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The carbon-constrained EOQ

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ABSTRACT

In this paper, we provide analytical support for the notion that it may be possible, via operational adjustments alone, to significantly reduce emissions without significantly increasing cost. Using the EOQ model, we provide a condition under which it is possible to reduce emissions by modifying order quantities. We also provide conditions under which the relative reduction in emissions is greater than the relative increase in cost and discuss factors that affect the difference in the magnitude of emission reduction and cost increase. We discuss the applicability of the results to systems under a variety of environmental regulations, including strict carbon caps, carbon tax, cap-and-offset, and cap-and-price.

1. Introduction

In pursuing carbon emission reduction efforts, firms have focused for the most part on reducing emissions due to the physical processes involved (e.g., replacing energy inefficient equipment and facilities, redesigning products and packaging, deployment and use of less polluting sources of energy); see [3], the unabridged version of the paper. These efforts are clearly valuable. However, they can overlook a potentially significant source of emissions, one that is driven by business practices and operational policies. In this paper, we examine the extent to which operational adjustments alone could indeed be effective in reducing emissions. We also examine the extent to which such adjustments could take place without significantly increasing cost. This is important because resistance to environmental regulation has often been based on concerns that such regulation would lead to significantly higher costs.

Our analysis is in part motivated by a recent paper [1], in which the authors observe that it is possible to significantly reduce carbon emissions without significantly increasing cost by making only operational adjustments. Their observations are based on numerical results obtained for a lot sizing problem in which a firm decides on production/procurement quantities over a finite planning horizon consisting of discrete periods. In this paper, we use the framework of the economic order quantity (EOQ) model to provide analytical support for similar observations. We provide a condition under which it is possible to reduce emissions by modifying order quantities. We also provide conditions under which the relative reduction in emissions is greater than the relative increase in cost and describe when the difference between the two is maximized. We discuss the applicability of these results to systems operating under a variety of regulatory policies and to other operational models. We show that, the key requirements are that the cost and emission functions yield different optimal solutions, implying that the cost tradeoffs are different from the emission tradeoffs, and that the cost function is flat around the optimal solution but can be steep elsewhere. We show that significant reductions in emissions can indeed be achieved without significant increases in cost whenever the flat region of the cost function coincides with the steep region of the emission function. In the unabridged version of the paper [3], we show that these features are present in other operational models, including the facility location and newsvendor models, among others.

The results in this paper indicate that the opportunity for reducing carbon emissions via operational adjustments exists whenever the operational drivers of emissions are different from the operational drivers of costs. In settings where this is not the case (e.g., operational decisions that reduce cost tend to also reduce emissions), operational adjustments will obviously be ineffective. In that case, investments in efforts that modify the emission function (e.g., investments in efforts or technologies that lead to reductions in the emission parameters of underlying processes and activities) would be necessary.

Although there is growing literature that is concerned with issues of sustainability in operations (see [1] for a review), papers that explicitly consider emissions are relatively few. Hua et al. [6] consider a model similar to the cap-and-price model

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we consider in Section 3. They compare the cost and order quantity obtained under cap-and-price to those obtained without carbon considerations. Cachon [2] studies the emission tradeoffs associated with the size and location of retail facilities and shows that carbon pricing would have little impact on these decisions; see also [5,8] for related analysis.

2. Problem formulation and results

Consider a firm that faces a constant demand with rate D per unit time. Each time the firm places an order (either with its internal production facility or with an external supplier), it incurs a fixed cost A per order. The firm also incurs a holding cost h per unit kept in inventory per unit time, and a cost c per unit purchased or produced. Without loss of generality, we assume that orders are delivered with zero lead time (a positive lead time can be included and does not affect the solution to the problem); we also assume that the firm must satisfy all the demand (the analysis can be easily extended to settings with backorders). Total cost per unit time is then given by

$$\frac{AD}{Q} + \frac{hQ}{2} + cD.$$

Similar to cost, emissions are associated with ordering, inventory holding, and production/purchasing, with $\hat{A}$, $\hat{h}$ and $\hat{c}$ denoting the amount of carbon emissions associated per order initiated, per unit held in inventory per unit time, and per unit purchased or produced. Total emission per unit is therefore

$$\frac{\hat{A}D}{Q} + \frac{\hat{h}Q}{2} + \hat{c}D.$$

The parameterization of emissions under the above formulation is flexible and can be used to capture a variety of settings. For example, if emission from holding inventory depends only on the maximum amount of inventory held (which would correspond to Q), then total emission can be expressed as $\frac{\hat{A}D}{Q} + \hat{h}Q + \hat{c}D$, which is equivalent to the original expression but with an inventory holding emission parameter equal to $2\hat{h}$. Similarly, if emission from holding inventory is invariant to the amount of inventory held, then total emission reduces to $\frac{\hat{A}D}{Q} + \hat{h} + \hat{c}D$, which is equivalent to the original model but with an inventory holding emission parameter equal to 0. If the emission associated with initiating an order has both a fixed and a variable component, say of the form $\hat{A}_1 + \hat{A}_2 Q$, then the corresponding total emission is $\frac{\hat{A}_1 D}{Q} + \frac{\hat{h}Q}{2} + (\hat{c} + \hat{A}_2)D$, which again has the same form as the original model. Note that, depending on the setting, $\hat{A}$ can be higher or lower than $\hat{h}$. For example, for some products transportation-related emissions are high but storage emissions are low or even negligible (e.g., canned foods) while for others the reverse may be true (e.g., refrigerated foods). Walmart recently discovered that the refrigerants used in grocery stores accounted for a larger percentage of Walmart's greenhouse gas footprint than its truck fleet (http://www.walmartstores.com/sites/responsibility-report/2012/sustainableFacilities.aspx). Tesco, the largest retailer in the UK, found that 26 percent of its direct emissions were due to refrigerant leakage while only 12 percent were due to transportation (http://www.tesco.com/climatechange/carbonFootprint.asp). Our analysis and results are applicable in all cases (see the end of this section for additional discussion).

