“Only death and tax are certain in life.”

In this part of the lecture, we will elaborate on how tax considerations can alter the economic decisions.

In order to understand how taxes are actually computed, one has to understand a concept called *depreciation*.

Governments use depreciation as an incentive for innovation.

Depreciation of physical properties plays a key role in engineering economic decision making.
So, what is depreciation?

It is a notion which tries to capture the idea that a property will lose its value over time.

In its heart, depreciation has a very mechanical role to play in accounting.

First of all, we note that not all properties are depreciable, e.g. land, or investment goods such as gold.

Depreciable property: property that has a determinable life, loses its value in the cause of its life.
Summarizing, a depreciable asset/property:

- must be used in business, or produce income
- must have a determinable useful life
- must lose its value in the course of its useful life
- must not be inventory, or investment
Examples of depreciable assets/properties: machines, cars, office facilities, houses, etc.

The following concepts are useful.

- tangible property
  - personal property
  - real property

- intangible property: copyright or patent

We will only discuss tangible properties in this course.
Consider an asset or a property.

Some terminology:

- **Cost basis:** cost to make the asset available.
- **Adjusted cost basis:** improvements added.
- **Annual depreciation deduction:** value decreased in a year.
- **Book value:** at the end of year $k$

$$
(book \text{ value})_k = \text{adjusted cost basis} - \sum_{j=1}^{k} (\text{depreciation deduction})_j
$$
- Market value.
- Salvage value.
- Recovery period: the number of years over which the basis of an asset is recovered.
- Recovery rate: the percentage of depreciation deduction over the cost basis in each year.
- Useful life (or depreciable life).
Traditional Depreciation Methods.

(1). Straight-Line Method.

\[ d_k = \frac{(B - SV_N)}{N} \]
\[ BV_k = B - kd_k \]

Example: A new electric saw has a cost basis of $4,000 and a 10-year depreciable life. The estimated salvage value is zero. Determine the annual depreciation amount and book value.

\[ d_k = \frac{4,000 - 0}{10} = 400 \]

and

\[ BV_k = 4,000 - 400 \times k \]

for \( k = 1, \ldots, 10 \).
(2). Declining Balance Method (constant percentage method or the Matheson formula).

\[
\begin{align*}
    d_1 &= BR \\
    d_k &= B(1 - R)^{k-1}R \\
    d_k^* &= B[1 - (1 - R)^k] \\
    BV_k &= B(1 - R)^k \\
    BV_N &= B(1 - R)^N
\end{align*}
\]

Example: The previous example with 200% declining reduction \((R = 2/N)\). In that case \(R = 2/10 = 0.2\),

\[
d_6 = $4,000 \times (1 - 0.2)^5 \times 0.2 = $262.14
\]

and

\[
d_k^* = $4,000 \times (1 - (1 - 0.2)^6) = $2,951.42.
\]
(3). Sum-of-the-years-digits Method (SYD).

\[ d_k = (B - SV_N) \frac{2(N - k + 1)}{N(N + 1)} \]

\[ BV_k = B - \left[ \frac{2(B - SV_N)}{N} \right] k + \left[ \frac{B - SV_N}{N(N + 1)} \right] k(k + 1) \]

*Example:* Use the same example as before. Then

\[ d_4 = \$4,000 \frac{2(10 - 4 + 1)}{10 \times 11} = \$509.09 \]

and

\[ BV_4 = \$4,000 - \frac{2(\$4,000)}{10 \times 11} \times 4 + \frac{\$4,000}{10 \times 11} \times 4 \times 5 \]

\[ = \$1,527.27. \]

Depreciation per unit of production

\[
= \frac{B - SV_N}{\text{Total units in life time}}
\]

Example: A piece of equipment in a business has a basis of $50,000, and is expected to have a $10,000 salvage value when replaced after 30,000 hours of use. Find its book value after 10,000 hours of use.

Depreciation per unit is

\[
\frac{50,000 - 10,000}{30,000} = 1.33 \text{ per hour}
\]

After 10,000 hours of use

\[
BV = 50,000 - 1.33 \times 10,000 = 36,700.
\]
Switch over from DB to SL.

In the beginning of each year, calculate both the DB depreciation reduction for that year and the SL depreciation reduction, and then take the larger one.

Example:
<table>
<thead>
<tr>
<th>200% DB</th>
<th>SL</th>
<th>True Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>800.00</td>
<td>400.00</td>
<td>800</td>
</tr>
<tr>
<td>640.00</td>
<td>355.56</td>
<td>640.00</td>
</tr>
<tr>
<td>512.00</td>
<td>320.00</td>
<td>512.00</td>
</tr>
<tr>
<td>409.60</td>
<td>292.57</td>
<td>409.60</td>
</tr>
<tr>
<td>327.68</td>
<td>273.07</td>
<td>327.68</td>
</tr>
<tr>
<td>262.14</td>
<td>262.14</td>
<td>262.14</td>
</tr>
<tr>
<td>209.72</td>
<td>262.14</td>
<td>262.14</td>
</tr>
<tr>
<td>167.77</td>
<td>262.14</td>
<td>262.14</td>
</tr>
<tr>
<td>134.22</td>
<td>262.14</td>
<td>262.14</td>
</tr>
<tr>
<td>107.37</td>
<td>262.14</td>
<td>262.14</td>
</tr>
</tbody>
</table>
The Modified Accelerated Cost Recovery System (MACRS)

*Two subsystems*: GDS and ADS.

