount of carbon emissions per unit in each order (e.g., emissions due to the handling or the production of each unit), and \hat{h}_t the amount of carbon emissions per unit of inventory held per period (emissions involved in the storage of each unit)⁵. The decision variables associated with the problem are denoted by y_t , q_t , I_t and B_t , where $y_t = 1$ if an order is placed in period t and $y_t = 0$ otherwise, q_t corresponds to the order quantity in period t , I_t is the amount of inventory carried from period t to period $t + 1$ and B_t is the amount of backorders also carried from period t to period $t + 1$. Finally, we let d_t refer to the demand in period t .

We consider a setting where the firm must adhere to a fixed cap C on emissions over the entire planning horizon. This cap could be mandated by a regulatory body external to the firm or could correspond to a decision made internally by management to adhere to specific limits on emissions (we later consider alternative regulatory policies under which emissions caps can be relaxed).

The problem faced by the firm can now be formulated as the following mixed integer linear program (MILP):

$$\text{Problem P1: Minimize } \sum_{t=1}^T (f_t y_t + c_t q_t + h_t I_t + b_t B_t) \quad (1)$$

subject to

$$I_t - B_t = I_{t-1} - B_{t-1} + q_t - d_t, \quad \text{for } t = 1, \dots, T, \quad (2)$$

$$\sum_{t=1}^T (\hat{f}_t y_t + h_t I_t + \hat{c}_t q_t) \leq C, \quad (3)$$

$$q_t \leq \left(\sum_{t'=1}^T d_{t'} \right) y_t, \quad \text{for } t = 1, \dots, T, \quad (4)$$

$$I_t, B_t, q_t \geq 0, \quad \text{for } t = 1, \dots, T, \quad (5)$$

$$y_t \in \{0, 1\}, \quad \text{for } t = 1, \dots, T. \quad (6)$$

⁵ These emission parameters may correspond to either direct emissions from fuel consumption (scope 1 emissions) or indirect emissions from the consumption of electricity (scope 2 emissions) or the sum of both. These emission parameters do not account for other emissions by the firm that are not affected by operational decisions. They also do not account for emissions elsewhere in the firm's supply chain (at its upstream suppliers and/or downstream customers) or other indirect emissions (scope 3 emissions). We introduce later in this section models that consider supply chain emissions (models P5-P7).

The objective function in (1) minimizes the sum of fixed and variable ordering costs, holding costs, and backordering costs over the entire planning horizon. Constraints (2) are net inventory balance equations. Constraint (3) ensures that the cap on carbon emissions over the planning horizon is not exceeded. Constraints (4) ensure that $y_t = 1$ whenever $q_t > 0$. The remaining constraints are standard integrality and non-negativity constraints. The model in (1)-(6) and its many variants can be solved using standard MILP solution approaches; see for example Pochet and Wolsey (2006) for an extensive treatment and discussion of recent advances. In the numerical results we describe in Section 3, we use the commercial solver ILOG CPLEX to generate solutions for the example problems we consider (see the Appendix for details).

In the above formulation, we assume that there are no constraints on order sizes and that there are no supply leadtimes. We also assume that unfulfilled demand is backordered. It is of course possible to model systems with order size constraints or positive leadtimes. It is also possible to model systems where demand in each period must be fulfilled; otherwise, it is considered lost and incurs a lost sales cost. The model could also be extended to systems with multiple stages and to multiple products. The above formulation assumes that the carbon emission cap is over the entire planning horizon, but it is possible to model settings in which the cap is over smaller subsets of periods or is associated with each period (see additional discussion in Section 3). The emission cap could also be associated with each unit produced or ordered. For example, firms may want to market products whose carbon footprint per unit does not exceed a certain threshold⁶. This can be accommodated by modifying constraints (3) as follows:

$$\sum_{t=1}^T (\hat{f}_t y_t + h_t I_t + \hat{c}_t q_t) \leq C \sum_{t=1}^T d_t, \quad (7)$$

where C now denotes the cap on carbon emissions per unit.

In defining the carbon emission parameters, we assume that emissions are linearly increasing in the associated decision variables. We make this assumption for the sake of demonstrating how such models can be constructed and to obtain in Section 3 insights for this important special case⁷. There may be of

⁶ Various firms are starting to attach carbon footprint labels to many of their products and to position these products as greener alternatives; see for example Edwards-Jones et al. (2009), Ball (2009) and Brenton et al. (2008) for discussion and examples; see also various case studies reported by the Carbon Trust at www.carbontrust.org. Two of the leading retailers in Europe, Tesco in the UK and Casino in France, have already embarked on aggressive labeling efforts.

⁷ This assumption appears to be consistent with assumptions used in other methodologies, such as economic input-output modeling, where emissions are assumed to be proportional to volumes produced or consumed (Matthews et al., 2008). It is also applicable in several important industrial settings such as steel and cement manufacturing and dairy and food processing (see for example Benjaafar et al. 2009 and Kondili et al. 1993).

course settings where this may not hold and where emissions could increase in a non-linear fashion (e.g., convex or concave). In that case, similar formulations could still be constructed, albeit with constraints that are non-linear (note that a non-linear function can always be approximated by a piece-wise linear function). As we discuss in Appendix 2, many of our results, including the insights we discuss in Section 3, continue to hold under more general assumptions.

In the above formulation, we also assumed that the carbon emission parameters are readily available. However, a prerequisite for using the models described in this paper is estimating these parameters. Indeed, any public policy regarding carbon emissions implicitly assumes that the emissions are measurable and quantifiable. Fortunately, significant effort is being made by various firms in documenting the carbon footprint of their activities, as this would be required in documenting their compliance with regulatory policies or in communicating the carbon footprint of their products to consumers⁸. These efforts are being supported by the emergence of standards for measuring and attributing emissions to products and processes and the increased availability of independent third parties for emission verification and certification⁹. They are also being supported by the increased availability of emission data on standard processes, materials, fuels, and modes of transportation (see for example calculation tools provided by the GHG Protocol <http://www.ghgprotocol.org/calculation-tools/sector-toolsets>, and emission factors prepared by the EPA <http://www.epa.gov/ttn/chief/ap42/index.html>) and for different sectors of the economy (see for example emission data available from the Office of National Statistics in the UK for over 90 sectors in the economy <http://www.statistics.gov.uk/statbase/ssdataset.asp?vlnk=5695&More=Y>). The cost and time involved in

⁸ Over 11,000 industrial facilities in Europe are subject to the ETS system and are required to document and report their emissions (Ellerman and Buchner, 2008). The French government has recently introduced legislation that would mandate that all products sold to consumers carry a carbon footprint label (Journal Officiel of 13, 2010. http://www.bureauveritas.com/wps/wcm/connect/39aad600434ada5e81958fa518eb85d7/Bulletin_10B-149.pdf?MOD=AJPERES&CACHEID=39aad600434ada5e81958fa518eb85d7). In the US, large emitters are increasingly being required to report their emissions (see for example the new EPA reporting requirement: <http://epa.gov/climatechange/emissions/ghgrulemaking.htm>). Hundreds of firms worldwide are choosing to report their emissions to the Carbon Disclosure Project (CDP), an independent organization that has the backing of major institutional investors (Sullivan 2010; Kolk and Pinkse 2005). This reporting includes scope 1 and scope 2 emissions (and scope 3 in some cases). These firms are also providing details about efforts they have undertaken to reduce their emissions. The reported data from each firm is publicly available and can be accessed at www.cdproject.net.

⁹ Various *standards* have recently emerged on how the carbon footprint for products and supply chains should be documented, measured, and reported; examples include the GHG Protocol by the World Resource Institute and the World Business Council for Sustainable Development, ISO 14064 by the International Organization for Standardization, and PAS 2050 by BSI British Standards, the Carbon Trust, and the Department for Environment, Food and Rural Affairs in the United Kingdom; for references see BSI (2008a, 2008b), Carbon Trust (2007), WRI (2009), and ISO (2006). There are ongoing efforts to harmonize these standards.

estimating emission parameters would of course vary from firm to firm. The potential usefulness of models such as the ones we describe in this paper could provide an additional impetus for firms to collect these data.

Finally, it might be tempting to view *carbon capacity* limits (caps on carbon emissions) as being similar to capacity limits on production (or ordering), which are common in many models in operations managements. The differences are however significant. First, carbon capacity may cover multiple periods or even the entire planning horizon. In contrast, production capacity typically applies to each period. Second, carbon capacity consumed in one period affects the available carbon capacity in future periods, while production usually regenerates in each period. Third, carbon capacity is consumed not only by production, but also by order processing, transportation, and inventory holding. This means that carbon capacity can be consumed by simply holding inventory even if no production, procurement or distribution activity takes place. It also means that operating changes in one area (e.g., inventory) may induce or demand changes in other areas (e.g., production or transportation) due to the joint carbon capacity limit.

Model II: A Single Firm with Carbon Tax, Carbon Cap and Trade, or Carbon Offsets

An alternative to strict caps on emissions is not to restrict emissions but instead to penalize emissions using a *carbon tax*. A carbon tax can take on a variety of forms. In its simplest, the tax is a financial penalty linear in the number of carbon units emitted. To illustrate how a carbon tax would modify the problem formulation in (1)-(6), let α denote the amount of tax paid on each unit emitted (the carbon unit price), then the problem facing the firm can be restated as

$$\text{Problem P2: Minimize } \sum_{t=1}^T (f_t y_t + c_t q_t + h_t I_t + b_t B_t) + \alpha \sum_{t=1}^T (\hat{f}_t y_t + \hat{c}_t q_t + h_t I_t) \quad (8)$$

subject to (2), (4), (5) and (6). The objective function in (8) can be rewritten as

$$\text{Minimize } \sum_{t=1}^T (\tilde{f}_t y_t + \tilde{c}_t q_t + \tilde{h}_t I_t + b_t B_t), \quad (9)$$

where $\tilde{f}_t = f_t + \alpha \hat{f}_t$, $\tilde{c}_t = c_t + \alpha \hat{c}_t$, and $\tilde{h}_t = h_t + \alpha \hat{h}_t$. Hence, the problem reduces to one of pure cost minimization, albeit with cost parameters that reflect the cost of emissions.

Numerous variations on this formulation are possible. It is possible to incorporate alternative tax schemes, such as those in which tax penalties are non-linear in the emission quantities, making the tax either progressive (e.g., increasing convex) or regressive (e.g., increasing concave). It is also possible to incorporate tax schedules in which the unit penalty changes in discrete steps as a function of the emission

quantity, including the case where taxes are not levied if emissions fall below a certain threshold. Some of these variations could lead to non-linear optimization problems which, depending on the assumption, may be possible to linearize by approximating the tax penalties with a piece-wise linear function.

An alternative policy to either imposing strict caps or applying a carbon tax is a system whereby firms are allowed to emit more than their prescribed caps but are penalized for doing so, with penalties increasing in the amount of emissions that exceed the cap. Firms are also rewarded for emitting less than their caps by receiving payments increasing in the difference between their caps and their actual emissions. This system of penalties and rewards can be implemented directly by a regulating agency or indirectly via a trading market for carbon emissions, in which firms can buy and sell the right to emit¹⁰. Both cases can be viewed as allowing the sale and purchase of emission permits at a price. An important difference is that in the case of carbon trading, (1) price is affected by market dynamics and (2) the total amount of carbon that can be bought and sold is limited by the sum of the caps imposed on the participating firms. Variations on these two schemes are possible, including hybrid policies, where price is allowed to fluctuate but the regulating agency guarantees a price ceiling, a price floor, or both; see Jacoby and Ellerman (2004), Goulder and Parry (2008), Philibert (2009), and Burtraw et al. (2010) for further discussion. Note that when the price ceiling is set sufficiently low, the price at which carbon is bought and sold becomes essentially fixed. This is also the case when there are both a price ceiling and price floor and the difference between the two is relatively small.