The objective of the firm is to choose an order quantity Q that minimizes its cost per unit time subject to the constraint on the amount of carbon emitted (this cap can reflect either government regulations imposed on the firm or a voluntary effort by the firm to reduce its emissions by a specified amount). The amount of carbon emitted is constrained to be less than a certain cap C. The problem can then be formally stated as follows:

$$\text{Minimize } Z(Q) = \frac{AD}{Q} + \frac{hQ}{2} + cD \tag{1}$$

$$\text{subject to } \frac{\hat{A}D}{Q} + \frac{\hat{h}Q}{2} + \hat{c}D \le C. \tag{2}$$

Let $\hat{Q}_{\min}$ denote the order quantity that minimizes carbon emission (the *emission-optimal* solution), then it is easy to verify that $\hat{Q}_{\min} = \sqrt{\frac{2\hat{A}D}{\hat{h}}}$ and the corresponding emission level is $E_{\min} = \sqrt{2\hat{A}\hat{h}D} + \hat{c}D$. Consequently, the problem admits a feasible solution if and only if $C \ge E_{\min}$. In the remainder, we assume that this condition is always satisfied. Also, let Q^* denote the order quantity that minimizes the total cost while ignoring the carbon emission constraint (the *cost-optimal* solution). Then, it is easy to see that $Q^* = \sqrt{\frac{2AD}{h}}$, which corresponds to the standard EOQ solution. The following theorem characterizes the optimal solution to (1)–(2).

Theorem 1. *Let*

$$Q_1 = \frac{\hat{C} - \sqrt{\hat{C}^2 - 2\hat{A}\hat{h}D}}{\hat{h}} \quad \text{and} \quad Q_2 = \frac{\hat{C} + \sqrt{\hat{C}^2 - 2\hat{A}\hat{h}D}}{\hat{h}}$$

where $\hat{C} = C - \hat{c}D$. Then the optimal solution to problem (1)–(2) is

$$\hat{Q}^* = \begin{cases} Q^* & \text{if } Q_1 \le Q^* \le Q_2, \\ Q_1 & \text{if } Q^* \le Q_1, \\ Q_2 & \text{if } Q^* \ge Q_2. \end{cases}$$

Furthermore, the emission level under the optimal order quantity is

$$E(\hat{Q}^*) = \begin{cases} E_{\max} & \text{if } Q_1 \le Q^* \le Q_2 \\ C & \text{otherwise,} \end{cases}$$

where

$$E_{\max} = \hat{A}\sqrt{\frac{hD}{2A}} + \hat{h}\sqrt{\frac{AD}{2h}} + \hat{c}D$$

and corresponds to the emission level in the absence of the carbon constraint (also corresponds to the emission level when the optimal order quantity is Q^*).

Proof. From constraint (2), we can show that the optimal order quantity must satisfy $Q_1 \le \hat{Q}^* \le Q_2$. If $Q_1 \le Q^* \le Q_2$, then obviously $\hat{Q}^* = Q^*$ and the corresponding emission is

$$E(Q^*) = \frac{\hat{A}D}{Q^*} + \frac{\hat{h}Q^*}{2} + \hat{c}D = \hat{A}\sqrt{\frac{hD}{2A}} + \hat{h}\sqrt{\frac{AD}{2h}} + \hat{c}D.$$

If $Q^* \le Q_1$ then $\hat{Q}^* = Q_1$ because $Z(Q)$ is convex in Q and choosing a higher value for $\hat{Q}^*$ will lead to a higher cost. Similarly, if $Q^* \ge Q_2$ then $\hat{Q}^* = Q_2$ because choosing a lower value for $\hat{Q}^*$ will lead to higher cost. In both of these cases, constraint (2) is binding and, therefore, $E(\hat{Q}^*) = C$. □

In the following proposition, we show that cost is indeed decreasing and convex in the emission cap C while emission is linearly increasing in C, implying that reducing the emission cap leads initially to a larger relative emission reduction than the relative cost increase (e.g., in the example illustrated in Fig. 1, an emission reduction of 20% leads only to a 4% increase in cost).

Proposition 1. *For $C \ge E_{\min}$, emission is linearly increasing in C while cost is decreasing and convex in C.*

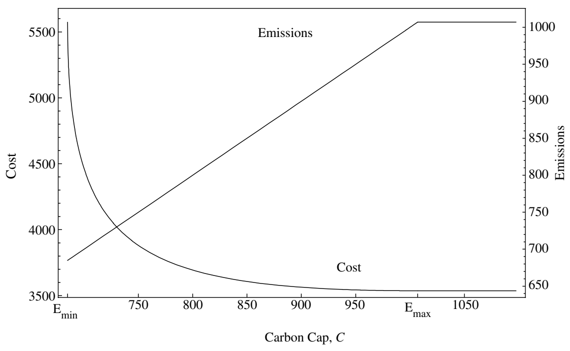


Fig. 1. The impact of the carbon cap on emission and cost ($D = 600$, $A = 120$, $h = 2$, $c = 5$, $\hat{A} = 2$, $\hat{h} = 3$, $\hat{c} = 1$).

Proof. For emission, the result follows immediately from Theorem 1. For cost, consider first the case where $Q^* = \hat{Q}^*$. In this case, cost and emissions are unaffected by the cap C and the results hold. Next, consider the case where $Q_1 \geq Q^*$. In this case, we have

$$Z(\hat{Q}^*) = Z(Q_1) = \frac{AD\hat{h}}{\hat{C} - \sqrt{\hat{C}^2 - 2\hat{A}\hat{h}D}} + \frac{h(\hat{C} - \sqrt{\hat{C}^2 - 2\hat{A}\hat{h}D})}{2\hat{h}} + cD.$$

Therefore,

$$\frac{\partial Z(\hat{Q}^*)}{\partial C} = \frac{\partial Z(\hat{Q}^*)}{\partial \hat{C}} = \frac{\partial}{\partial \hat{C}}\left(\frac{AD\hat{h}}{\hat{C} - \sqrt{\hat{C}^2 - 2\hat{A}\hat{h}D}} + \frac{h(\hat{C} - \sqrt{\hat{C}^2 - 2\hat{A}\hat{h}D})}{2\hat{h}}\right) = \frac{A\hat{h} + \hat{A}h}{2\hat{A}\hat{h}} + \frac{A\hat{h} - \hat{A}h}{2\hat{A}\hat{h}}\frac{\hat{C}}{\sqrt{\hat{C}^2 - 2\hat{A}\hat{h}D}}.$$

Consequently,

$$\frac{\partial^2 Z(\hat{Q}^*)}{\partial C^2} = \frac{\partial^2 Z(\hat{Q}^*)}{\partial \hat{C}^2} = \frac{A\hat{h} - \hat{A}h}{2\hat{A}\hat{h}}\frac{-2\hat{A}\hat{h}D}{(\sqrt{\hat{C}^2 - 2\hat{A}\hat{h}D})^3} = \frac{(\hat{A}h - A\hat{h})D}{(\sqrt{\hat{C}^2 - 2\hat{A}\hat{h}D})^3}.$$