**ADS**: Straight-line method based on the recovery period.

**GDS**: 
- 3, 5, 7 and 10 recovery years: 200% DB (switching to SL).
- 15 and 20 recovery years: 150% DB (switching to SL).
- Non-residential real and residential rental property: SL with the GDS recovery period.

Half year convention always applies.
Income Taxes.

Taxable income

\[\text{Taxable income} = \text{gross income} - \text{all expenses} - \text{depreciation deduction}\]

Terminology:

- Net income before tax (NIBT)
- Net income after tax (NIAT)

\[\text{NIAT} = \text{NIBT} - \text{income taxes}\]
**Corporate Federal Income Tax Rates.**

Tax rate table, 2006:

<table>
<thead>
<tr>
<th>Interval</th>
<th>Tax Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 50K]</td>
<td>15%</td>
</tr>
<tr>
<td>(50K, 75K]</td>
<td>25%</td>
</tr>
<tr>
<td>(75K, 100K]</td>
<td>34%</td>
</tr>
<tr>
<td>(100K, 335K]</td>
<td>39%</td>
</tr>
<tr>
<td>(335K, 10M]</td>
<td>34%</td>
</tr>
<tr>
<td>(10M, 15M]</td>
<td>35%</td>
</tr>
<tr>
<td>(15M, 18.3M]</td>
<td>38%</td>
</tr>
<tr>
<td>(18.3M, ...]</td>
<td>35%</td>
</tr>
</tbody>
</table>
Let

\[ R_k = \text{revenues in year } k \]
\[ E_k = \text{expenses in year } k \]
\[ d_k = \text{deduction in year } k \]
\[ t_k = \text{tax rate paid in year } k \]
\[ T_k = \text{income tax paid in year } k \]

Then,

\[ T_k = -t_k(R_k - E_k - d_k) \]

and

\[ NIAT_k = R_k - E_k - d_k - t_k(R_k - E_k - d_k) \]
\[ = (1 - t_k)(R_k - E_k - d_k) \]
We now simplify the analysis by assuming that the tax rate is flat. In order to calculate the ‘real’ cash flow, we need to take the income tax into account.

Remember that the depreciation deduction is not really a cash flow. So, the ATCF (after tax cash flow) in year $k$ is

$$ATCF_k = NIAT_k + d_k$$

$$= (1 - t)(R_k - E_k) + td_k$$

At the same time, obviously, the BTCF (before tax cash flow) in year $k$ is

$$BTCF_k = R_k - E_k$$

The difference between the BTCF and ATCF is the tax:

$$T_k = ATCF_k - BTCF_k$$
Example: If the revenue from a project is $10,000 during a tax year, expenses are $4,000, and depreciation deductions for income tax purposes are $2,000, what is the ATCF when \( t = 0.4 \)?

We have

\[
ATCF = (1 - 0.4)(10,000 - 4,000 - 2,000) + 2,000
\]
\[
= 4,400.
\]

On the other hand,

\[
NIAT = 4,400 - 2,000 = 2,400.
\]
Example: Suppose that an asset has a cost basis of $100,000, and an ADS recovery period of 5 years applies. The MACRS depreciation is shown as follows:

<table>
<thead>
<tr>
<th>Year</th>
<th>Depreciation Deduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$10,000</td>
</tr>
<tr>
<td>2</td>
<td>$20,000</td>
</tr>
<tr>
<td>3</td>
<td>$20,000</td>
</tr>
<tr>
<td>4</td>
<td>$20,000</td>
</tr>
<tr>
<td>5</td>
<td>$20,000</td>
</tr>
<tr>
<td>6</td>
<td>$10,000</td>
</tr>
</tbody>
</table>
If the tax rate is 40%, what is the PW of after tax savings from the depreciation? Assume that MARR=10%.

\[ PW(10\%) = 40\% \times \sum_{k=1}^{6} \frac{d_k}{1.1^k} = $28,948. \]
Since

\[ ATCF \approx (1 - t)BTCF \]

therefore

\[ MARR_{AT} \approx (1 - t)MARR_{BT} \]

The formula is exact if \( d_k = 0 \).

Example: Suppose that the bank deposit rate is 8% (the MARR before taxes). Suppose that the income tax rate for Mr. Lee is 15%. Then, Mr. Lee’s MARR after taxes is

\[ (1 - 15\%)8\% = 6.8\%. \]
Example: Suppose that a company wants to install a new machine. The useful life is 10 years. The cost is $180,000 and the machine can reduce the operating cost of the company up to $36,000 annually. It is estimated that the machine has a MV $30,000 at the end of the 10th year. One wonders whether this machine is worthwhile if its after-tax MARR is 10%. 
*Example:* An engineering consulting firm can purchase a fully configured Computer-Aided Design (CAD) workstation for $20,000. It is estimated that the useful life of the workstation is seven years, and its MV in seven years should be $2,000. Operating expenses are estimated to be $40 per eight-hour workday, and maintenance will be performed under contract for $8,000 per year. The MACRS (GDS) property class is five years, and the effective income tax rate is 40%.

As an alternative, sufficient computer time can be leased from a service company at an annual cost of $20,000. If after-tax MARR is 10%, how many workdays per year must the workstation be needed in order to justify leasing it?