If we assume that price is relatively stable over the firm's planning horizon and is exogenous to decisions made by individual firms (as in the scenarios discussed above), then we can reformulate the problem in (1)-(6) as follows:

$$\text{Problem P3: Minimize } \sum_{t=1}^T (f_t y_t + c_t q_t + h_t I_t + b_t B_t + p(e_t^+ - e_t^-)) \quad (10)$$

subject to

$$\sum_{t=1}^T (\hat{f}_t y_t + h_t I_t + \hat{c}_t q_t + e_t^-) \leq C + \sum_{t=1}^T e_t^+, \quad (11)$$

¹⁰ There are several active carbon trading markets. The annual global market for carbon was recently valued at over \$130 billion and expected to rapidly grow. The most important market is the European Trading System (ETS), which covers over 50% of all emissions in the European Union (EU). In the US, a cap-and-trade system is part of the US climate bill that was recently passed by the House of Representatives (it is also part of the California Assembly Bill 32 that regulates greenhouse gas emissions in the state of California). There are several regional trading markets already in place in the US, including the Regional Greenhouse Gas Initiative (www.rggi.org) involving utility companies in the Northeast and Mid-Atlantic States. The Chicago Climate Exchange is a voluntary trading market with participation by companies from North America, US municipalities, states, and universities. For recent references on emission trading markets see Fankhauser and Hepburn (2010a, 2010b), Ellerman and Trotignon (2009), Stavins (2009), Grubb and Neuhoff (2006), Kossoy and Ambrosi (2010), and Ellerman et al. (2010).

$$e_t^+, e_t^- \geq 0, \quad \text{for } t=1, \dots, T, \quad (12)$$

along with (2), (4), (5), and (6), where p is the price per unit of carbon emission and e_t^+ and e_t^- denote respectively the amount of carbon credit the firm buys and sells in period t ¹¹. Note that the carbon credits bought serve to relax the effective cap on emissions, although it is costly to do so, while the carbon credits sold represent a new source of revenue. In case there is a limit on how much carbon credit a firm can buy or sell in a period, (12) is modified to include an upper bound on e_t^+ and e_t^- . In case there is a need to differentiate between the price of buying and selling of carbon, the objective function is modified by associated prices p^+ and p^- - with e_t^+ and e_t^- respectively.

In the above formulation, we have assumed that the market price for carbon is fixed. However, under cap-and-trade without price control, price would be subject to volatility. This volatility could be assumed away if it is primarily of a short-term nature and if the long-term trend can be reliably forecast (a predictable increase or decrease in price can be easily incorporated into the model). It can also be assumed away if the firm employs financial options, such as those commonly used in the procurement of commodities, which guarantee the firm the option to buy or sell at a specified price. However, there are many settings where it is necessary to construct a model that explicitly accounts for price volatility and/or for the dynamics of the carbon exchange market. In particular, unless artificially fixed, price would be sensitive to the supply and demand for carbon, which is itself dependent on the imposed carbon caps. Hence, a more complete model would need to capture the dependency of price on the carbon caps (see Section 3 for further discussion).

A third policy alternative to strict caps is to impose caps but to allow firms to invest in so-called *carbon offsets*. Offsets are investments a firm would make in carbon-reducing projects, typically offered by a third party, to offset emissions in excess of its specified cap¹². Hence, it is essentially the same as

¹¹ Because the cap is imposed on the entire planning horizon, the formulation of P3 can be simplified by defining variables E^+ and E^- corresponding respectively to the total amount of carbon purchased and sold over the entire planning horizon such that $E^+ = \sum_{t=1}^T e_t^+$ and $E^- = \sum_{t=1}^T e_t^-$. We intentionally introduce period-specific emission trading variables to show how readily the formulation can be extended to settings where the emission price is period-dependent and to settings where the cap is imposed over intervals shorter than the entire planning horizon.

¹² The use of offsets is one of the main mechanisms available to countries under the Kyoto Protocol (and also under the pending US Climate Bill) for fulfilling their emission reduction commitments. In particular, industrialized countries can earn emission reduction credits from emission-reduction projects in developing countries (under the *Clean Development Mechanism*) or in other industrialized countries (under the *Joint Investment* system). For details, see NCEP (2009), Stern (2007), and Carbon Trust (2009).

the purchasing of emission credits in a cap-and-trade system, except that the underlying market mechanism is different (in a cap-and-trade system, the availability and pricing of emission credits are determined by a carbon exchange market, while the availability and pricing of offsets is determined by independent suppliers of such offsets). If we let p now denote the price per unit of carbon offset and e_t^+ denote the amount of carbon offset (in units of carbon emissions) the firm purchases, then the problem in (1)-(6) can be reformulated as

$$\text{Problem P4: Minimize } \sum_{t=1}^T (f_t y_t + c_t q_t + h_t I_t + b_t B_t + p e_t^+) \quad (13)$$

subject to

$$\sum_{t=1}^T (\hat{f}_t y_t + h_t I_t + \hat{c}_t q_t) \leq C + \sum_{t=1}^T e_t^+, \quad (14)$$

$$e_t^+ \geq 0, \quad \text{for } t = 1, \dots, T, \quad (15)$$

along with (2), (4), (5), and (6). This formulation is similar to the one for cap and trade, except that a firm does not benefit from emitting less than its specified cap.

Model III: Multiple Firms with and without Collaboration

Let us now consider the problem faced by firms that operate within a supply chain consisting of other firms that serve as either suppliers or customers. In particular, consider a serial supply chain consisting of N firms, where firm i orders from firm $i + 1$, with $i = 1, \dots, N - 1$. Each firm must determine when and how much to order from its “supplier firm” so that it minimizes its total costs over the entire planning horizon. Note that for firms, other than firm 1, demand corresponds to orders generated by its “customer firm” (in other words, the demand of firm i corresponds to the orders generated by firm $i - 1$). If each firm is subject to a strict emission cap, C_i for firm $i = 1, \dots, N$, and if the firms operate independently, then the problem faced by firm i , for $i = 2, \dots, N$, can be formulated as

$$\text{Problem P5: Minimize } \sum_{t=1}^T (f_{i,t} y_{i,t} + c_{i,t} q_{i,t} + h_{i,t} I_{i,t}) \quad (16)$$

subject to

$$I_{i,t} = I_{i,t-1} + q_{i,t} - q_{i-1,t}^*, \quad \text{for } t = 1, \dots, T, \quad (17)$$

$$\sum_{t=1}^T (\hat{f}_{i,t} y_{i,t} + h_{i,t} I_{i,t} + \hat{c}_{i,t} q_{i,t}) \leq C_i, \quad (18)$$

$$q_{i,t} \leq \left(\sum_{t'=t}^T q_{i-1,t'}^* \right) y_{i,t}, \quad \text{for } t = 1, \dots, T, \quad (19)$$

$$I_{i,t}, q_{i,t} \geq 0, \quad \text{for } t = 1, \dots, T, \quad (20)$$

$$y_{i,t} \in \{0, 1\}, \quad \text{for } t = 1, \dots, T. \quad (21)$$

where the decision variables and cost and carbon emission parameters are indexed by i and t to indicate the fact that they are now firm-specific. We use the notation $q_{i,t}^*$ to indicate the optimal order quantity for firm i in period t obtained by firm i solving the optimization problem above. Note that the vector of orders $(q_{i,1}^*, \dots, q_{i,T}^*)$ constitutes the vector of demands that firm $i + 1$ must satisfy.

For firm 1, the problem can be formulated similarly as

$$\text{Problem P6: Minimize } \sum_{t=1}^T (f_{1,t}y_{1,t} + c_{1,t}q_{1,t} + h_{1,t}I_{1,t} + b_{1,t}B_{1,t}) \quad (22)$$

subject to

$$I_{1,t-1} - B_{1,t-1} + q_{1,t} - d_{1,t} = I_{1,t} - B_{1,t}, \quad \text{for } t = 1, \dots, T, \quad (23)$$

$$q_{1,t} \leq \left(\sum_{t'=1}^T d_{t'} \right) y_{i,t}, \quad \text{for } t = 1, \dots, T, \quad (24)$$

along with (18), (20) and (21).

In the above formulation, we assume that backordering is allowed only for firm 1. Other firms must satisfy orders in the same period the orders are received. It is of course possible to allow for backorders by firms other than firm 1. It is also possible to model settings in which firms can resort to an outside supplier to fulfill orders that they cannot (or prefer not to) fulfill themselves. This becomes important when the carbon constraint can lead to infeasibilities.

We also assume that firms make decisions about ordering and production independently of each other and ignore each other's capabilities and carbon emission constraints. The firms could obviously reduce the total supply chain cost by making these decisions jointly. Such collaboration is not uncommon in industry and there are industry initiatives, such as the Collaborative Forecasting, Planning, and Replenishment (CFPR) initiative, whose goal is to support such collaboration. As we explore in Section 3, the presence of carbon footprint constraints can provide additional impetus for such collaboration. In its simplest form, if firms were to collaborate, they would jointly solve the following problem:

$$\text{Problem P7: Minimize } \sum_{i=1}^N \sum_{t=1}^T (f_{i,t}y_{i,t} + c_{i,t}q_{i,t} + h_{i,t}I_{i,t}) + \sum_{t=1}^T b_{1,t}B_{1,t} \quad (25)$$

subject to

$$I_{1,t} - B_{1,t} = I_{1,t-1} - B_{1,t-1} + q_{1,t} - d_{1,t}, \quad \text{for } t = 1, \dots, T, \quad (26)$$

$$I_{i,t} = I_{i,t-1} + q_{i,t} - q_{i-1,t}, \quad \text{for } t = 1, \dots, T, \quad i = 2, \dots, N, \quad (27)$$

$$\sum_{t=1}^T (\hat{f}_{i,t}y_{i,t} + h_{i,t}I_{i,t} + \hat{c}_{i,t}q_{i,t}) \leq C_i, \quad \text{for } i = 1, \dots, N, \quad (28)$$

$$q_{i,t} \leq \left(\sum_{t'=1}^T d_{t'} \right) y_{i,t}, \quad \text{for } t = 1, \dots, T, \quad i = 1, \dots, N, \quad (29)$$

$$I_{i,t}, B_{1,t}, q_{i,t} \geq 0, \quad \text{for } t = 1, \dots, T, \quad i = 1, \dots, N \quad (30)$$

$$y_{i,t} \in \{0, 1\}, \quad \text{for } t = 1, \dots, T, \quad i = 1, \dots, N. \quad (31)$$

The above formulation assumes that, although the firms collaborate, emission caps continue to be imposed individually on each firm. If it were possible for firms to share their emission caps, constraints (28) would be replaced by the following constraint:

$$\sum_{i=1}^N \sum_{t=1}^T (\hat{f}_{i,t} y_{i,t} + h_{i,t} I_{i,t} + \hat{c}_{i,t} q_{i,t}) \leq \sum_{i=1}^N C_i. \quad (32)$$

Such sharing of the emission caps might be possible if the firms are not independent entities but are divisions owned by a single large firm. It might also be possible if the environmental regulation allows for carbon trading between members of the same supply chain (e.g., in the absence of an open market for carbon trading). Sharing of the emission cap is also possible when the cap is voluntary and the objective for the supply chain is to eventually certify that the end-product has a carbon footprint that does not exceed a certain threshold. For example, several large retailers (e.g., Wal-Mart in the US (Wal-Mart 2009) and Tesco in the UK (Tesco 2009)) are working with their suppliers to reduce the overall carbon footprint of the products they sell and to market these products as greener alternatives to those sold by competitors.

Finally, we note that the models incorporating a carbon tax, carbon cap-and-trade, and carbon offsets, discussed earlier for the case of a single firm can be readily extended to the case of multiple firms, with or without collaboration and with or without the sharing of the emission caps.

3. Insights from the Models

In this section, we illustrate how the models presented in the previous section can be used to obtain useful insights. The insights, presented in the form of a series of observations, are based on numerical results generated from solving the models for examples of problems with varying parameter values. The details of the experimental setup and the examples can be found in the Appendix. Representative subsets of these numerical results are summarized in Figures 1-15. Each of these figures is used to illustrate one or more qualitative effects that can arise when carbon emission considerations are incorporated into operational models. In Appendix 2, we comment on the robustness of the insights we obtain and provide additional analytical support.

The rest of this section is organized into three subsections: §3.1 presents a series of observations based on model P1 and deals with a setting in which there is a strict cap on emissions; §3.2 presents observations based on models P2, P3, and P4 and compares the impact of different regulatory policies; and finally §3.3 presents observations based on models P5, P6 and P7 and examines issues that arise from the interaction of multiple firms within a supply chain.

3.1 Systems with Strict Emission Caps

Observation 1: *It is possible to significantly reduce emissions without significantly increasing cost.*

This observation is illustrated in Figure 1 which shows the impact of varying the emission cap on the total cost and total emissions for the examples considered. As expected, reducing the emission cap increases total cost and reduces total emissions. However, what is perhaps surprising is the fact that the emission cap can be significantly reduced without significantly affecting the total cost. In the example shown, reducing the emission cap from 2100 to 1785 reduces the average total amount of emissions by 15% but increases the average total cost by only 3%. Note that this reduction is obtained through adjustments in operational decisions only, namely modifying order quantities in each period, could alone lead to significant reductions in carbon emissions while not significantly compromising overall cost.