Since $Q_1 \geq Q^*$ implies $\sqrt{\frac{2AD}{h}} \leq Q_1 \leq \sqrt{\frac{2\hat{A}D}{\hat{h}}}$, we have $\hat{A}h - A\hat{h} \geq 0$. Therefore, $\frac{\partial^2 Z(\hat{Q}^*)}{\partial C^2} \geq 0$. In the remaining case of $Q_2 \leq Q^*$ we can show using similar arguments that $\frac{\partial^2 Z(\hat{Q}^*)}{\partial C^2} \geq 0$. Consequently, the optimal cost function is convex with respect to the carbon cap C. □

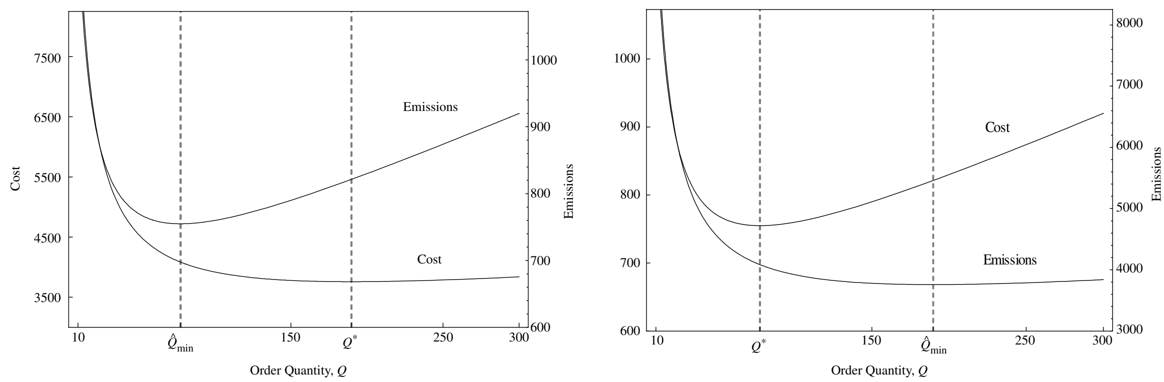
From Proposition 1, we can see that imposing an emission cap leads to an emission reduction only if the cap is sufficiently small, namely $C \leq E_{\max}$. Otherwise, the cost-optimal solution is feasible and the corresponding emission is $E_{\max}$. If $C \leq E_{\max}$, emission is reduced by adjusting the order quantity (by either increasing it or decreasing it from the cost-optimal order quantity). However, for this to be possible, the cost-optimal order quantity must be different from the emission optimal solution. This leads to the following important corollary.

Corollary 1. *Reducing emissions by adjusting order quantities is possible if and only if* $\frac{A}{h} \neq \frac{\hat{A}}{\hat{h}}$.

This corollary follows from the fact that if $\frac{A}{h} = \frac{\hat{A}}{\hat{h}}$ then the cost-optimal solution is also emission-optimal (i.e., $Q^* = \hat{Q}_{\min}$). In that case, emissions are already at their minimum and there is no operational adjustment that could further reduce them. On the other hand, if $\frac{A}{h} \neq \frac{\hat{A}}{\hat{h}}$, there is an opportunity to reduce emissions by either increasing or decreasing the order quantity. In particular, if $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$ ($\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$) then increasing (decreasing) the order quantity decreases emissions; see Fig. 2 for a graphical illustration.

The broader implication of Corollary 1 is that operational adjustments can lead to emission reductions only if the cost parameters are not strongly correlated to the emission parameters, so that what drives the cost more is not what also drives the emissions more. This is the case for example when the fixed costs are higher than the inventory holding costs but the fixed emissions are lower than the inventory-related emissions.

Although Corollary 1 identifies settings where it is possible to reduce emissions by adjusting order quantities, it does not specify the extent to which this reduction can be realized without significantly increasing cost. The examples shown in Figs. 1 and 2 do suggest that indeed a modest reduction in the order quantity (away from the cost-optimal order quantity) leads to a modest increase in cost but a significant reduction in emission (both cost and emission have similar functional forms, with both being convex in Q and approaching ∞ as Q approaches either 0 or ∞). More importantly, the cost function for the EOQ is *flat* in the region around the optimal solution. This means that in this region a relative change in the order quantity leads to a lower relative increase in cost. In what follows, we characterize this flatness and identify a condition under which the reduction in emission is



(a) ($D = 600$, $A = 120$, $h = 4$, $c = 5$, $\hat{A} = 10$, $\hat{h} = 2$, $\hat{c} = 1$). (b) ($D = 600$, $A = 10$, $h = 2$, $c = 1$, $\hat{A} = 120$, $\hat{h} = 4$, $\hat{c} = 5$).

Fig. 2. Cost versus emissions.

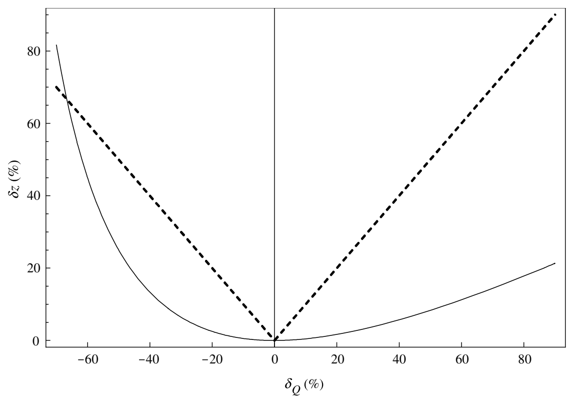


Fig. 3. The impact of changes in order quantity on cost (Values of δ_Z below the 45 degree lines (shown in dashes) correspond to cases where δ_Z is smaller than δ_Q).

greater than the increase in cost (for further discussion and related results see [4,7,9]).

Let $Z'(Q)$ and $E'(Q)$ refer to the components of cost and emission that are affected by order quantity. That is, $Z'(Q) = AD/Q + hQ/2$ and $E'(Q) = \hat{A}D/Q + \hat{h}Q/2$. Also, let

$$\delta_Q = \frac{Q - Q^*}{Q^*}$$

denote the relative change in the order quantity (with respect to the cost-optimal order quantity), and

$$\delta_Z = \frac{Z'(Q) - Z'(Q^*)}{Z'(Q^*)} \quad \text{and} \quad \delta_E = \frac{E'(Q^*) - E'(Q)}{E'(Q^*)}$$

denote respectively the corresponding relative change in cost and emission. Then, we can show that

$$\delta_Z = \frac{\delta_Q^2}{2(1+\delta_Q)} \quad \text{and} \quad \delta_E = -\frac{(1-\alpha)\delta_Q + \delta_Q^2}{(1+\alpha)(1+\delta_Q)}, \qquad (3)$$

where $\alpha = \frac{\hat{A}/\hat{h}}{A/h}$ is the ratio of the emission parameters to the cost parameters.

First, it is easy to see that even relatively large values of $|\delta_Q|$ lead to relatively small values of δ_Z. For example, increasing the order quantity by 30% ($\delta_Q = 0.3$) leads to an increase in cost of only 3.46% ($\delta_Z = 0.0346$); see Fig. 3 for a full characterization of δ_Z as a function of δ_Q. Mathematically, we can show that

$$\frac{\partial \delta_Z}{\partial \delta_Q} = \frac{\delta_Q(2+\delta_Q)}{2(\delta_Q+1)^2},$$

which is equal to 0 for $\delta_Q = 0$. Moreover, we can show that if $\delta_Q > 0$, then $\delta_Z \le \delta_Q$ (in other words, the relative increase in cost is always lower than the relative increase in the order quantity regardless of the size of the increase). If $\delta_Q < 0$, then $\delta_Z \le |\delta_Q|$ as long as $\delta_Q \ge -2/3$ (i.e., we can reduce order quantity by as much as $2/3$ without increasing cost by as much). In contrast,

$$\frac{\partial \delta_E}{\partial \delta_Q} = \frac{\alpha - (1+\delta_Q)^2}{(1+\alpha)(1+\delta_Q)^2}$$

and takes on a value equal to $\frac{\alpha-1}{\alpha+1} \ne 0$ for $\alpha \ne 1$ when $\delta_Q = 0$. That is, while the cost function is always flat around Q^*, the emission function can be quite steep (i.e., $|\frac{\alpha-1}{\alpha+1}|$ can be large). The fact that the flat region of the cost function coincides with the steep region of the emission function means that there is an opportunity to achieve more relative emission reductions than the corresponding relative increase in cost.

Next, we describe, the range over which the order quantity can be adjusted while guaranteeing that the relative increase in cost is less than the relative reduction in emission, that is $\delta_Z \le \delta_E$ and $\delta_E \ge 0$.

Proposition 2. *For $\alpha > 1$, $\delta_Z \le \delta_E$ if $0 \le \delta_Q \le \frac{2(\alpha-1)}{3+\alpha}$, and $\delta_Z \ge \delta_E$ otherwise. For $\alpha < 1$, $\delta_Z \le \delta_E$ if $\frac{2(\alpha-1)}{3+\alpha} \le \delta_Q \le 0$, and $\delta_Z \ge \delta_E$ otherwise.*

The proof immediately follows from the expressions of δ_Z and δ_E in Eq. (3) and for brevity, we omit the details. As we can see, the interval over which the order quantity can be varied depends solely on α with its width increasing in the absolute value of the difference between $\frac{\hat{A}}{\hat{h}}$ and $\frac{A}{h}$. In the limit cases of either $\alpha \to 0$ or $\alpha \to \infty$ the order quantity can be adjusted by as much as a factor of 3. When $\delta_Z = \delta_E$ (and $\delta_Q = \frac{2(\alpha-1)}{3+\alpha}$) the resulting relative decrease in emission and in cost is given by

$$\delta_{E=Z} = \frac{2(\alpha-1)^2}{(1+3\alpha)(3+\alpha)}$$

which is also increasing in the absolute value of the difference between the ratios $\frac{\hat{A}}{\hat{h}}$ and $\frac{A}{h}$. In the limit, as either $\alpha \to 0$ or $\alpha \to \infty$, $\delta_{E=Z} \to 2/3$, implying that emissions could be reduced by up to $2/3$ without increasing cost by as much.

The following proposition further characterizes the tradeoff between cost and emission reductions.

Proposition 3. *Let $\delta_E(\delta_Z, \alpha)$ denote the relative emission reduction as a function of the relative cost increase δ_Z and α. Then,*

$$\delta_E(\delta_Z,\alpha) = \begin{cases} -\dfrac{(\delta_Z + \sqrt{2\delta_Z + \delta_Z^2})(1 - \alpha + \delta_Z + \sqrt{2\delta_Z + \delta_Z^2})}{(1+\alpha)(1+\delta_Z + \sqrt{2\delta_Z + \delta_Z^2})}, & \text{if } \alpha > 1, \\ -\dfrac{(\delta_Z - \sqrt{2\delta_Z + \delta_Z^2})(1 - \alpha + \delta_Z - \sqrt{2\delta_Z + \delta_Z^2})}{(1+\alpha)(1+\delta_Z - \sqrt{2\delta_Z + \delta_Z^2})}, & \text{if } \alpha < 1. \end{cases} \qquad (4)$$

Moreover,

- *$\delta_E(\delta_Z, \alpha)$ is concave in δ_Z, and it achieves its maximum value (the emission optimal solution) for $\delta_Z = \frac{(1-\sqrt{\alpha})^2}{2\sqrt{\alpha}}$ leading to an emission reduction of $\delta_E = \frac{(1-\sqrt{\alpha})^2}{1+\alpha}$,*
- *for $\alpha > 1$, $\delta_E(\delta_Z, \alpha)$ is increasing concave in α (the same percentage increase in cost leads to a lower decrease in emissions for a greater value of α), and*
- *for $\alpha < 1$, $\delta_E(\delta_Z, \alpha)$ is decreasing convex in α (the same percentage increase in cost leads to a lower decrease in emissions for a lower value of α).*

Proof. Expressing δ_Q as function of δ_Z leads to:

$$\delta_Q = \begin{cases} \delta_Z + \sqrt{2\delta_Z + \delta_Z^2} \ge 0, \\ \delta_Z - \sqrt{2\delta_Z + \delta_Z^2} \le 0. \end{cases}$$

Substituting into the expression of δ_E leads to (4). It is easy to verify that

$$\frac{\partial \delta_E(\delta_Z,\alpha)}{\partial \alpha} = \begin{cases} \dfrac{2\sqrt{\delta_Z(\delta_Z+2)}}{(1+\alpha)^2} \ge 0, & \alpha > 1, \\ -\dfrac{2\sqrt{\delta_Z(\delta_Z+2)}}{(1+\alpha)^2} \le 0, & \alpha < 1, \end{cases} \quad \text{and}$$

$$\frac{\partial^2 \delta_E(\delta_Z,\alpha)}{\partial \alpha^2} = \begin{cases} -\dfrac{4\sqrt{\delta_Z(\delta_Z+2)}}{(1+\alpha)^3} \le 0, & \alpha > 1, \\ \dfrac{4\sqrt{\delta_Z(\delta_Z+2)}}{(1+\alpha)^3} \ge 0, & \alpha < 1. \end{cases}$$