To provide intuition to how this might arise, first note that in the absence of constraints on emissions, the optimal solution would only minimize cost. In doing so, the main tradeoff is between the fixed cost f_i and the holding cost h_i . When the ratio f_i / h_i is large, the solution favors ordering in large quantities. This reduces the frequency of ordering even though it may result in more inventory being held over longer periods. The reverse is true when f_i / h_i is small. However, the order quantities that minimize cost may not necessarily minimize emissions. For example, the ratio f_i / h_i could be high (favoring large order quantities from a cost perspective) but the ratio \hat{f}_i / \hat{h}_i is low (favoring small order quantities from an emission perspective). In other words, minimizing emissions is subject to similar tradeoffs to those involved in minimizing cost, except that the relevant ratio is \hat{f}_i / \hat{h}_i . This has the following important implication. Reducing emissions via operational adjustment (an adjustment in the order quantity) is not possible if \hat{f}_i / \hat{h}_i and f_i / h_i are equal. In that case, order quantities that minimize cost also minimize emissions. However, if these ratios are different (as they are in the examples illustrated), then it is possible to reduce emissions at the expense of increasing cost¹³.

Moreover, as illustrated in Appendix A.2, for lot sizing problems such as the ones we consider in this paper, the optimal cost is known to be relatively insensitive to the value of the order quantity in the region

¹³ The above also means that in illustrating the qualitative behavior of cost emission what matters most are the relative values of the ratios \hat{f}_i / \hat{h}_i and f_i / h_i and not the specific values of the individual parameters. Since Problem P1 serves as a building block to most of the other models discussed in section 2, similar insights apply to the other models. In particular, the relative values of the ratios \hat{f}_i / \hat{h}_i and f_i / h_i continue to play a dominant role in whether or not operational adjustments could have a significant impact on emissions.

around the optimal quantity (i.e., the cost function is *flat* around the optimal solution) but quite sensitive to the value of the order quantity otherwise. Given that the cost and emission functions have similar forms, this explains why it is possible to make a significant change in the order quantity and affect cost very little but affect emissions significantly; see the discussion of the EOQ model in Appendix A.2 and the associated Figures A.1 and A.2.

Observations 2: *Emission caps could be met more cost-effectively by adjusting operational decisions than by investing in costly more energy-efficient technology.*

An alternative to adjusting operational decisions is for a firm to lower its carbon emission parameters, \hat{f}_t , \hat{c}_t , and \hat{h}_t , by investing in more energy-efficient technology in the production, transportation and warehousing of its products. Figure 2 illustrates the impact of varying emission caps on total cost for technologies with varying levels of energy efficiency. As we can see, there is very little difference in total operational costs as emission caps are initially lowered for the different technologies, and only when the emission cap is significantly lower than the unconstrained emission values (more than 15% in the example shown) does investing in more energy-efficient technology begin to yield significant cost savings.

Figure 2 also illustrates rather dramatically that in the absence of operational models, such as the ones we describe in this paper, that can capture the potential of mitigating carbon emissions via operational adjustments, it is difficult, for both firms and policy makers, to assess the impact of lower emission caps on the economic welfare of firms and their consumers. Without such models, it is also difficult for a firm to decide whether or not investing in more energy-efficient facilities and processes is economically advantageous.

Observation 3: *Tighter caps on emissions can paradoxically lead to higher total emissions.*

Figure 3 shows the impact of varying the emission cap when a cap is imposed on each period instead of the entire planning horizon. Perhaps surprisingly, lower emission caps in this case can lead to higher total emissions in some cases. This is due to the fact, that imposing emission caps on a period by period basis prevents firms from the possibility of emitting more in one period if this allows significantly less emissions in future periods. For example, consider a setting in which the fixed emissions (emissions associated with initiating an order such as transportation) are relatively high. A firm could reduce its

overall emissions by reducing the frequency of orders, but carrying more inventory in the periods immediately following the placement of an order. Because of the carbon emissions associated with carrying inventory, this could mean higher carbon emissions in those periods, potentially violating emission caps if they are imposed on a period by period basis.

This observation has two important implications. First, policy makers (and also firms) need to be aware that the specifics of how emission caps are implemented can have very different impacts on operational costs. Second, devising policies that provide firms with more flexibility in how and when they fulfill the required cap could allow the fulfillment of these caps at significantly lower costs. Examples of such policies are those that allow firms to borrow, to a certain extent, against their future emission quotas or to bank unused quotas for future use (see for example Tietenberg 2006 and Ellerman et al. 2010 for related discussion and references)

3.2 Systems with Carbon Offsets, Carbon Tax, and Cap-and-Trade

Observation 4: *Carbon offsets enable tighter emission caps by mitigating the impact of lowering emission caps on cost.*

Figure 4 illustrates the impact of varying the emission cap when a firm has the option of purchasing carbon offsets. The option to offset is valuable even when the unit carbon price is relatively high; firms resort to offsetting selectively and only to mitigate an even higher operational cost. This observation points to the importance for policy makers of supporting the emergence of a competitive market for offsets, which could drive the price of these offsets down achieving the dual benefit of maintaining low cost for the consumer and low carbon for the environment.

Observation 5: *Under cap and trade when the price is fixed, emission levels are not affected by emission caps and are affected only by the price for carbon.*

Figure 5 shows the impact of varying carbon caps and carbon price on total emissions. The somewhat surprising result that emission levels are unaffected by emission caps can be explained as follows. For problem P3, the emission constraint in (11) is binding because

$$\sum_{t=1}^T (e_t^+ - e_t^-) = \sum_{t=1}^T (\hat{f}_t y_t + h_t I_t + \hat{c}_t q_t) - C.$$

Consequently, Problem P3 can be reformulated as

$$\text{Minimize } \sum_{t=1}^T \left(f_t y_t + c_t q_t + h_t I_t + b_t B_t + p \left(\hat{f}_t y_t + h_t I_t + \hat{c}_t q_t \right) \right) - pC \quad (33)$$

subject to (2), (4), (5), and (6), from which we can see that the optimal solution is always independent of the carbon cap C . This result perhaps points to a limitation of a cap-and-trade policy. The emission cap cannot be used as a direct lever to control emissions and to ensure that the desired emission limits are achieved in the way other policies, such as a strict cap on emissions can. Emissions can be indirectly controlled by varying the price of carbon, but the resulting emissions are more difficult to predict as they depend on the cost and emission parameters of each firm. From (33), we can also see that the amount of carbon emissions would be the same as it would be under a carbon tax if the tax rate $\alpha = p$ ¹⁴.

In settings where the price of carbon is not fixed and instead is determined by the market place, the only lever available to the policy maker is the carbon cap. However, in this case, the caps imposed on the participating firms affect the total supply of carbon credits that can be traded and, therefore, the price of carbon. Hence, a policy maker can affect emissions by varying the carbon cap. In Figure 6, we illustrate the interaction between the carbon cap and carbon price by considering a simple linear carbon pricing model where $p = a - bC$, leading to the following modified objective function:

$$\text{Minimize } \sum_{t=1}^T (f_t y_t + c_t q_t + h_t I_t + b_t B_t) + (a - bC) \left(\sum_{t=1}^T (\hat{f}_t y_t + h_t I_t + \hat{c}_t q_t) - C \right) \quad (34)$$

As we can see from Figure 6, a tighter cap now leads to lower carbon emissions. Moreover, the effect of the carbon cap on cost is no longer monotonic, with initial decreases in the carbon cap actually benefiting the firm due to the revenue associated with selling carbon.

In practice, predicting the market dynamics for carbon can be difficult, a difficulty compounded by the inherent uncertainty and volatility in how demand and supply affect price. In turn, this makes it difficult for both firms and policy makers to easily assess the impact of a cap-and-trade system. This perhaps supports the argument that between cap-and-trade and a carbon tax, a carbon tax provides a simpler mechanism for reducing carbon emissions quickly and reliably (see Figure 7).

Observation 6: *Under cap-and-trade, a higher carbon price can lead to lower total cost.*

¹⁴ Letting $\alpha = p$, problems P2 and P3 have the same solution. That is firms would make the same operational decisions. However, the net cost to firms is lower under cap-and-trade because of the revenue they generate from selling carbon. This perhaps explains some of the resistance among the business community to legislation involving a carbon tax. This also illustrates the fact that introducing a cap and trade system can lead to a windfall for some firms (see for example Stavins (2008) and Ellerman and Buchner (2008) for further discussion of this windfall effect).

As we can see from Figure 8¹⁵, the effect of carbon price, given a fixed carbon cap, on the total cost is not monotonic, with cost initially increasing and then decreasing. When carbon price is relatively low, the firm is mostly engaged in the buying of carbon. Therefore, higher carbon prices increase its carbon purchasing cost. When the carbon price is sufficiently high, the firm becomes engaged in the selling of carbon, as the firm finds it advantageous to adjust its operations and emit less carbon (this of course means that the operational cost increases but this increase is more than offset from the higher revenue generated by carbon selling).

Observation 7: *The benefit from more energy-efficient technology is affected by the type of emission control policy.*

This observation is illustrated in Figure 9 which shows how technologies with varying energy efficiency affect operational cost. As we can see, as energy efficiency initially increases, the benefit is higher under a strict emission cap policy. Under such a policy, firms have no alternatives to mitigating the impact of the cap, the way they have under cap and trade or cap and offset. The benefit eventually levels off once the energy efficiency is sufficient to essentially make the carbon constraint irrelevant. This diminishing effect is absent under cap-and-trade (and similarly under a carbon tax) since there is always an incentive to reduce emissions and generate corresponding income. The benefit of more energy-efficient technologies is lower under a cap-and-offset policy than under a cap-and-trade policy since more energy-efficient technology can only be used to avoid the cost of exceeding the cap.

These results suggest that certain policies could be more effective than others in promoting the adoption of greener technologies. In particular, when the energy gains from alternative technologies are relatively modest, then a policy of imposing strict emission caps may be the most effective. On the other hand, if the gains from alternative technologies can be substantial, then a cap-and-trade policy or a carbon tax may be more effective in motivating the firms to adopt the more energy-efficient technologies.

3.3 Systems with Multiple Firms with and without Collaboration

Observation 8: *The presence of carbon constraints can increase the value of supply chain collaboration.*

¹⁵ The figure is based on the original model P3 where the carbon price is fixed and independent of the cap.

Figures 10 and 11 show the effect of varying the carbon cap on the percentage reduction in cost achieved from collaboration in a supply chain consisting of two firms under a policy of strict caps¹⁶. The curves show the cost reductions associated with each firm and with the entire supply chain. As we can see, total supply chain cost can be significantly reduced by having the firms collaborate. This benefit derives of course from having firms adjust their decisions to take into account the implications of these decisions on the cost of other firms. The benefits of these adjustments are more important in the presence of carbon caps than without them, as they allow firms to achieve these caps more cost effectively (the right-most segments of the curves correspond to settings where the emission cap is very high and where the benefit from collaboration is not due to the presence of the emission cap). The benefit from collaboration is not monotonic in the carbon cap level, with the benefit highest when the carbon cap is in the mid-range and less significant when the cap is either very low or very high. When the cap is very low, firms have less room to make adjustments in their operations and collaboration may lead to little or no cost reduction. On the other hand, when the cap is very high, the supply chain has less need for firms to make adjustments, although there continues to be some value to collaboration.

Observation 9: *Collaboration can lead to increases in the cost and carbon emissions of some of the firms.*

As we can see from Figure 10, although total supply chain cost is lower with collaboration, the cost of firm 1 can be higher. This is because firm 1, which faces end demand in this two-firm example, adjusts its order sizes to reduce the cost of firm 2. The resulting reduction in the cost of firm 2 more than offsets the increase in the cost of firm 1. As shown in Figure 12, these adjustments in order sizes mean that the emissions could also increase with collaboration, although always remaining within the cap. The fact that cost for some of the firms can increase with collaboration would require that contractual arrangements are in place that would suitably compensate these firms. In principle, such compensation is always possible since there is a net surplus for the supply chain. However, the specifics of how the surplus is divided among the firms can affect the long term success of the collaboration (see Section 4 for further discussion). With collaboration, the operational responsibilities within the supply chain of the different firms could also be significantly affected. Figure 13 shows how, when firms collaborate, the amount of inventory held by each firm is affected by the difference in the inventory holding carbon footprint.

¹⁶ Percentage cost reduction due to collaboration = $100\% \times (\text{cost without collaboration} - \text{cost with collaboration}) / \text{cost without collaboration}$.

Collaboration allows the responsibility for holding inventory to be shifted to the firm that is more carbon efficient. In practice, this may require further adjustments to the physical infrastructure of the supply chain to accommodate this shift (e.g., investment in more warehousing facilities by one of the firms).

Observation 10: *Imposing supply chain-wide emission caps achieves lower emissions at lower costs; it also increases the value of collaboration.*

Figure 11 illustrates the impact of imposing a shared emission cap on the entire supply chain instead of individual caps on each firm. As we can see, the reduction in cost to the supply chain can be substantial, particularly when the emission caps are tight. A shared cap provides firms within the same supply chain with the flexibility of having some firms emit more than their individual cap if it can be offset with less emissions from other firms. This allows firms that are more cost effective at reducing their carbon footprint to take on a greater responsibility in meeting the overall carbon cap. As with individual caps, cooperation among the firms could mean that some firms would see their individual costs increase. Here too, a compensation scheme would need to be put in place first for firms to agree to cooperate.