Similarly,

$$\frac{\partial^2 \delta_E(\delta_Z,\alpha)}{\partial \delta_Z^2} = \begin{cases} \dfrac{(1-\alpha)}{(1+\alpha)(\delta_Z(\delta_Z+2))^{3/2}} \le 0, & \alpha > 1, \\ \dfrac{(\alpha-1)}{(1+\alpha)(\delta_Z(\delta_Z+2))^{3/2}} \le 0, & \alpha < 1. \end{cases}$$

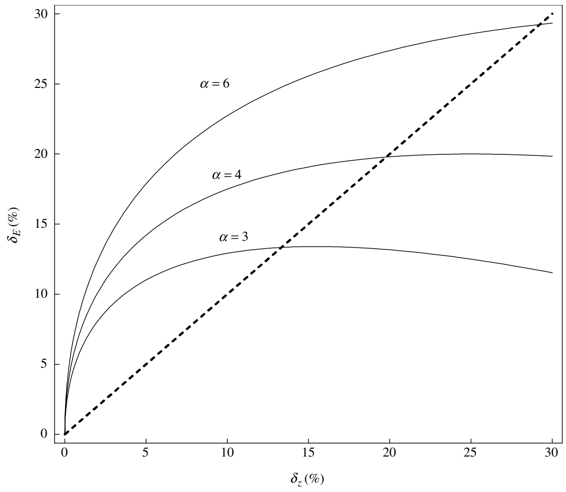


Fig. 4. Cost–emission tradeoff.

Furthermore, solving for $\frac{\partial \delta_E(\delta_Z,\alpha)}{\partial \delta_Z} = 0$ leads to $\delta_Z = \frac{(1-\sqrt{\alpha})^2}{2\sqrt{\alpha}}$ and $\delta_E(\frac{(1-\sqrt{\alpha})^2}{2\sqrt{\alpha}}, \alpha) = \frac{(1-\sqrt{\alpha})^2}{1+\alpha}$, which completes the proof. □

The cost–emission tradeoff is illustrated in Fig. 4, where values of δ_E above the 45 degree lines (shown in dashes) correspond to cases where δ_E is larger than δ_Z. Fig. 4 suggests that there is a value of δ_Z (and corresponding order quantity) which maximizes the difference $\delta_E - \delta_Z$. This value of δ_Z, to which we refer as $\delta_Z^{\max}$, is given by

$$\delta_Z^{\max} = \frac{2(1+\alpha)}{\sqrt{(3+\alpha)(3\alpha+1)}} - 1,$$

achieved for $\delta_Q^{\max} = \sqrt{\frac{3\alpha+1}{3+\alpha}} - 1.$

The associated decrease in emission $\delta_E^{\max}$ and the maximum difference $(\delta_E^{\max} - \delta_Z^{\max})$ are respectively given by

$$\delta_E^{\max} = \frac{1}{1+\alpha}\left(1 - \sqrt{\frac{3+\alpha}{3\alpha+1}}\right)\left(\sqrt{\frac{3\alpha+1}{3+\alpha}} - \alpha\right), \quad \text{and}$$

$$\delta_E^{\max} - \delta_Z^{\max} = \frac{\sqrt{(3+\alpha)(3\alpha+1)}}{2(1+\alpha)} \times \left(1 - \sqrt{\frac{3+\alpha}{3\alpha+1}}\right)\left(\sqrt{\frac{3\alpha+1}{3+\alpha}} - 1\right).$$

These values can be viewed as maximizing the benefit derived from the operational adjustments (the most environmental bang for the cost buck).

Proposition 4. *$\delta_E^{\max} - \delta_Z^{\max}$ is increasing in $\alpha > 1$ and decreasing in $\alpha < 1$ and ranges from* 0 *to* $(2-\sqrt{3}) \approx 26.8\%$.

Proof. It is easy to show that

$$\frac{\partial(\delta_E^{\max} - \delta_Z^{\max})}{\partial \alpha} = \frac{2(\alpha-1)}{(1+\alpha)^2\sqrt{(3+\alpha)(3\alpha+1)}} \times \begin{cases} > 0 & \text{for } \alpha > 1 \\ < 0 & \text{for } \alpha < 1 \end{cases}, \quad \text{and}$$

$$\text{limit}_{\alpha\to 0}(\delta_E^{\max} - \delta_Z^{\max}) = \text{limit}_{\alpha\to\infty}(\delta_E^{\max} - \delta_Z^{\max}) = 2 - \sqrt{3}. \quad \square$$

In the special cases of $\hat{A} = 0$ and $\hat{h} \neq 0$, $\delta_Z^{\max} = \frac{2-\sqrt{3}}{\sqrt{3}} \approx 15.5\%$ and $\delta_E^{\max} = \frac{\sqrt{3}-1}{\sqrt{3}} \approx 42.3\%$ (in other words, we achieve a 42.3% reduction in emissions with only a 15.5% increase in cost). An example of how $\delta_E^{\max} - \delta_Z^{\max}$ changes with α is illustrated in Fig. 5.

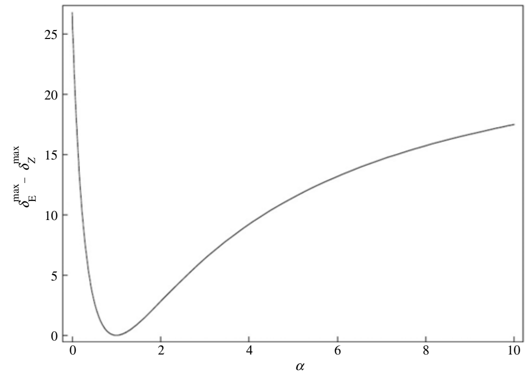


Fig. 5. The impact of α on the maximum difference between δ_E and δ_Z.

We conclude this section by noting that the key results and insights from this section are applicable to settings where either $\hat{A} = 0$ or $\hat{h} = 0$—that is, settings in which emissions are due only to initiating orders or only to keeping inventory. In both cases, it is easy to see that the opportunity to reduce emissions without significantly increasing cost is even greater; for brevity, we omit the details.

3. Extensions to systems with carbon prices

In this section, we briefly describe how the analysis can be extended to settings where emissions are regulated using carbon prices, instead of strict caps, or a combination of caps and prices. In particular, we consider settings with a *carbon tax*, *cap-and-offset*, and *cap-and-price*. In each case, we discuss the extent to which the possibility of an operational adjustment can be leveraged to either reduce emission costs or to generate additional revenue.

3.1. Carbon tax

The pricing of carbon can take on a variety of forms. A simple mechanism is to impose a financial penalty, a tax, per unit of carbon emitted. If we let $t > 0$ denote the penalty per unit of carbon emitted (the tax rate), then the total cost incurred by the firm, given a choice of order quantity Q, can be expressed as follows

$$Z_t(Q) = Z(Q) + tE(Q)$$

where,

$$\begin{cases} Z(Q) = \frac{AD}{Q} + \frac{hQ}{2} + cD \text{ is the direct operational cost associated with order quantity } Q, \\ E(Q) = \frac{\hat{A}D}{Q} + \frac{\hat{h}Q}{2} + \hat{c}D \text{ is the corresponding emission.} \end{cases}$$

To assess the extent to which operational adjustments can mitigate the cost of emissions we compare two strategies available to a firm. The first is for the firm to continue *business as usual* and to choose the order quantity that minimizes its operational cost $Z(Q)$ and then to pay the resulting tax. This means that the firm chooses order quantity $Q^* = \sqrt{\frac{2AD}{h}}$. The second strategy is for the firm to adjust its order quantity so that it minimizes the sum of its operational and emission costs $Z_t(Q)$. In this case, the firm chooses

$$Q_t^* = \sqrt{\frac{2(A + t\hat{A})D}{(h + t\hat{h})}}.$$

Note that $Q_t^* \neq Q^*$ only if $\frac{\hat{A}}{\hat{h}} \neq \frac{A}{h}$. Otherwise, the "operational cost"-optimal solution is also "total cost"-optimal.

Proposition 5. *Let $\delta_{Z_t'} = \frac{Z_t'(Q^*) - Z_t'(Q_t^*)}{Z_t'(Q^*)}$ and $\delta_{E'} = \frac{E'(Q^*) - E'(Q_t^*)}{E'(Q^*)}$ denote respectively the relative reduction in cost and emission due to the* *adjustment in the order quantity, where $Z'_t(Q)$ and $E'(Q)$ refer to the components of cost and emission that are affected by order quantity, then when $\frac{\hat{A}}{\hat{h}} \neq \frac{A}{h}$*

- *Both $\delta_{Z'_t}$ and $\delta_{E'}$ are positive and strictly increasing in t ($\delta_{E'}$ is concave in t), with $\delta_{Z'_t} < \delta_{E'}$ and $\lim_{t\to\infty} \delta_{Z'_t} = \lim_{t\to\infty} \delta_{E'} = \frac{(1-\sqrt{\alpha})^2}{1+\alpha}$, and*
- *Both $\delta_{Z'_t}$ and $\delta_{E'}$ are strictly increasing (decreasing) in $\alpha > 1 (\alpha < 1)$.*

Proof. First note that $\delta_{Z'_t}$ and $\delta_{E'}$ are respectively given by

$$\delta_{Z'_t} = 1 - \frac{\sqrt{(1+ut)(1+vt)}}{1+\frac{t}{2}(u+v)} \quad \text{and}$$

$$\delta_{E'} = 1 - \left(\frac{u}{u+v}\sqrt{\frac{1+vt}{1+ut}} + \frac{v}{u+v}\sqrt{\frac{1+ut}{1+vt}}\right),$$

where $u = \frac{\hat{A}}{A}$, $v = \frac{\hat{h}}{h}$, so that $\alpha = u/v$. Then, we can show that

$$\frac{\partial \delta_{Z'_t}}{\partial t} = \frac{(u-v)^2 t}{\sqrt{(1+ut)(1+vt)}(2+ut+vt)^2} > 0 \quad \text{and}$$

$$\begin{cases} \dfrac{\partial \delta_{E'}}{\partial t} = \dfrac{(u-v)^2}{2(u+v)((1+ut)(1+vt))^{3/2}} > 0 \\ \dfrac{\partial^2 \delta_{E'}}{\partial t^2} = -\dfrac{(u-v)^2(u+v+2uvt)}{4(u+v)((1+ut)(1+vt))^{5/2}} < 0. \end{cases}$$

Moreover, $Z'_t(Q^*)^2 - Z'_t(Q^*_t)^2 = \frac{AhD}{2}(u-v)^2t^2 > 0$, $E'(Q^*)^2 - E'(Q^*_t)^2 = \frac{AhD}{2}(u-v)^2\frac{(u+v+uvt)t}{(1+ut)(1+vt)} > 0$, and $\delta_{E'} - \delta_{Z'_t} = \frac{(a-b)^2 t}{(a+b)\sqrt{(1+ut)(1+vt)}(2+t(u+v))} > 0$, therefore, $0 < \delta_{Z'_t} < \delta_{E'}$. It is easy to verify that $\lim_{t\to\infty} \delta_{Z'_t} = \lim_{t\to\infty} \delta_{E'} = \frac{(1-\sqrt{\alpha})^2}{1+\alpha}$. Finally, we can show that

$$\begin{cases} \dfrac{\partial \delta_{E'}}{\partial u} = \dfrac{(u-v)}{2(u+v)^2}\dfrac{t(1+vt)(u+3v+2uvt)}{((1+ut)(1+vt))^{3/2}} > 0\ (<0) \\ \quad \text{for } u > v (u < v), \\ \dfrac{\partial \delta_{Z'_t}}{\partial u} = \dfrac{(u-v)t^2}{(2+t(u+v))^2}\sqrt{\dfrac{1+vt}{1+ut}} > 0\ (<0) \\ \quad \text{for } u > v (u < v), \end{cases}$$

and

$$\begin{cases} \dfrac{\partial \delta_{E'}}{\partial v} = \dfrac{(v-u)}{2(u+v)^2}\dfrac{t(1+ut)(v+3u+2uvt)}{((1+ut)(1+vt))^{3/2}} > 0\ (<0) \\ \quad \text{for } u < v (u > v), \\ \dfrac{\partial \delta_{Z'_t}}{\partial v} = \dfrac{(v-u)t^2}{(2+t(u+v))^2}\sqrt{\dfrac{1+vt}{1+ut}} > 0\ (<0) \\ \quad \text{for } u < v (u > v). \end{cases}$$

Therefore, $\delta_{Z'}$ and $\delta_{E'}$ are strictly increasing (decreasing) in $\alpha > 1 (\alpha < 1)$. □

The results of Proposition 5 show that operational adjustment can indeed lower a firm's emission cost (its tax burden) and for this reduction to be sufficient to offset the associated increase in its operational cost. The relative benefit from this adjustment (both in terms of cost and emission) increases with the tax rate, as a higher tax rate justifies moving further away from the "operational cost"-optimal order quantity. The fact that the emission reduction is concave in the tax rate implies that there is a diminishing effect to increasing the tax rate; see Fig. 6. It also implies that a modest tax rate can lead to a significant reduction in emissions. This would be the case for example when α is large (small) for $\alpha > 1 (\alpha < 1)$, or equivalently when the absolute difference between the ratios $\frac{A}{h}$ and $\frac{\hat{A}}{\hat{h}}$ is large.