Observation 11: *The benefit derived from collaboration can be significantly affected by the type of regulatory policy that is in effect.*

As we can see from Figure 14, the benefit derived from supply chain collaboration is sensitive to the type of regulatory policy that is in place. Some policies provide greater incentives than others for collaboration. In particular, collaboration is most beneficial when there is a strict cap and this cap is in the middle range. The benefit under cap and offset is less significant since firms have the option of meeting their carbon caps via the purchase of offsets. Therefore, having a supply chain partner that can help them meet their cap requirements becomes less important. Under cap and trade, collaboration can be greatly beneficial when the cap is high. In this case, the benefit from collaboration derives primarily from the ability of the supply chain to sell higher amounts of carbon to the market.

Observation 12: *Collaboration, if it does not involve all members of the supply chain, can increase the cost and emissions of those firms left out.*

This could occur under a variety of scenarios. One such example is illustrated in Figure 15 for a supply chain consisting of three firms, where firm 2 is the supplier of firm 1 and firm 3 is the supplier of firm 2. Firms 1 and 2 collaborate but firm 3 makes decisions on its own. The cost to firm 3 can be significantly

higher than when firms 1 and 2 do not collaborate. In this example, when firms 1 and 2 collaborate, changes in the order sizes from firm 2 to firm 3 lead firm 3 to incur higher shortage costs. Similar effects can be observed with respect to carbon emissions. Hence, partial collaboration can be harmful to the firms that do not participate, in terms of cost, and to the environment in terms of carbon emissions.

4. Concluding Comments and Directions for Future Research

In this paper, we presented a series of models that illustrate how carbon footprint considerations could be incorporated into operational models. We showed how important insights can be obtained to inform decision making by both operating firms and policy makers. These insights highlight the impact of operational decisions on carbon emissions and the extent to which adjustments to operations can mitigate emissions. Our results show that operational adjustments alone can lead in some cases to significant emission reductions without significant increases in cost. The results also show that the full impact of different regulatory policies cannot be assessed without accounting for the operational adjustments firms could make in response to the regulation. More significantly, the results highlight the opportunity that exists to leverage collaboration across the supply chain to mitigate the cost of reducing emissions; they also highlight important implications of such collaboration (e.g., shift in roles among members of the same supply chain and the shift in associated costs and emissions).

Some of the insights we have obtained appear to be consistent with effects observed in countries where emissions have been subject to regulation. For example, Grubb and Brewer (2009), in a recent report on lessons learned from the EU experience, make the following observations: (1) introducing a cap-and-trade system has had a significant impact on carbon emissions, with reductions of up to 5% since 2005 in the covered sectors (see also Ellerman and Buchner 2008); (2) this reduction in emissions has had a limited economic impact, with the imposed caps estimated to cost less than 1% of total GDP by 2020; (3) firms in nearly all sectors covered by emission caps have been able to profit from the introduction of a cap-and-trade system, either from the selling of some of their emission allowances, undertaking cost-efficient emission reduction measures, or by passing the cost of carbon to consumers; and (4) despite generous emission caps, firms in many sectors, most notably the cement industry, have found simple and cheap ways to significantly reduce their energy consumption. There are undoubtedly factors other than those captured in our models that contribute to these observations. However, the observations seem to support the robustness of some of the insights we obtained from the models.

Avenues for future research are numerous. We highlight a few of these below. In doing so, our goal is not to provide a comprehensive list but simply to emphasize the richness of operational problems that could be revisited with carbon footprint in mind. As highlighted in this paper, there are numerous facets to how environmental concerns and government policies might affect operations. The analysis we carried out in this paper highlights some of these interactions. However, each of the issues raised in the paper, as well as others, is worthy of more comprehensive and more rigorous treatment.

The analysis we carried out in this paper was based on specific models of operations management, namely variants of traditional lot sizing models. Similar analysis could be carried out using other common models of operations such as newsvendor models, economic order quantity models, or models for stochastic dynamic inventory control. Using these models, it might be possible to characterize analytically the impact of carbon emission limits and carbon prices on the structure of optimal policies.

In this paper, we have focused on decisions regarding production and procurement. There are of course other operational decisions that are affected by concerns for carbon emissions, including facility location, supplier selection, capacity planning, choice of transportation mode, and raw material and component selection, among many others. Constructing “carbon-enhanced” models for these decisions could uncover new ways in how operational adjustments can be used to mitigate carbon emissions.

Moreover, it would be useful to extend the modeling to more complex supply chain structures, such as those involving assembly or distribution, and to supply chains with multiple products with shared components and resources. With these more complex supply chains, there are additional opportunities for supply chain collaboration that arise from the possibility of joint replenishment and transportation and/or production coordination. This increased collaboration could however make it more difficult to accurately assign the carbon footprint to the various parties or the various products involved. For example, if the products were to be assigned a carbon footprint label, schemes must be devised to correctly and fairly attribute carbon emissions to products that share the same production facilities, warehouses or transportation vehicles.

Insights described in this paper revealed the importance of interactions between firms in determining overall emission levels and corresponding costs. There is an opportunity to build models to analyze both cooperative and competitive interactions that may result from carbon emissions. For example, it would be useful to study how collaborative coalitions might form, how costs and revenues might be shared among members of these coalitions, and the characteristics of cost and revenue sharing schemes that lead to

coalition stability. It would also be useful to study how competing firms or supply chains modify their decisions to take into account emission levels of other firms, when these levels affect market share (because of environmental concerns by customers) or when they affect profitability (because emission levels affect the availability and price of carbon).

A few countries are discussing the possibility of charging carbon tariffs for imported goods that would take into account the carbon footprint of these goods. Once implemented, such tariffs would obviously have a significant impact on global supply chains and international trade. In particular, this could impact decisions firms make about where to locate facilities, from which suppliers to procure, and in which markets to sell. For policy makers, it is important that if tariffs are imposed, they do generate the desired positive impact on the environment while minimizing the cost to firms and consumers. Therefore, it would be important to develop models that could guide policy makers in devising the right set of tariffs and to assist operating firms in optimizing their operations in the presence of such tariffs.

Models such as the ones described in this paper could also be useful at the product design stage in identifying opportunities for designing products with features that account for both cost and emissions over the product's entire supply chain. This may require taking into account where products are to be produced and in what quantities, where they are to be sold and consumed, how far they are likely to be shipped and with what mode, how frequently they are to be ordered, and where and how long they will be stored at various stages of the supply chain. Such analysis, which may be integrated with the more typical lifecycle analysis, could affect choice of material, components, geometry, and packaging, among others (see for example the case studies discussed in Carbon Trust 2006).

In this paper, we have focused on the role of quantitative models in informing the decisions of both firms and policy makers. It would be useful to carry out empirical work that can be used to validate or enrich the results from the analytical models. For example, there is already carbon emission control legislation that has been in place in various countries, such as those in the EU, for several years now. It might be possible to further document the impact this legislation has had on the operations of various firms in those countries and on emission levels and carbon prices. In particular, it would be useful to identify the types of operational adjustments that firms have made in response to climate control legislation and the impact these adjustments have had on emissions and cost. It would also be of interest to compare how differences in legislation from country to country (e.g., those that have adopted a carbon

tax versus those with a cap-and-trade system) have affected differently operational decisions made in those countries.

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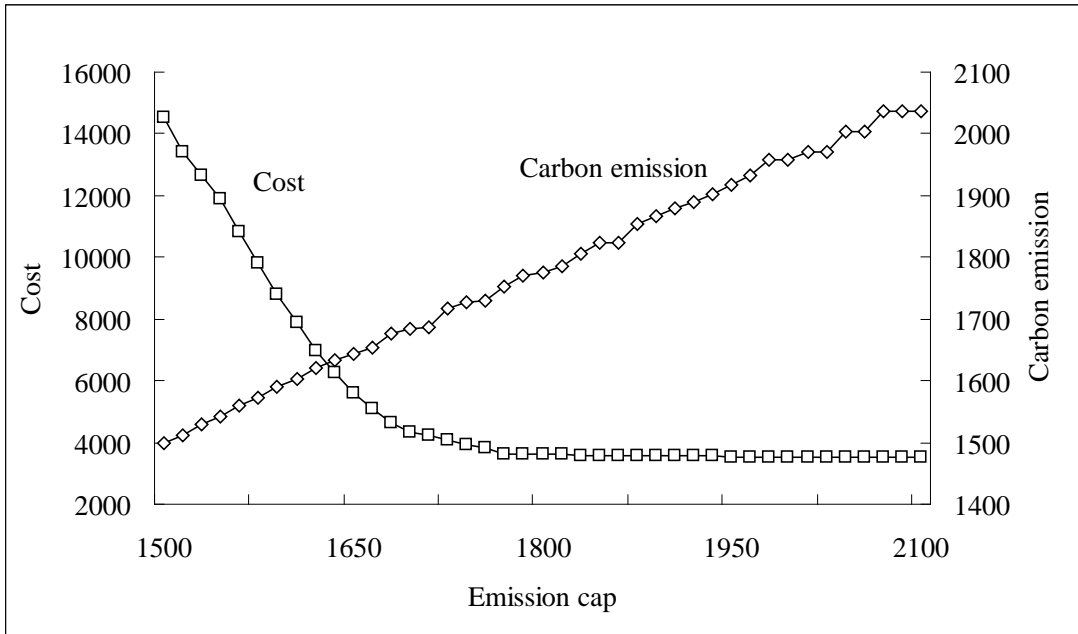


Figure 1: The effect of emission cap on cost and carbon emission

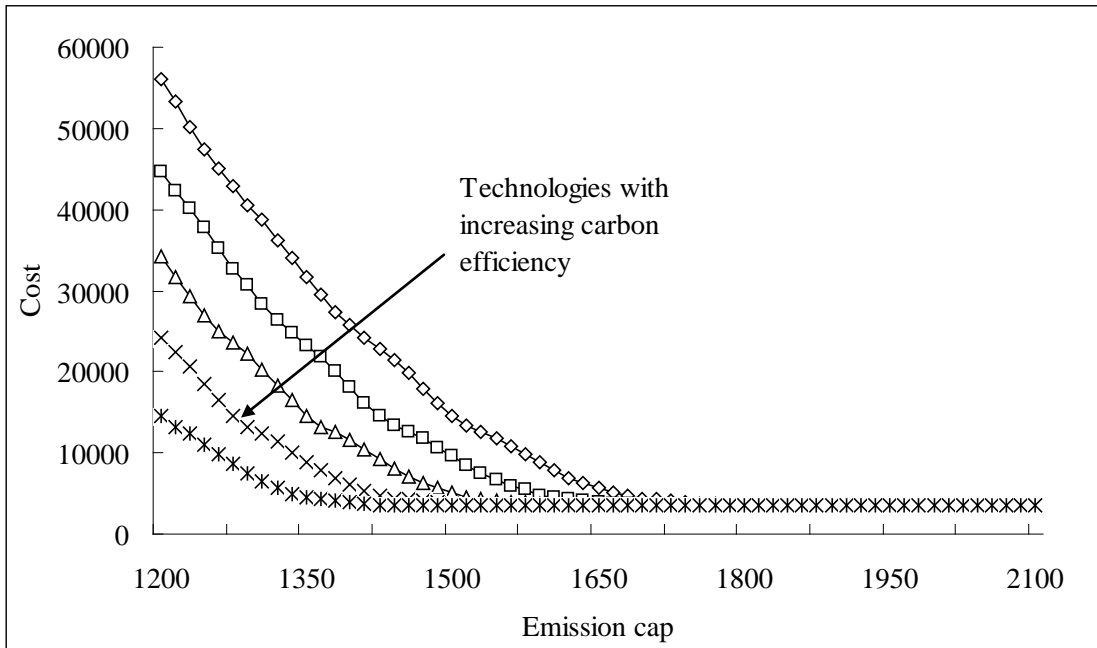


Figure 2: The effect of carbon efficiency and emission cap on cost (the cost shown does not include the investment cost needed to reduce the carbon emission parameters)