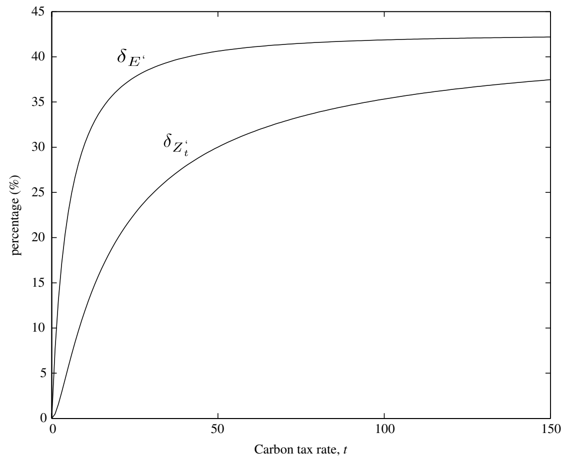


Fig. 6. The benefit of operational adjustment under carbon tax.

3.2. Cap-and-offset

An alternative to taxing all emissions is to tax only emissions that exceed a certain threshold. This can arise in settings where the regulatory agency sets an emission cap on the firm and imposes penalties if the cap is exceeded. It can also arise in settings where the regulatory agency sets an emission cap but allows the firm to *relax* its cap through the purchase of *emission offsets* through third parties (see [1] for examples and references). Note that systems with a strict cap and a tax on all emissions can be viewed as special cases of cap-and-offset, where in the first case the penalty for exceeding the cap is infinitely large and in the second the cap is set at zero.

Under a cap-and-offset system, the firm has again the option of either operating business-as-usual by choosing an order quantity that minimizes the sum of its fixed ordering and inventory holding costs (its direct operational costs) and then to pay any resulting penalties if it exceeds its cap, or of adjusting its order quantity by choosing one that minimizes the sum of its operational and emission costs. In the latter, the problem the firm faces can be formulated as follows:

$$\text{Minimize } Z_o(Q) = \frac{AD}{Q} + \frac{hQ}{2} + cD + t\left(\frac{\hat{A}D}{Q} + \frac{\hat{h}Q}{2} - \hat{C}\right)^+, \quad (5)$$

where $\hat{C} = C - \hat{c}D$, t is the penalty paid per unit emitted in excess of the cap, and $X^+ = \max(0, X)$.

Theorem 2. *The optimal order quantity that minimizes (5) is given by*

$$Q^*_o = \begin{cases} Q^*_t = \sqrt{\dfrac{2(A+t\hat{A})D}{h+t\hat{h}}}, & \hat{C} < \hat{C}_1, \\ Q_1 = \dfrac{\hat{C} - \sqrt{\hat{C}^2 - 2\hat{A}\hat{h}D}}{\hat{h}}, & \hat{C}_1 < \hat{C} < \hat{C}_2 \text{ and } \frac{A}{h} < \frac{\hat{A}}{\hat{h}}, \\ Q_2 = \dfrac{\hat{C} + \sqrt{\hat{C}^2 - 2\hat{A}\hat{h}D}}{\hat{h}}, & \hat{C}_1 < \hat{C} < \hat{C}_2 \text{ and } \frac{A}{h} > \frac{\hat{A}}{\hat{h}}, \\ Q^* = \sqrt{\dfrac{2AD}{h}}, & \hat{C} > \hat{C}_2, \end{cases}$$

where $\hat{C}_1 = \sqrt{\frac{D}{2}}(\hat{A}\sqrt{\frac{h+t\hat{h}}{A+t\hat{A}}} + \hat{h}\sqrt{\frac{A+t\hat{A}}{h+t\hat{h}}})$, *and* $\hat{C}_2 = \sqrt{\frac{D}{2}}(\hat{A}\sqrt{\frac{h}{A}} + \hat{h}\sqrt{\frac{A}{h}})$.

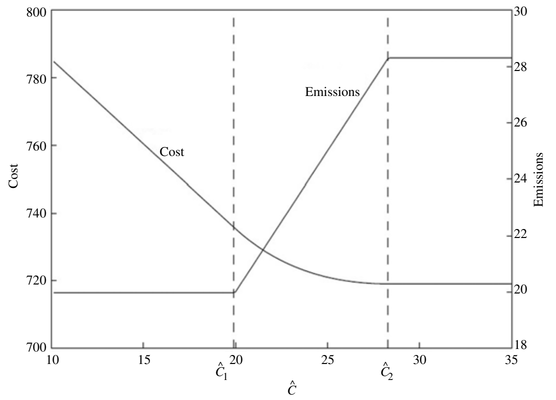


Fig. 7. The effect of the carbon cap under cap-and-offset ($D = 100$, $A = 120$, $h = 2$, $c = 5$, $\hat{A} = 1$, $\hat{h} = 0.5$, $\hat{c} = 0$, $t = 5$).

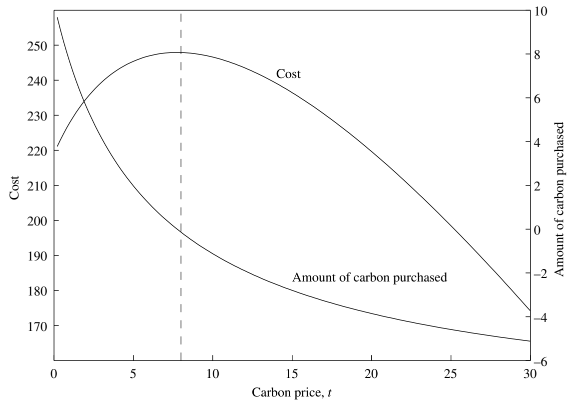


Fig. 8. The impact of carbon price on cost and carbon emissions purchased when $E_{min} \le C \le E_{max}$ ($D = 100$, $A = 120$, $h = 2$, $\hat{A} = 1$, $\hat{h} = 0.5$, $c = 0$, $\hat{c} = 0$).