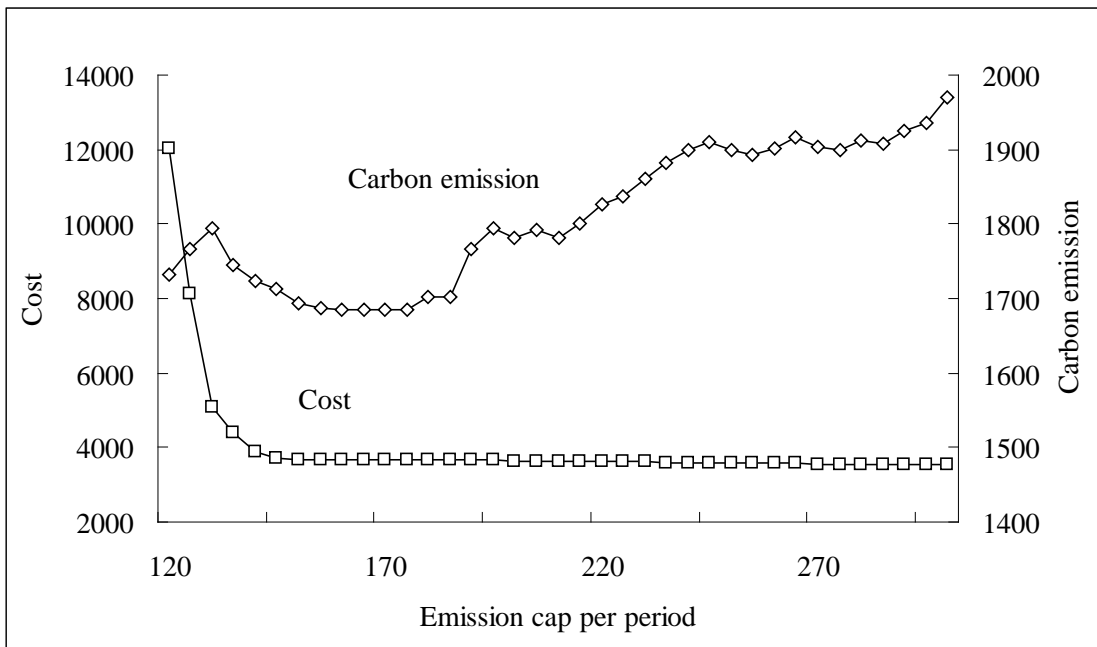


Figure 3: The effect of emission cap on cost and carbon emission when the emission cap is imposed on each period

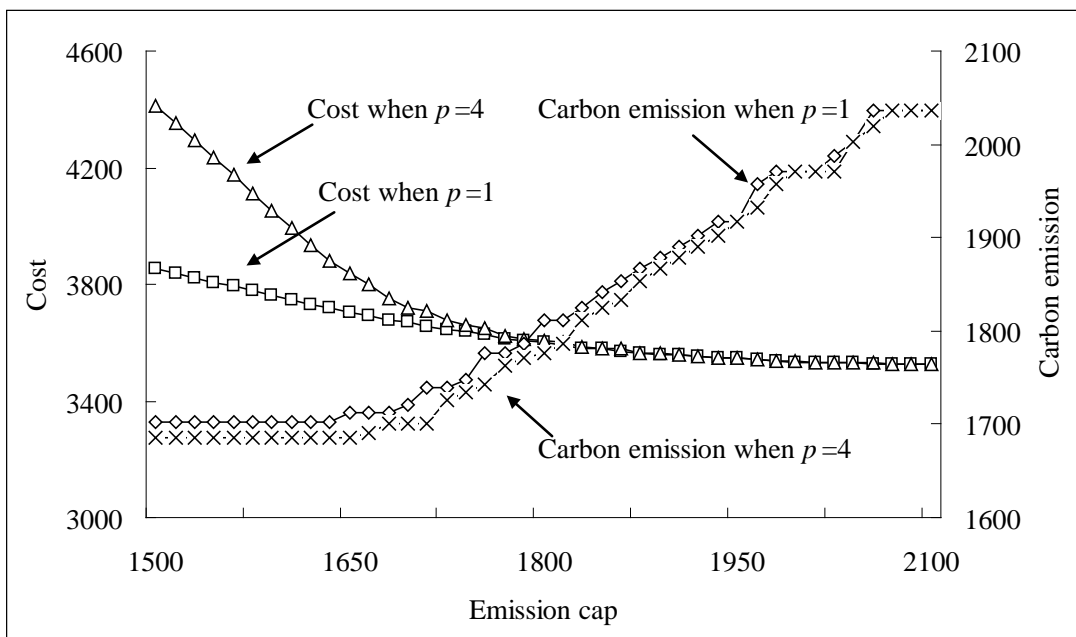


Figure 4: The effect of emission cap and carbon offset price on cost and carbon emission

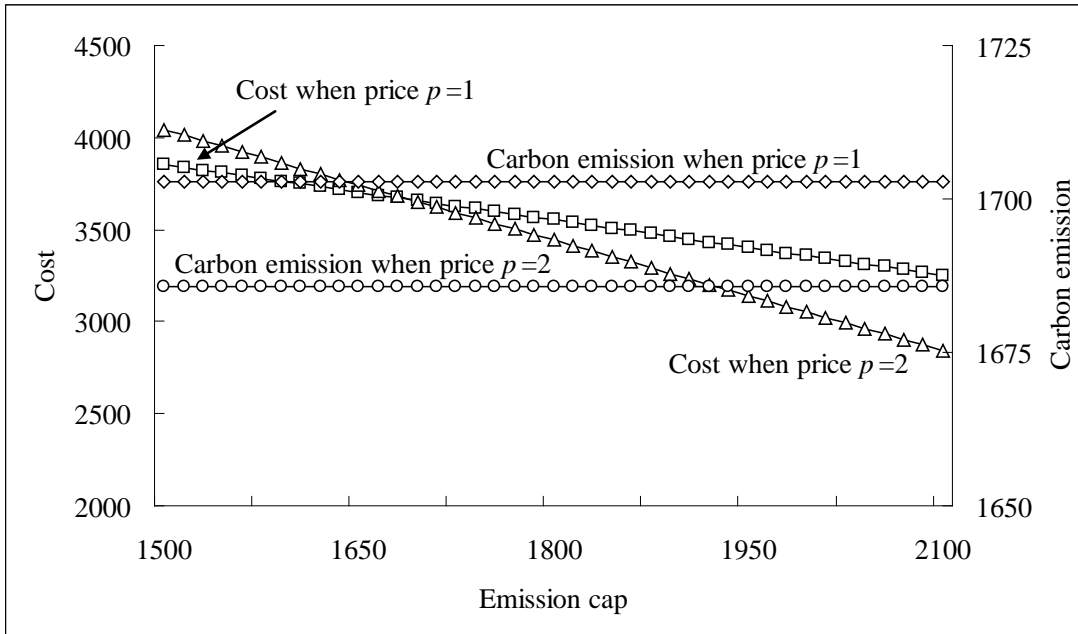


Figure 5: The effect of emission cap and carbon price on cost and carbon emission under cap and trade

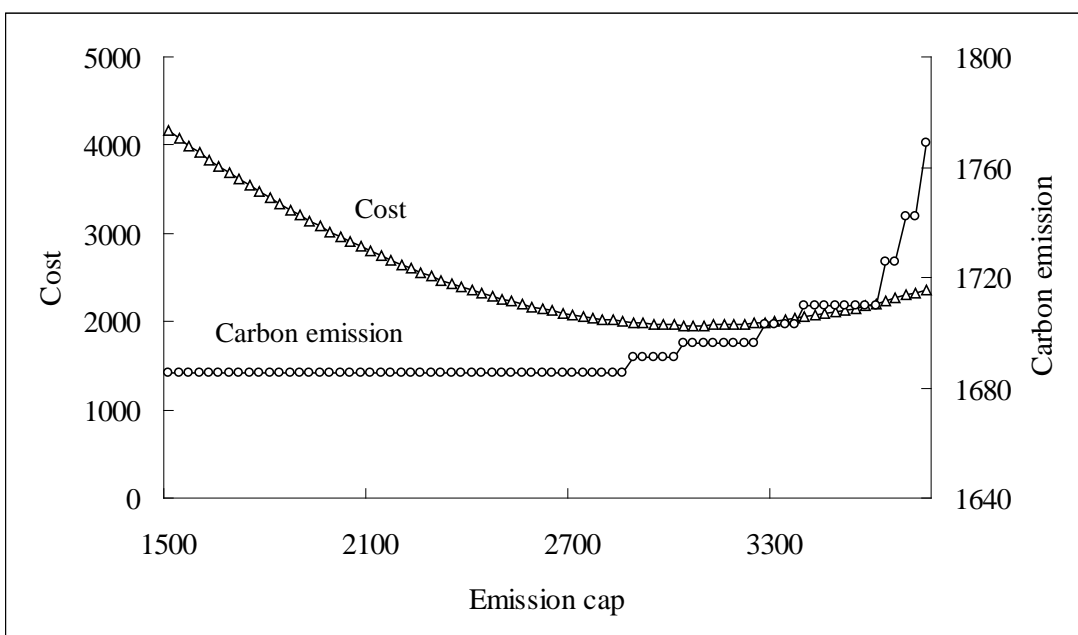


Figure 6: The effect of emission cap on cost and carbon emission under cap and trade when carbon price is dependent on the emission cap

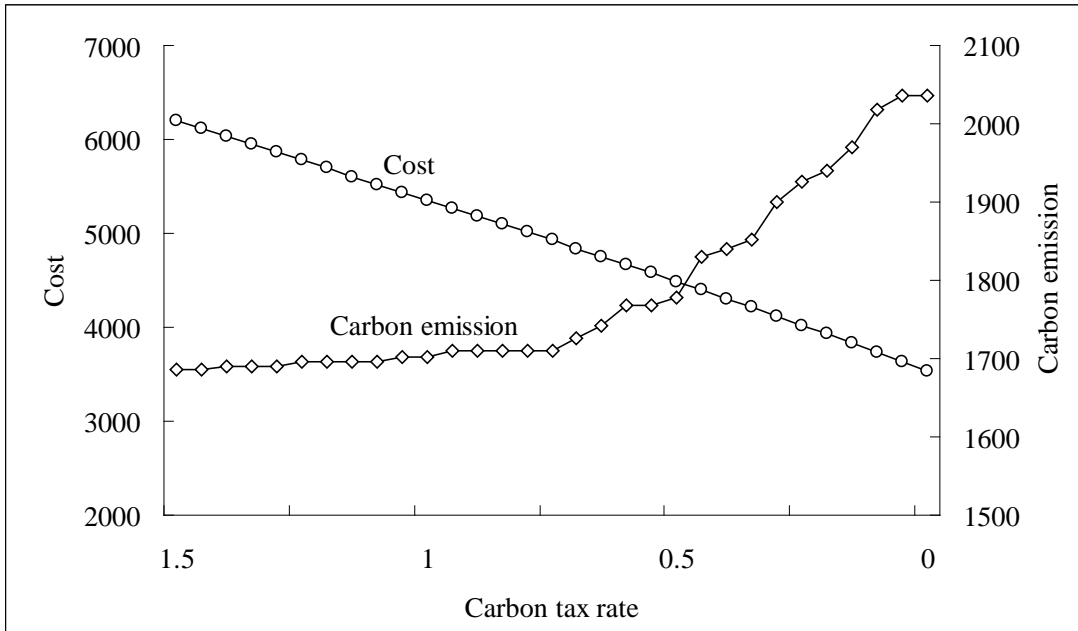


Figure 7: The effect of the carbon tax rate on cost and carbon emission

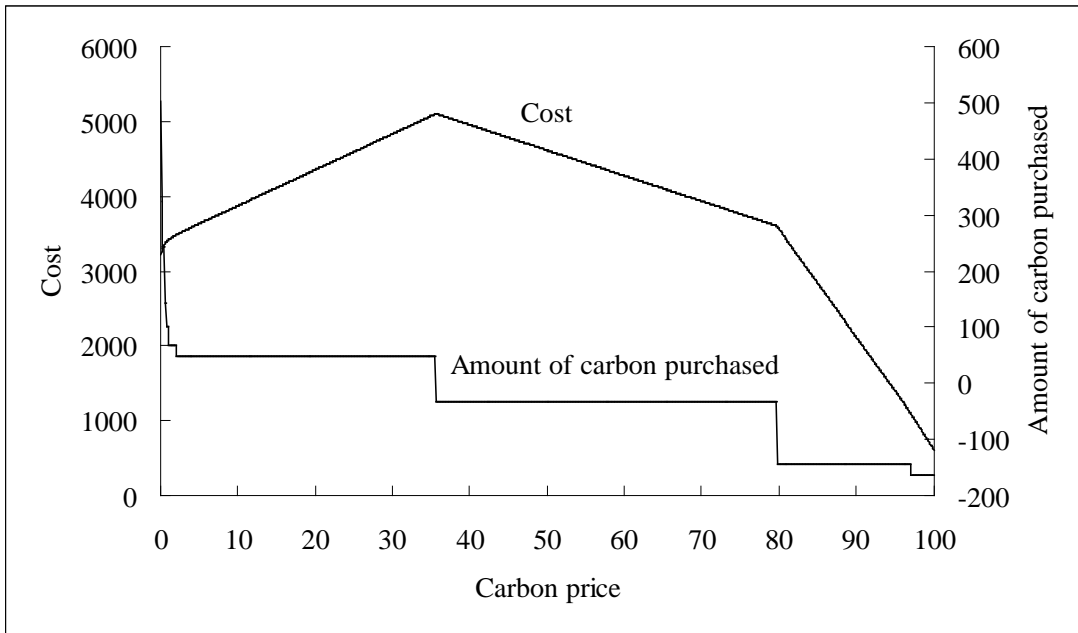


Figure 8: The effect of carbon price on cost and the amount of carbon purchased (a negative amount corresponds to carbon sold)

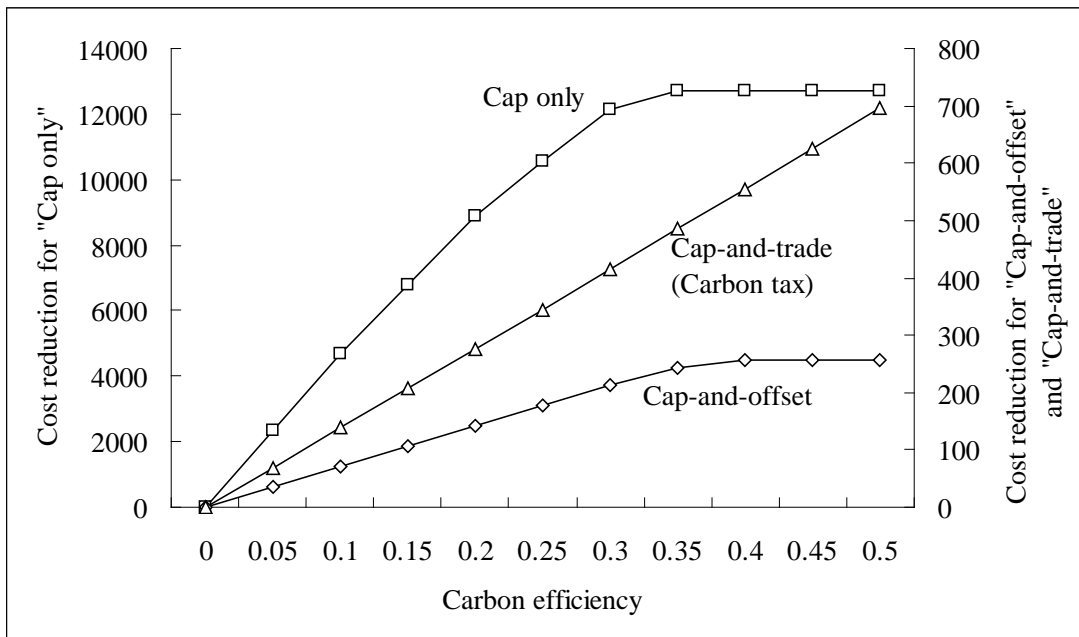


Figure 9: The benefit of carbon efficient technologies under different policy mechanisms (the benefit is measured by the cost reduction relative the base case of technology with zero carbon efficiency)

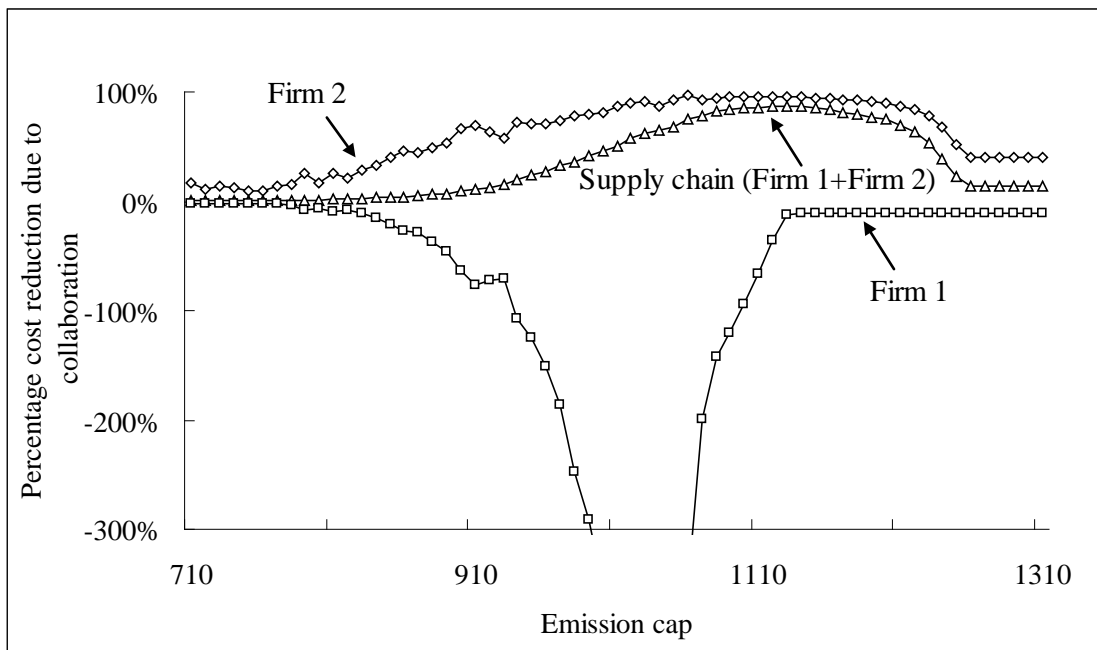


Figure 10: The impact of collaboration on cost when individual caps are imposed on each firm (a negative percentage corresponds to a cost increase; at its lowest point, which is not shown, the percentage reduction for firm 1 is approximately -400% for a cap of 1030)

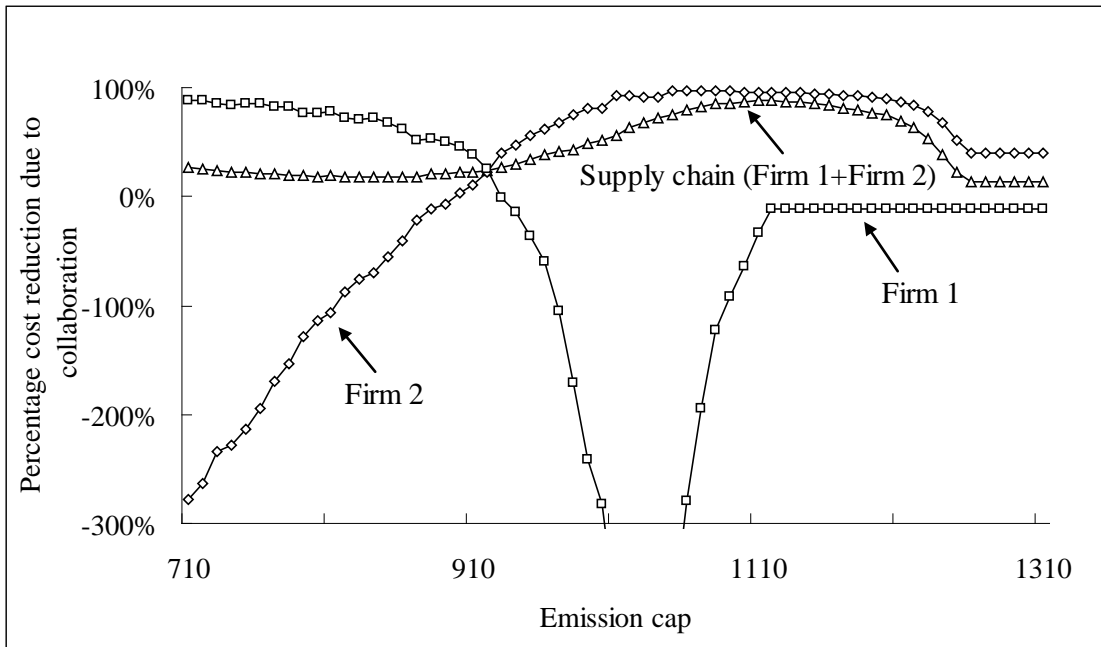


Figure 11: The impact of supply chain collaboration on cost when a shared cap is imposed on the entire supply chain (at its lowest point, which is not shown, the percentage reduction for firm 1 is approximately -500% for a cap of 1030)

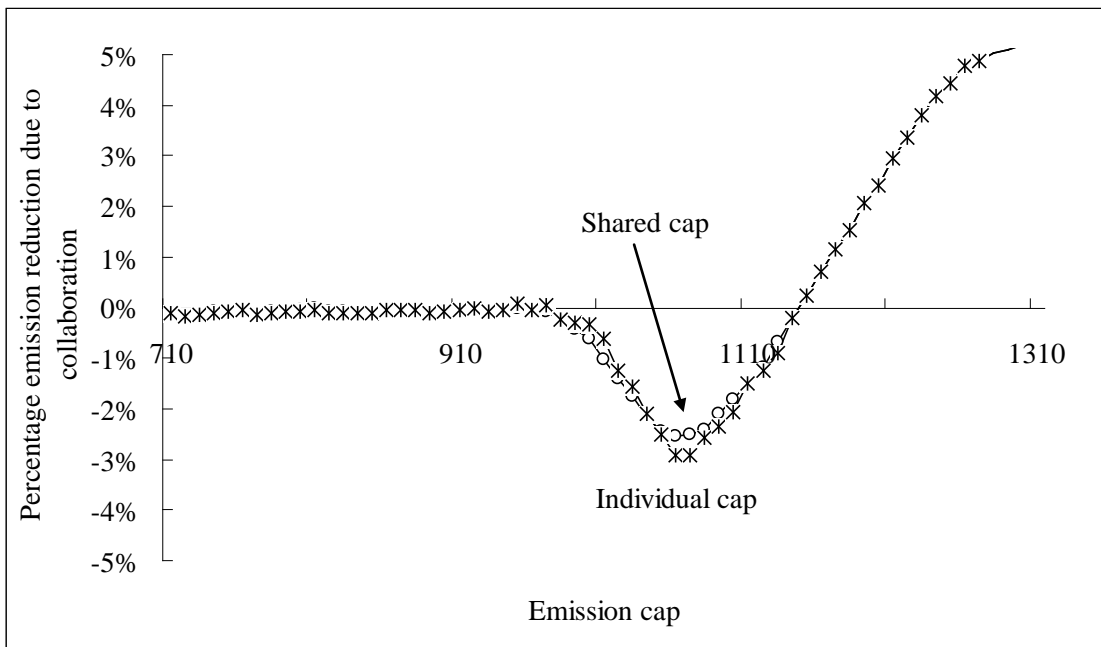


Figure 12: The impact of supply chain collaboration on carbon emissions for the entire supply chain, with and without a shared emission cap (a negative percentage corresponds to emission increases due to collaboration)

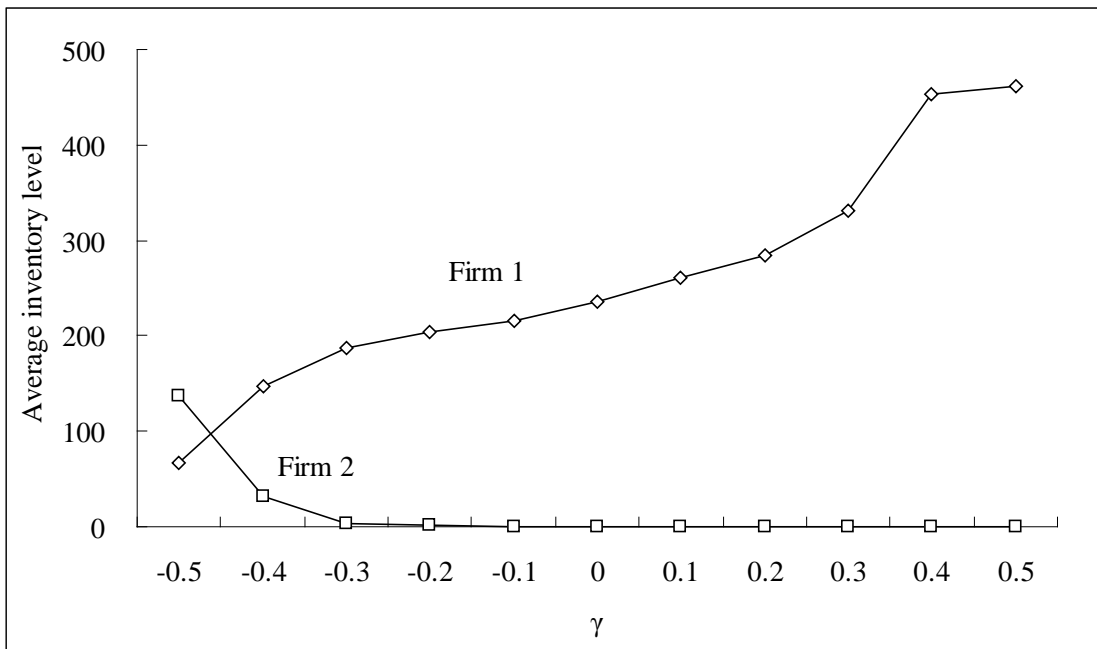


Figure 13: The impact of inventory holding carbon footprint on the average inventory held by the firms in the supply chain ($\hat{h}_1 = 1 - \gamma, h_2 = 1 + \gamma$)