Proof. $\hat{C}_1$ and $\hat{C}_2$ correspond to $E(Q_t) - \hat{c}D$, $E(Q^*) - \hat{c}D$ respectively. When $\hat{C} \ge \hat{C}_2$, we know that for any Q, $Z_o(Q^*) \le Z_o(Q)$, therefore $Q_o^* = Q^*$. When $\hat{C}_1 \le \hat{C} \le \hat{C}_2$ and $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$, we know that for any Q satisfying $E(Q) \ge C$, $Q \le Q_1 \le Q_t$. Therefore, $Z_o(Q_1) \le Z_o(Q)$. On the other hand, for any Q satisfying $E(Q) < C$, we know that $Q^* \le Q_1 \le Q$. Therefore, $Z_o(Q_1) \le Z_o(Q)$. Therefore, when $\hat{C}_1 \le C \le \hat{C}_2$, we have $Q_o^* = Q_1$. The case of $\hat{C}_1 \le C \le \hat{C}_2$ and $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$ is similar. When $\hat{C} \le \hat{C}_1$ and $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$, we know that for any Q satisfying $E(Q) \ge C$, $Z_o(Q_t^*) \le Z_o(Q)$. On the other hand, for any Q satisfying $E(Q) < C$, we know that $Q > Q_1 > Q_t^*$, because $\hat{C} < \hat{C}_1$, $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$. Therefore, $Z_o(Q) > Z_o(Q_1) > Z_o(Q_t^*)$. Thus, when $\hat{C} < \hat{C}_1$ and $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$, we have $Q_o^* = Q_t^*$. The case of $\hat{C} < \hat{C}_1$ and $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$ is similar. □

Theorem 2 indicates that, depending on the cap, there are three regions of operation. If the cap is sufficiently large (i.e., $\hat{C} \ge \hat{C}_2$), then the firm operates business-as-usual and does not make any adjustments in its order quantity. If $\hat{C}_1 \le \hat{C} \le \hat{C}_2$, the firm makes adjustments in its order quantity so that its emissions do not exceed the cap; the firm in this case does not incur any emission penalties. Note that here the firm operates as if it were subject to a strict emission cap and therefore all the results of Section 2 apply, including the opportunity to significantly reduce emissions without significantly increasing cost. Note also that in this region the firm finds it more preferable to incur the increase in its direct operational costs than to incur emission penalties. On the other hand, if $\hat{C} \le \hat{C}_1$, then the firm finds it more preferable to pay the emission penalty than to reduce its emissions below $C_1 = \hat{C}_1 + \hat{c}D$. In this region, the firm emits exactly C_1 and pays the penalty $t(C - C_1)$ associated with the difference. The three regions are illustrated for an example system in Fig. 7. Note that for the strategy of not adjusting order quantity, cost is linear in the cap for $\hat{C} \le \hat{C}_2$. Hence, the difference in cost between adjusting and not adjusting order quantity can be significant in the region where $\hat{C}_1 \le \hat{C} \le \hat{C}_2$. It can also be significant when $\hat{C} \le \hat{C}_1$, where the difference is constant and equal to $t(C_2 - C_1)$.

We conclude this section by highlighting the effect of the emission penalty t. In particular, an increase in t has the effect of decreasing the value of $\hat{C}_1$ and, therefore, increasing the width of the interval $[\hat{C}_1, \hat{C}_2]$. This means that a larger t increases the region in which incurring higher operational costs is preferable to incurring emission penalties.

3.3. Cap-and-price

Instead of only penalizing emissions that exceed the specified cap, as in cap-and-offset, the regulating agency may choose to also encourage emissions that are lower than the cap by rewarding firms that emit less than their cap. Rewarding lower emissions can take on a variety of forms. At its simplest, it consists of a reward (penalty) t' (t) per unit of emission below (above) the cap C. We refer to this scheme as cap-and-price since there is now monetary value for emissions regardless of whether or not the cap is exceeded (this is similar to the so-called *cap-and-trade* system, except that in that case prices are established through a market mechanism for carbon trading). All the previous schemes can be viewed as special cases of cap-and-price (for example cap-and-offset corresponds to the case where $t' = 0$).

It is easy to show that when $t' = t$ the problem is similar to one with a carbon tax, with a similar expression for total cost except for an additional revenue term $-tC$ (this extra term is due to the fact that cap-and-price is equivalent to a scheme where firms receive a reward equal to tC and are then taxed for each unit of emission with rate t). Therefore, many of the results obtained for the case of carbon tax continue to apply. However, in contrast to a system with a carbon tax, the effect of t on cost is not monotonic. In particular, if the cap C is sufficiently large (larger than the emission that would be incurred if the firm maintained business as usual), then cost would be strictly decreasing in t, eventually becoming negative with the firm realizing a net profit. On the other hand, if the cap is sufficiently small (smaller than the emission that would be incurred if the firm chose the emission-optimal order quantity), then cost would be strictly increasing in t since the amount of emissions would always exceed the cap. If the cap falls between these two thresholds, then cost is not monotonic in t; it initially increases and then decreases. In particular, when t is small, the firm chooses to exceed the emission cap and, therefore, incurs the corresponding emission penalties. However, when t is large, the firm chooses to emit less than the cap and realizes the corresponding revenue. This effect is illustrated in Fig. 8. Note that the benefit from adjusting the order quantity to take into account of carbon price can be significant, in terms of both cost and emission, when carbon price is high and the firm can emit less than its cap.

4. Conclusion

In the unabridged version of the paper [3], we illustrate the broader applicability of our key results to two other important classes of operational models, namely facility location models, where costs and emissions are driven by a fixed component and a distance-dependent component; and newsvendor models, where costs and emissions are driven by inventory overages and inventory shortages. In both cases, we show that the cost function is remarkably flat around the optimal, providing an opportunity under some conditions to significantly reduce emissions without significantly increasing cost.

In this paper, we have assumed that the parameters of the emission regulation are exogenous. It is however possible to take the perspective of a social planner who decides on the parameters of the regulation to maximize social welfare—social welfare is the difference between the sum of the producer surplus, consumer surplus and tax revenue on one hand and the environmental damage on the other. In the unabridged version of the paper [3], we show that for the setting described in this paper, a strict emission cap and an emission tax, when designed optimally, both achieve the same level of social welfare, including the same level of emissions. However, a strict emission cap is preferred by the producer since it does not involve any tax payments.

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