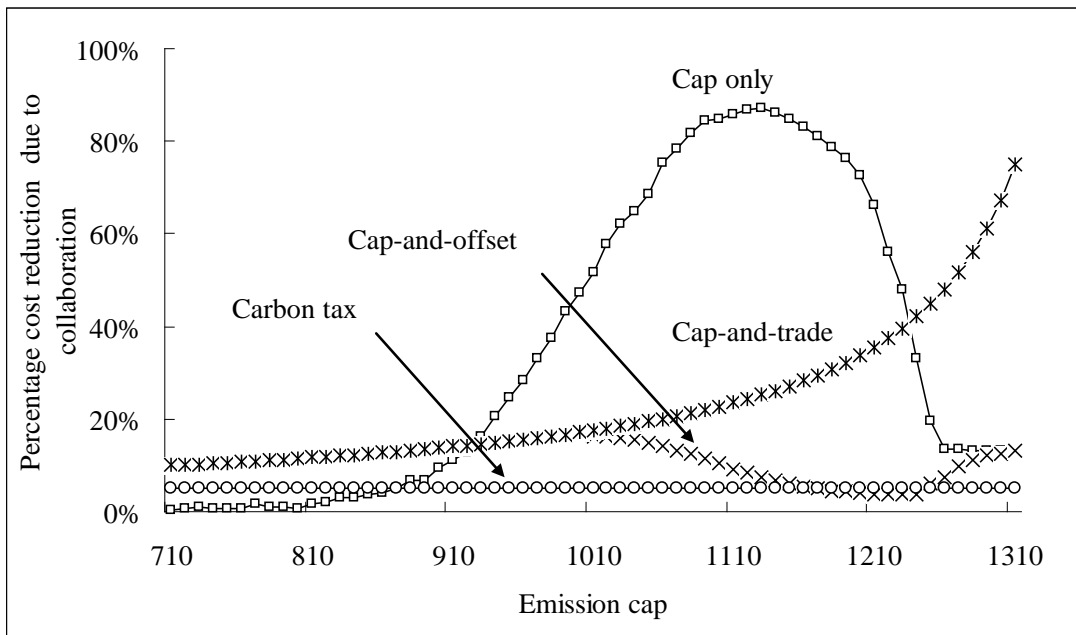


Figure 14: The effect of regulatory policies on the benefit of supply chain collaboration under varying emission cap levels

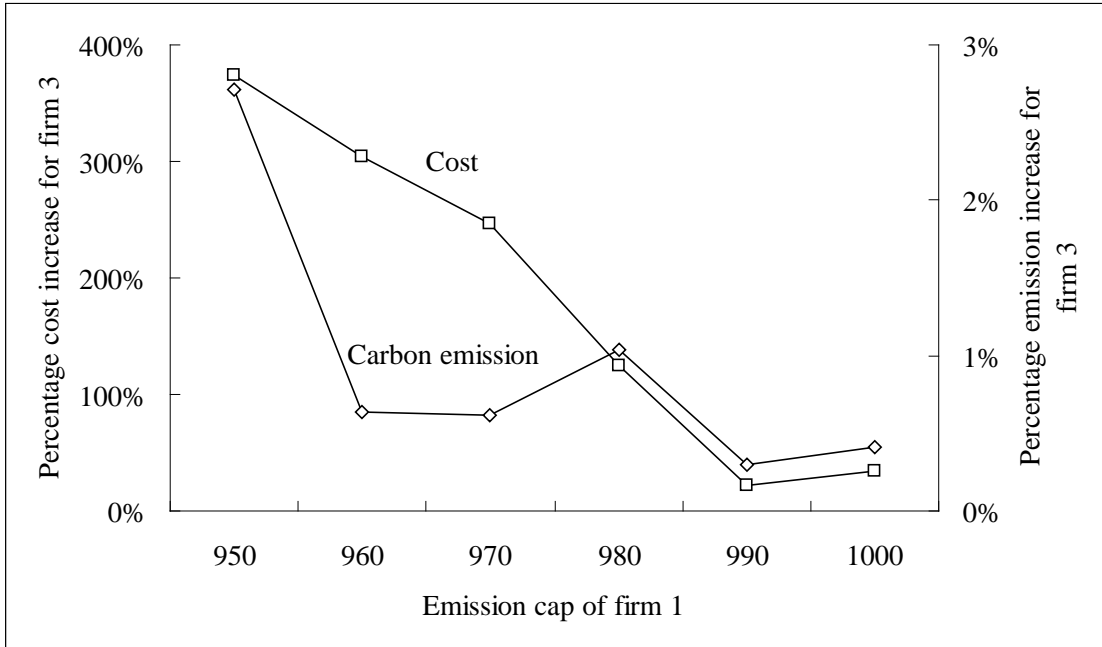


Figure 15: The impact of partial supply chain collaboration on the cost and emission of the firm that does not participate in the collaboration

Appendix 1: Experimental Setup for the Numerical Results

Unless stated otherwise, Figures 1-15 are based on a set of examples with similar parameter values. In particular, the planning horizon in all cases is 15 periods. End demand in each period is generated independently from a uniform distribution over the interval [20, 70]. Each data point shown in each figure corresponds to the average of 20 different demand series. The same demand series are used for all the figures. In all cases, and unless stated otherwise, cost and carbon emission parameters are stationary. The parameters are given by the following¹⁷:

$$h_t = 1; f_t = 60; c_t = 4; b_t = 100; \hat{h}_t = 2; \hat{f}_t = 20; \text{ and } \hat{c}_t = 2 \text{ for all } t,$$

in settings with a single firm, and by

$$h_{i,t} = 1; f_{i,t} = 30; c_{i,t} = 0; b_{i,t} = 100; \hat{h}_{i,t} = 1; \hat{f}_{i,t} = 40; \text{ and } \hat{c}_{i,t} = 1 \text{ for all } i \text{ and all } t,$$

in settings with multiple firms. All the results were obtained by solving the corresponding optimization problems as specified below. The problems were solved using the commercial solver ILOG CPLEX version 11.1 running on a personal computer workstation with an Intel CPU of 3.2 GHz and 1 GB of memory.

The following is additional information specific to each of the figures.

- Figure 1 is based on model P1.
- Figure 2 is based on model P1 with different plots corresponding to different carbon emission parameter values. These parameter values are varied by varying i from $i = 0$ to $i = 4$ as follows: $\hat{f}_i = 0.95^i f_0$, $\hat{c}_i = 0.95^i c_0$, and $h_i = 0.95^i h_0$. where the values $\hat{f}_0 = 20$, $\hat{c}_0 = 2$, and $h_0 = 2$ correspond to the base case.
- Figure 3 is based on a modified version of model P1 where constraint (3) is replaced by $\hat{f}_t y_t + h_t I_t + \hat{c}_t q_t \leq C$, for $t = 1, \dots, T$ (that is a carbon emission cap is imposed on each period instead of the entire planning horizon).
- Figure 4 is based on model P4.
- Figure 5 is based on model P3.
- Figure 6 is based on a modified version of model P3 where the objective function is given by (34)

¹⁷ As we discuss in Section 3, solutions to the lot sizing problem are primarily affected by the ratios f/h and \hat{f}/\hat{h} and less by the specific values of these parameters. When these ratios are different, there is an opportunity to adjust order quantities so that emissions are reduced, albeit with an increase in cost. The parameter values shown above are for a setting where such adjustment is possible. Settings where the ratios are similar are less interesting since in that case whatever order quantities minimize cost tend also to minimize emissions; see Appendix 2 for further discussion and analytical support.

with $p = 4 - 0.0009C$.

- Figure 7 is based on model P2.
- Figure 8 is based on model P3.
- Figure 9 is based on models P1, P2, P3, and P4 (for this figure, the emission cap is set to 1200 for the policies “cap only,” “cap-and-offset,” and “cap-and-trade;” similarly, carbon price (or carbon tax) is set to 2 for the policies “cap-and-offset,” “cap-and-trade,” and “carbon tax;” Carbon efficiency is measured relative to the base case $\hat{f}_t = 20$, $\hat{c}_t = 2$, and $h_t = 2$. For example, an efficiency of 10% means that the new carbon parameters are 10% smaller, i.e. $\hat{f}_0 = 18$, $\hat{c}_0 = 1.8$, and $h_0 = 1.8$).
- Figures 10-12 are based on modified versions of models P5, P6, and P7 involving two firms, firm 1 and 2 with firm 2 serving as the supplier to firm 1. To ensure that firm 2 is always able to fulfill orders from firm 1, in the experiments carried out, we replace constraint (17) with $I_{i,t} = I_{i,t-1} + B_{i,t} + q_{i,t} - q_{i-1,t}^*$, for $t = 1, \dots, T$; replace constraint (27) with $I_{i,t} = I_{i,t-1} + B_{i,t} + q_{i,t} - q_{i-1,t}$, for $t = 1, \dots, T$, $i = 1, \dots, N$; add the term $\sum_{t=1}^T b_{i,t} B_{i,t}$ to the objective function (16) and the term $\sum_{i=2}^N \sum_{t=1}^T b_{i,t} B_{i,t}$ to the objective function (25). The variables $B_{i,t}$ correspond here to the amounts of demand fulfilled through other means, such as outsourcing. A unit of demand fulfilled in this fashion incurs the higher fulfillment cost $b_{i,t}$.
- Figure 13 is based on the same modified version of model P7 used to generate Figures 10-12 and by assuming that firms are allowed to share their emission caps.
- Figure 14 is based on the same modified versions of models P5-P7 used to generate figures 10-12; these models are further modified to incorporate policy mechanisms, other than strict caps, as in models P2-P4; the results are for the case where the firms are subject to individual caps; and where the carbon price and the carbon tax factor are both set equal to 2.
- Figure 15 is based on similar models to those used to generate figures 10-12, except that in this case we have three firms (the parameters for all three firms are the same). Firms 1 and 2 collaborate, while firm 3 makes decisions independently. The emission cap of firm 1 is varied as shown in the figure. Emission caps of firm 2 and 3 are $C_2=1200$ and $C_3=950$.

Appendix 2: On the Robustness of the Numerical Observations¹⁸

In this appendix, we discuss the extent to which the observations we make in Section 3 may hold in general. We first discuss each observation individually. Then, we describe an analytical model, for which closed form solutions can be obtained, and which captures many of the effects observed for the discrete lot sizing model.

A.2.1 The Observations Revisited

In this section, we briefly comment on the extent to which each observation discussed in Section 3 might hold for problem parameters other than those used in the figures shown.

Observation 1: Per the discussion in Section 3, we expect Observation 1 to hold in settings where the ratios \hat{f}_i / \hat{h}_i and f_i / h_i are different. The extent to which significant emission reductions can be achieved with relatively limited impact on cost will generally depend on how the total cost and total emission functions are affected by varying the order quantity. In Section A.2.3, we show that for an important class of problems, which can be modeled using the classic EOQ model, this is indeed the case. As we discuss in that section, the result may apply to other classes of problems as long as certain condition on the cost and emission functions hold.

Observation 2: This observation would hold when the ratios \hat{f}_i / \hat{h}_i and f_i / h_i are different and the cost of introducing technology that reduces the emission parameters is high.

Observation 3: By taking away the ability of a firm to emit more in some periods (if this allows it to emit less in other periods), we limit the decision making flexibility of the firm. This loss of flexibility can prevent an efficient allocation of cost and emissions over the planning horizon. This effect is quite general and has been shown to arise using other models (see for example Tietenberg 2006).

Observation 4: This observation illustrates the fact that firms that are not efficient at reducing their emissions might find it more cost-effective to purchase offsets from a third party than attempting to reduce its own emissions. It is likely that this effect would arise for a wide range of problem parameter values and for a variety of settings.

¹⁸ We leave it to the discretion of the editors and the recommendations of the referees whether or not this appendix is included with the paper. We provide it here as a part of our response to comments from the referees. However, it might also be of interest to the general reader.

Observation 5: This observation is true as long as the assumptions of a fixed carbon price and no limits on carbon trading hold. It is easy to show its validity (as we do on page 17, equation 33) for any choice of problem parameter values.

Observation 6: With an increase in carbon price, firms that can adjust their order quantities to reduce emissions earn income from selling the resulting emission credits even though their cost increases. When the price of carbon is above a certain *threshold*, the income from selling carbon exceeds the higher cost. This result does not depend on the specific choice of problem parameters; only the value of the *price* threshold does.

Observation 7: Similar to observation 6, the qualitative effects shown in Figure 9 do not depend on the values of problem parameters, although the specific values of the cost reductions associated with each policy do. In fact, the limiting behaviors (i.e., when the carbon cap gets very large) of each policy shown in Figure 9 can be easily shown (analytically) to always hold.

Observation 8: This observation and the associated Figures 10 and 11 highlight the possibility of significant gains from collaboration but also show that it is possible to see no gains at all (e.g., when the emission cap is low).

Observations 9 & 12: Both observations are meant to provide a counter-example. Therefore, the numerical examples provided are sufficient to support the observations.

Observation 10: This observation is true regardless of the problem parameter values because any solution that is feasible when each firm is subject to an independent emission constraint is also feasible when the supply chain is subject to a shared constraint.

Observation 11: This observation illustrates in particular the broader principle of collaboration being more valuable if it is the only mechanism available to firms for relaxing the emission cap constraint.

A.2.2 The Economic Order Quantity Model

In this section, we briefly describe an analytical model that captures the essential cost and emission tradeoffs of the lot sizing models described in this paper. The model is based on the widely used and studied economic order quantity (EOQ) model; see Lowe and Schwarz (1983), Poertus (1985), Dobson (1987), Roundy (1989), Weng (1995), Zipkin (2000) and the references therein. The model uses simplifying assumptions, including the assumptions that demand occurs continuously over time with a

constant rate (time is treated as a continuous parameter and not discretized), and that all cost and emission parameters are stationary and not time-dependent. A major advantage of the EOQ model is that it is mathematically tractable and leads to closed form solutions. In what follows we briefly describe the model and its solution in the context of a problem similar to P1 (i.e., a single firm with a strict emission cap). Details and extensions can be found in a companion note (Benjaafar and Chen 2010) available upon request from the authors.

Under the standard EOQ assumptions, the firm faces known deterministic demand with rate d per unit time which must always be satisfied (for simplicity, we assume that backorders are not allowed but they can be easily included). Cost parameters are stationary with f , c and h referring respectively to the fixed order cost, variable order cost, and inventory holding cost. Emission parameters are also stationary with \hat{f} , \hat{c} , and \hat{h} denoting the amount of carbon emissions associated per order initiated, per unit purchased or produced, and per unit held in inventory per unit time. The objective of the firm is to choose an order quantity q that minimizes its cost per unit time subject to the constraint on the amount of carbon emitted per unit time, C .

Given the above assumption, we can show that the problem can be expressed as follows:

$$\text{Problem A.P1: Minimize } f d/q + h q/2 + cd \quad (\text{A.1})$$

subject to

$$\hat{f} d/q + \hat{h} q/2 + \hat{c}d \leq C. \quad (\text{A.2})$$

Let \hat{q}_{\min} denote the order quantity that minimizes carbon emission, then it is easy to verify that $\hat{q}_{\min} = \sqrt{2\hat{f}d/\hat{h}}$ and the corresponding emission level is $\hat{E}_{\min} = \hat{c}d + \sqrt{2\hat{f}\hat{h}d}$. Consequently, problem A.P1 admits a feasible solution only if $C \geq \hat{E}_{\min}$. In the remainder of this section, we assume that this condition is always satisfied. Also, let q^* denote the order quantity that minimizes the total cost while ignoring the carbon emission constraint (that is q^* is the solution to the standard EOQ problem). Then that $q^* = \sqrt{2fd/h}$. The following Theorem characterizes the optimal solution to A.P1 (a proof of this and all other results can be found in Benjaafar and Chen 2010).

Theorem A.1: *Let*

$$q_1 = \frac{\hat{C} - \sqrt{\hat{C}^2 - 2\hat{f}\hat{h}d}}{\hat{h}} \quad \text{and} \quad q_2 = \frac{\hat{C} + \sqrt{\hat{C}^2 - 2\hat{f}\hat{h}d}}{\hat{h}}, \quad (\text{A.3})$$

where $\hat{C} = C - \hat{c}d$. Then the optimal solution to problem A.P1 is given by

$$\hat{q}^* = \begin{cases} q^* & \text{if } q_1 \leq q^* \leq q_2 \\ q_1 & \text{if } q^* \leq q_1 \\ q_2 & \text{if } q^* \geq q_2. \end{cases} \quad (\text{A.4})$$

Furthermore, the emission level under the optimal order quantity is

$$\hat{E}^* = \begin{cases} \hat{E}_{\max} & \text{if } q_1 \leq q^* \leq q_2 \\ C & \text{otherwise,} \end{cases} \quad (\text{A.5})$$

where

$$\hat{E}_{\max} = \hat{f} \sqrt{\frac{hd}{2f}} + \hat{h} \sqrt{\frac{fd}{2h}} + \hat{c}d \quad (\text{A.6})$$

and corresponds to the emission level in the absence of the carbon constraint (also corresponds to the emission level when the optimal order quantity is q^*).

The results in Theorem A.1 are due to the fact that both the total cost and total emissions are convex functions in the order quantity q . From the results in Theorem A.1, we can see that the emission constraint in (A.2) is always binding when $C < \hat{E}_{\max}$. Therefore, emissions are linearly increasing in the cap C from \hat{E}_{\min} to \hat{E}_{\max} and are equal to \hat{E}_{\max} thereafter. We can also show that the optimal cost is decreasing and strictly convex in C when $C < \hat{E}_{\max}$ and is constant and equal to $cd + \sqrt{2fhd}$ thereafter. These results are illustrated in Figure A.1. These results are consistent with Observation 1 in the paper.

As we argued in section A.2.1, when $f/h = \hat{f}/\hat{h}$, the optimal order quantity that minimizes cost also minimizes emissions (in other words, we have $\hat{q}^* = q^* = \hat{q}_{\min}$). When these ratios are different, choosing a value for q in the interval (\hat{q}_{\min}, q^*) if $\hat{q}_{\min} < q^*$ or in the interval (q^*, \hat{q}_{\min}) if $q^* < \hat{q}_{\min}$ leads to an emission reduction. This is illustrated in Figure A.2. Note that for the change in q to lead to a significant reduction in emissions without a significant increase in cost, two conditions are necessary. First, cost must be relatively insensitive to the choice of q in the region around q^* . Second, emissions must be sensitive to changes in q in regions that are far from \hat{q}_{\min} (see Figure A.2). In our case, both conditions can be shown to be satisfied. In particular, the optimal cost function is relatively flat around q^* (see discussion below) and the total emission function is strictly convex in q with emissions approaching ∞ when q approaches either 0 or ∞ .

To show the insensitivity of the optimal cost to the choice of q , consider the ratio of the fixed ordering and inventory holding costs when the optimal order quantity is q^* to the fixed ordering and inventory holding costs when the order quantity is q where $q \neq q^*$. This ratio is given by

$$\frac{f d/q + h q/2}{f d/q^* + h q^*/2} = \frac{f d/q + h q/2}{\sqrt{2fdh}}$$

which can be rewritten as

$$\frac{1}{2} \left[\frac{\sqrt{2fd/h}}{q} + \frac{q}{\sqrt{2fd/h}} \right] = \frac{1}{2} \left[\frac{q^*}{q} + \frac{q}{q^*} \right].$$

from which we can see for example that a percentage difference between q and q^* equal to α leads to an increase in cost that is less than α (e.g., doubling the order quantity by choosing $q = 2q^*$ leads to only a 25% increase in cost).

In addition to validating Observation 1 in the paper, the results of theorem A.1 can be used to validate Observations 2. It is similarly straightforward to extend the formulation in A.P1 to settings with carbon tax, cap-and-offset, and cap-and-trade per the formulations P2-P4. The observations associated with these settings can be similarly validated analytically. Moreover, it is possible to extend the single firm models to models with a supply chain consisting of multiple firms per the formulations in P5-P7 and to analytically validate Observations 9-12. For the sake of brevity, we omit the details and refer to Benjaafar and Chen for the details.

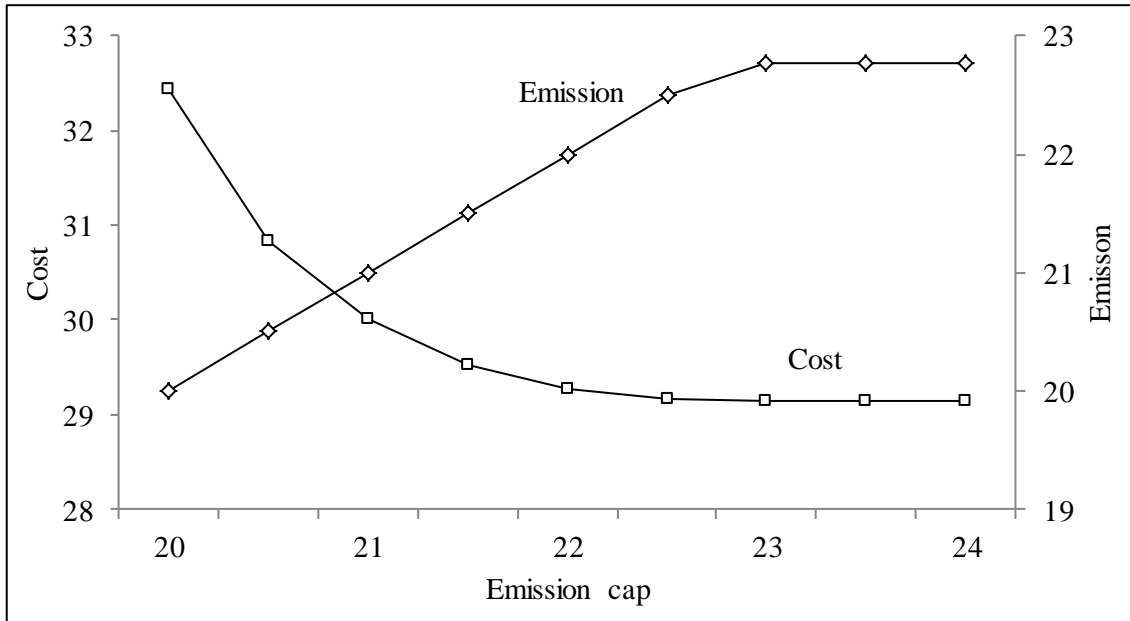


Figure A.1: The effect of emission cap on cost and carbon emissions
 ($f = 100$, $h = 0.1$, $d = 10$, $\hat{C} = 1.5$, $\hat{f} = 10$, $\hat{h} = 0.1$, $\hat{c} = 10$)

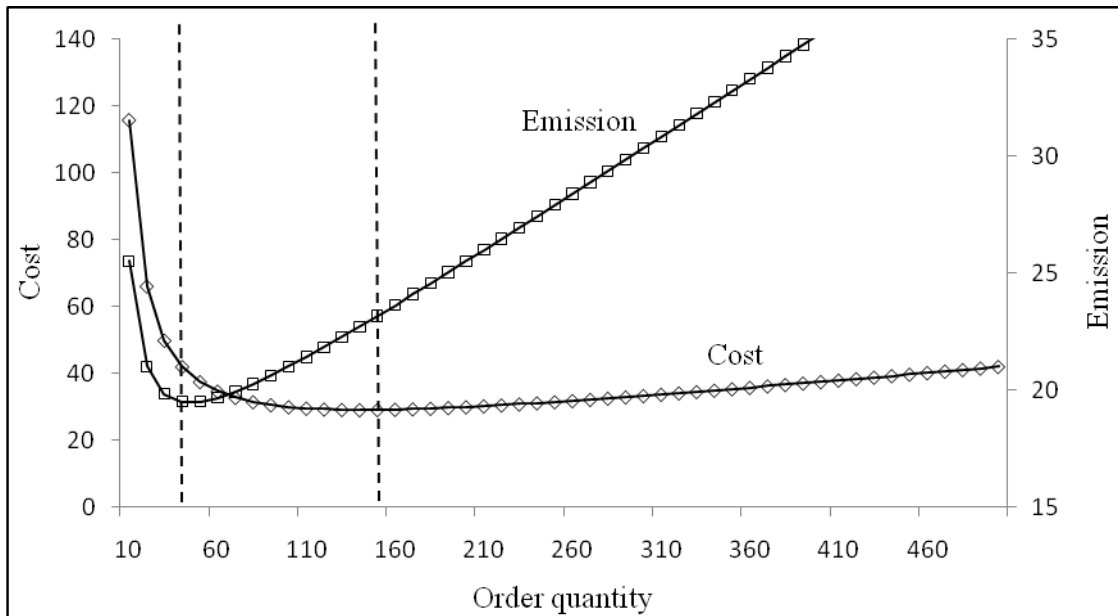


Figure A.2: Cost and emission versus order quantity
 the dashed lines represent respectively the cost-optimal and emission-optimal order quantities;
 ($f = 100$, $h = 0.1$, $d = 10$, $\hat{C} = 1.5$, $\hat{f} = 10$, $\hat{h} = 0.1$, $\hat{c} = 10$)